Modeling stellar pulsations (mostly correctly)

László Molnár

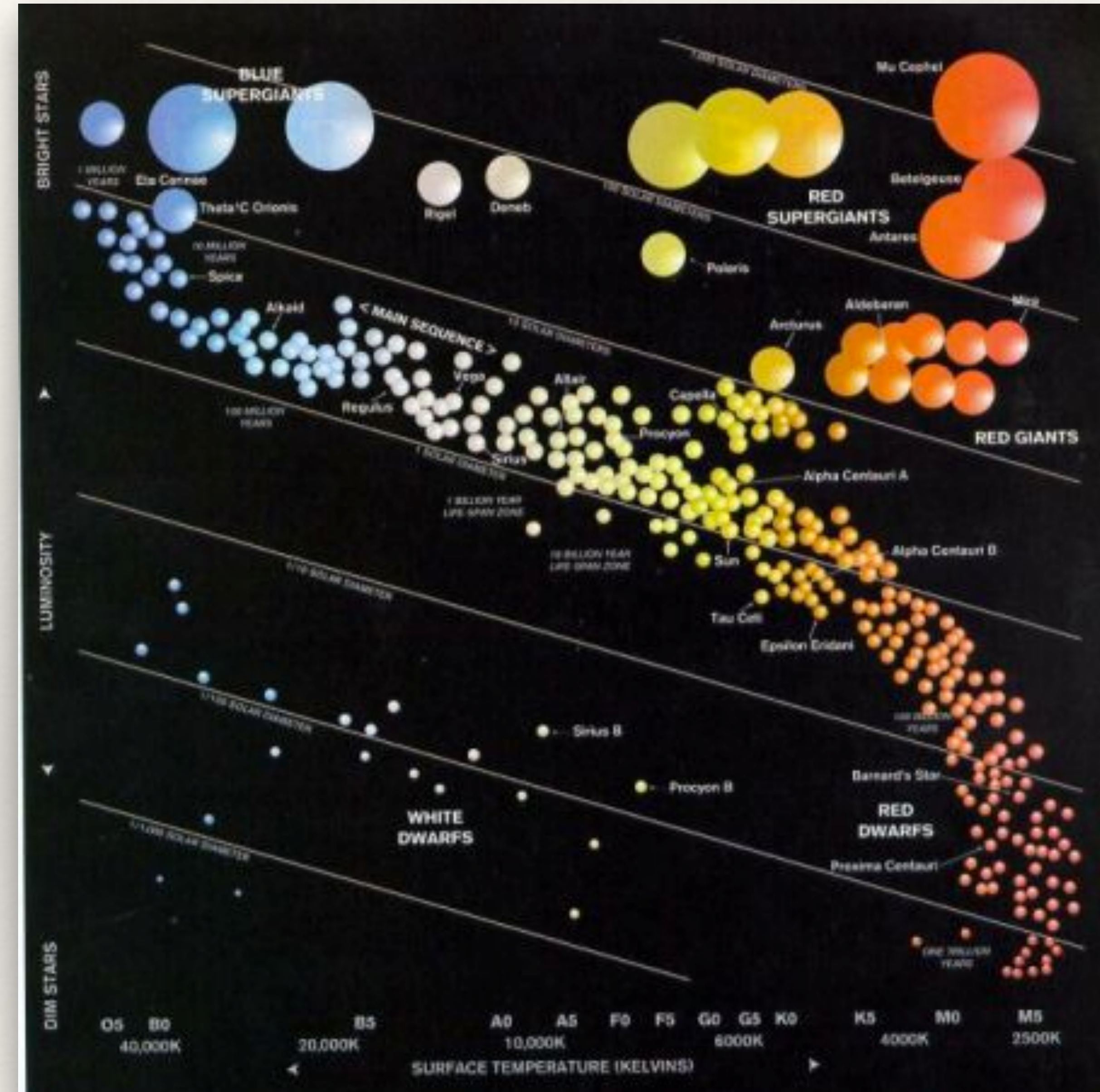
Konkoly Observatory,
MTA CSFK, Budapest

Motivations

- Stellar pulsations: related to stellar structure
- Goal: reproduce periods and amplitudes
- Need accurate physical models!
  - Models based on actual physics
  - Physics based on observed stellar properties

The beginnings

- Recognition of stellar pulsations
  - Cepheids could not be binaries
- Eddington: linear perturbation theory (1914-)
  - very simple stellar model: equilibrium, symmetry, etc
  - Limit cycle can be reached
  - period ~ avg. density -> stellar structure!


BLUE SUPERGIANTS
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RED GIANTS
Mu Cephei
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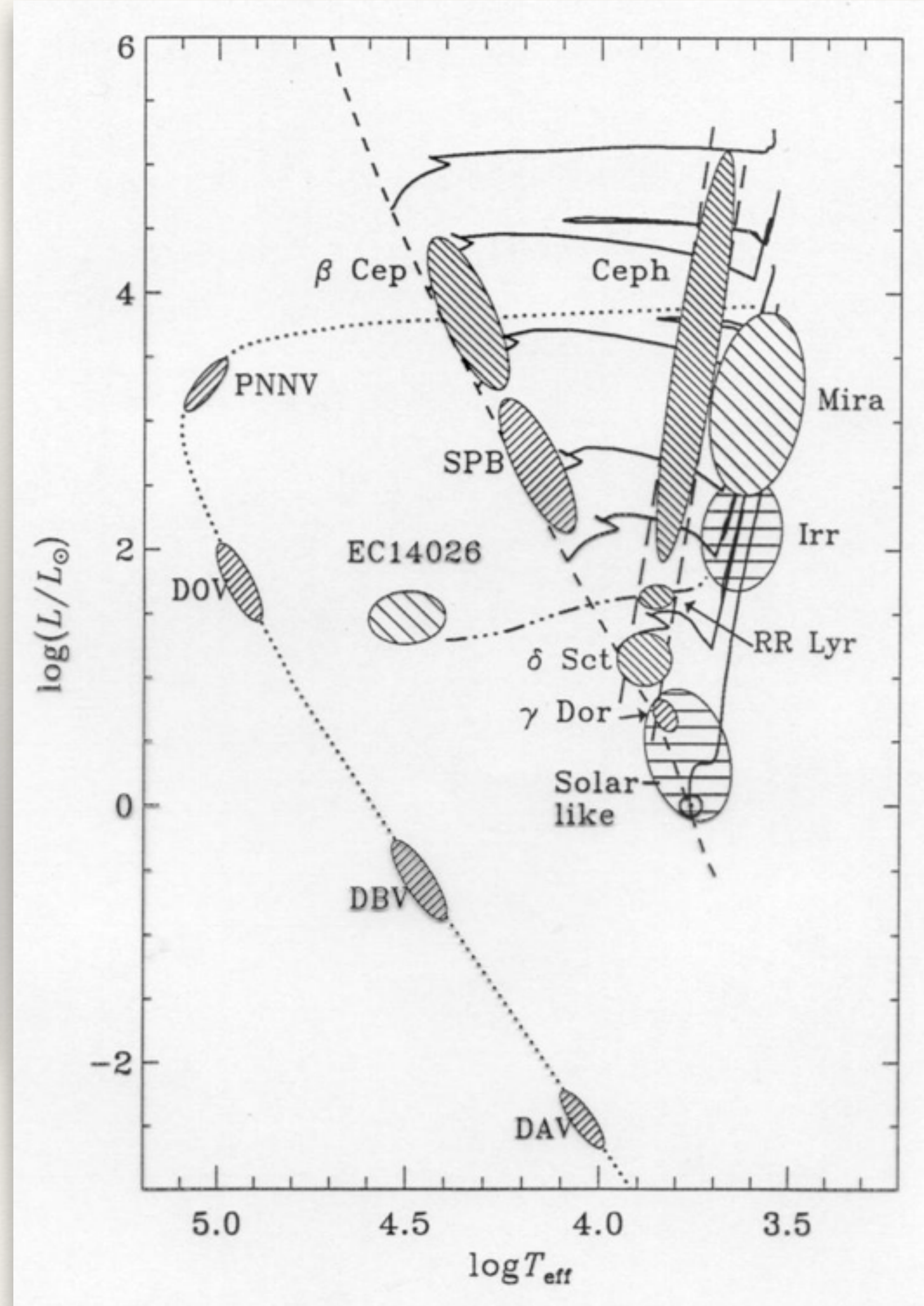
Theta'C Orionis
Spica
Alkaid
< MAIN SEQUENCE >
Arcturus
Aldebaran
Mira
Regulus
Vega
Altair
Capella
Procyon
Sirius
Alpha Centauri A
Sun
Alpha Centauri B
Tau Ceti
Epsilon Eridani
Sirius B
Procyon B
Barnard's Star
Proxima Centauri
WHITE DWARFS
RED DWARFS
BRIGHT STARS
LUMINOSITY
DIM STARS
O5 B0 B5 A0 A5 F0 F5 G0 G5 K0 K5 M0 M5
40,000K 20,000K 10,000K 6000K 4000K 2500K
SURFACE TEMPERATURE (KELVINS)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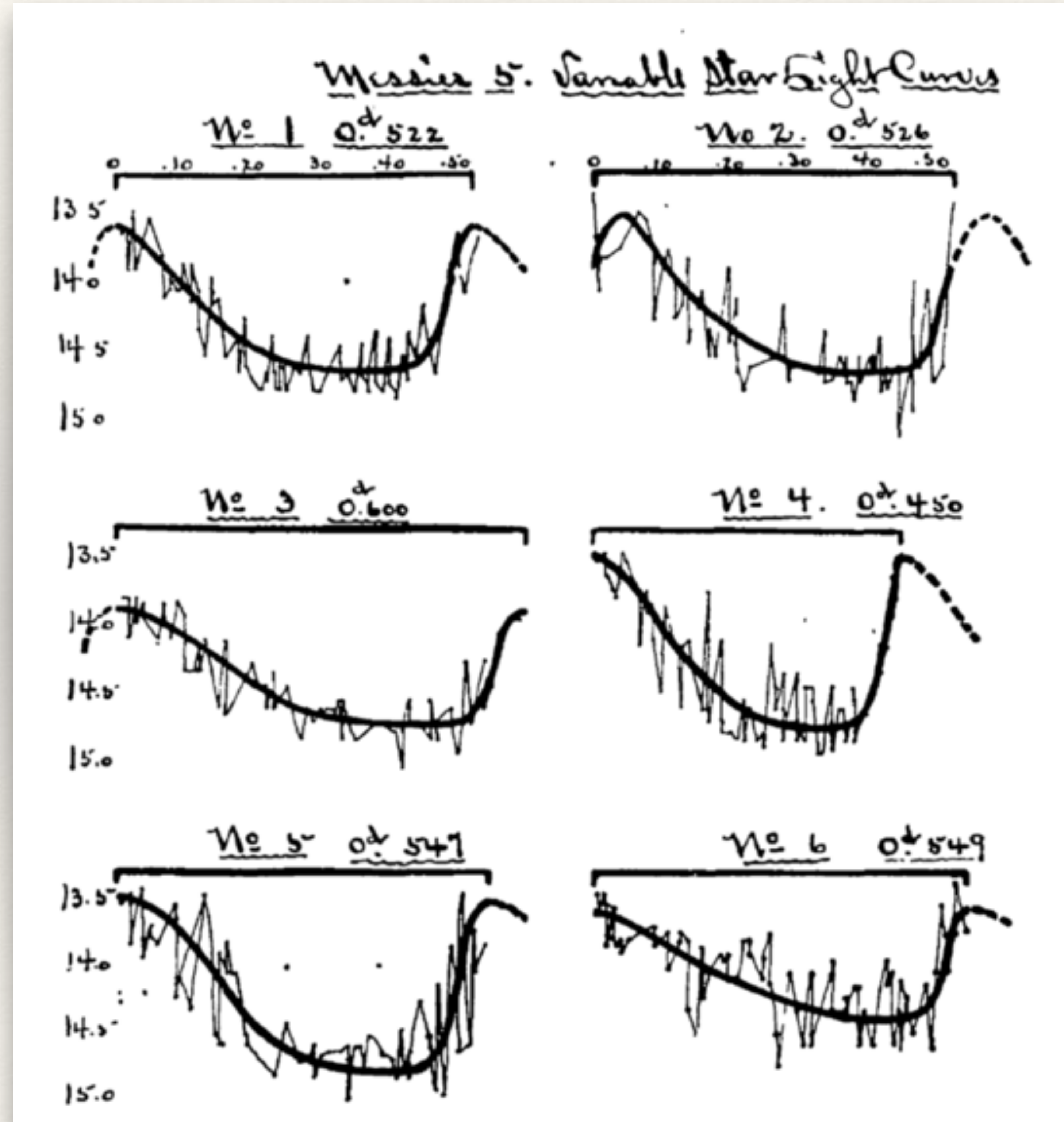
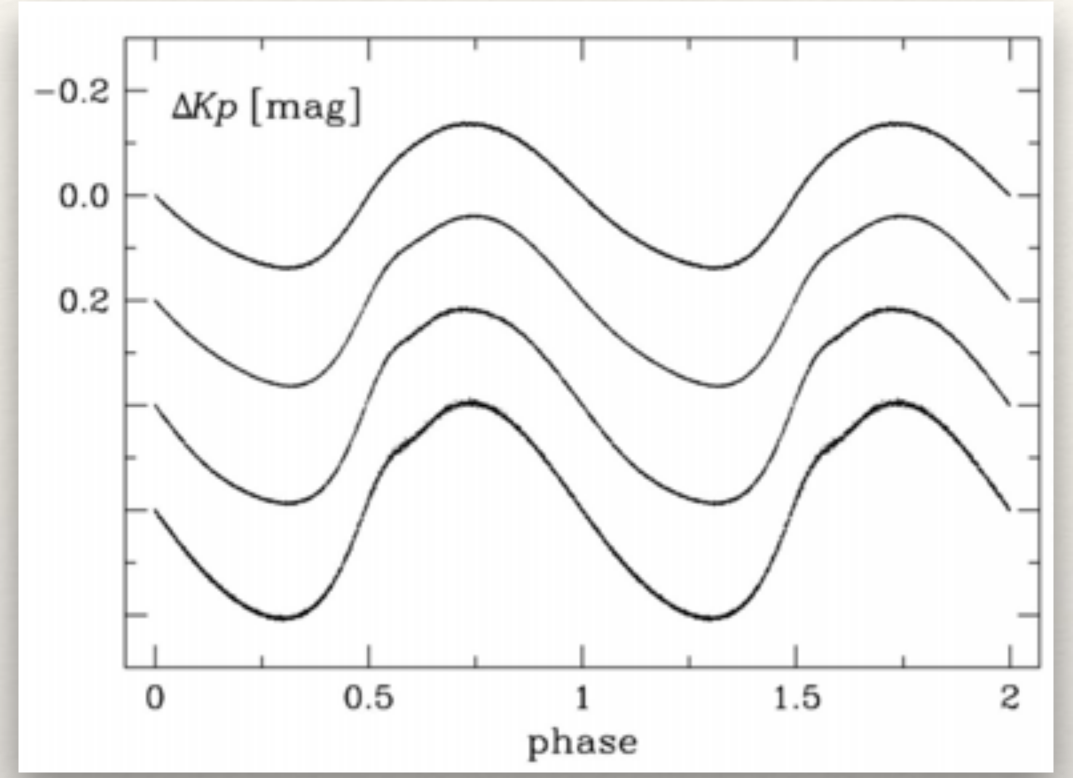
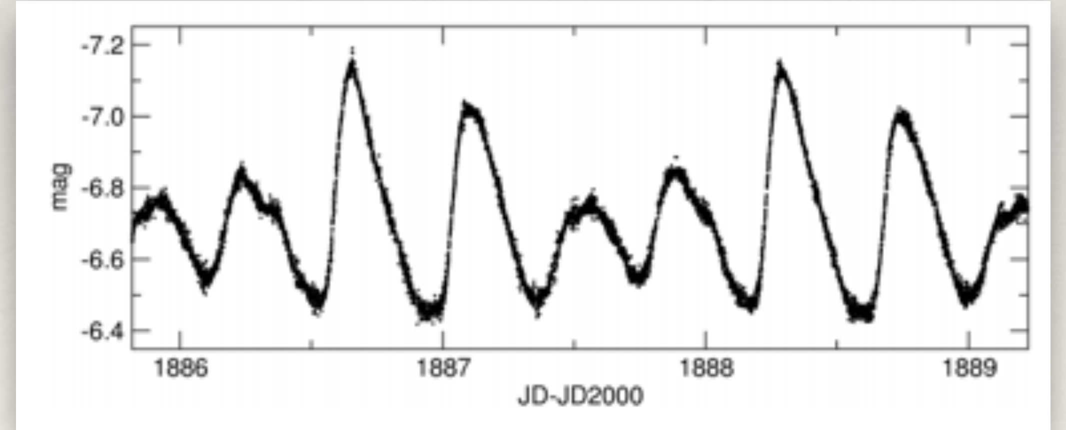
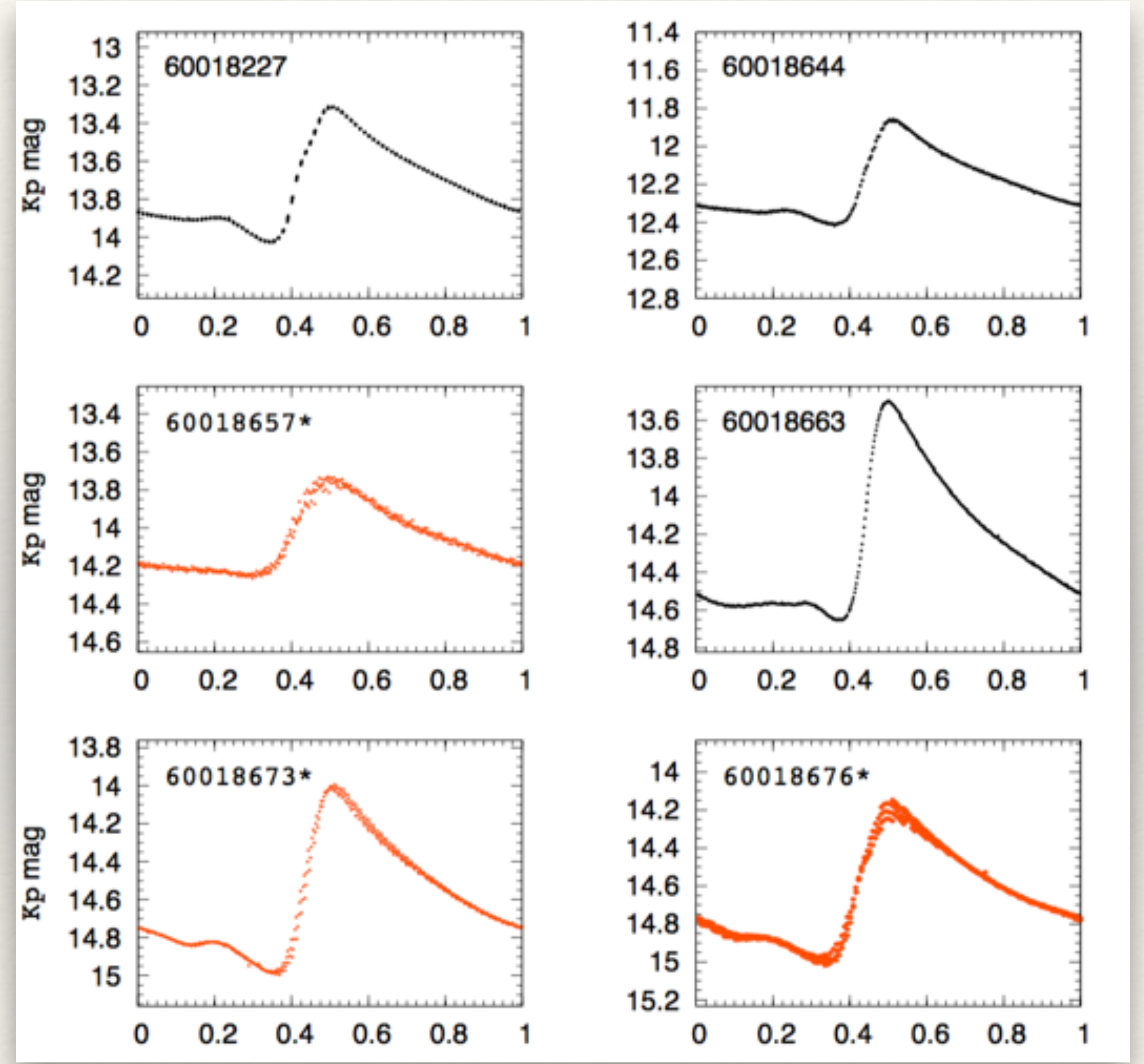
RR Lyrae stars



RR Lyrae stars

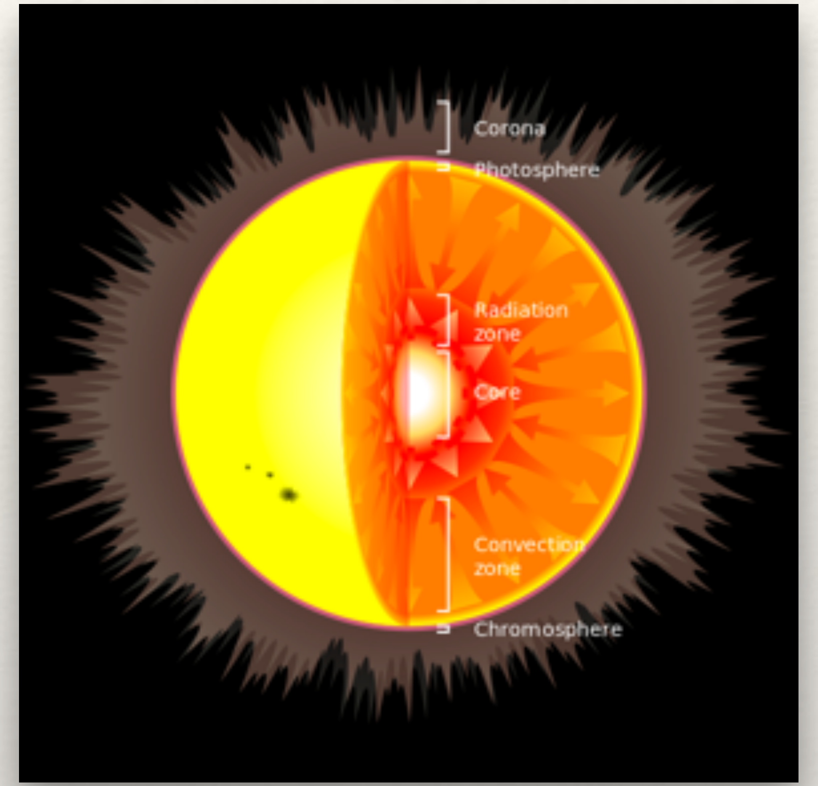






The beginnings

- What drives the pulsation?
- Composition and energy production still poorly understood!
- Two important advancements:
  - Pulsation restricted to the envelope
  - Internal composition: H, He partial ionization layers
    - Zhevakin (1953), Cox (1958)



Early radiative models

- 1D, non-adiabatic, non-linear models, no convection
- Basic structure
  - Heat source at the bottom
  - Envelope divided into movable zones (one or more)
  - Some atmosphere on top (grey)
- Did well for single-mode models
  - Cepheids, RR Lyrae stars, giants…

Pressing problems

- Cepheid mass problem
  - Evolutionary and pulsation masses differ
- Double-mode pulsations
  - Only in resonant cases - far more exist in nature
- Red edge of the instability strip(s)
- Need for artificial viscosity to temper shockwaves
  - Reduces the mode amplitudes

Turbulent convective models

- Convection is important for the pulsation!
  - Essentially 3D - need a 1D approximation
- Mixing-length theory (Böhm-Vitense)
  - Modified for stellar models
    (e.g. time dependence)
  - New term: eddy viscosity

Turbulent convective models

- Three state-of-the-art 1D models: Warsaw, Vienna and Florida-Budapest models
- Similar in several characteristics
- Lagrangian mesh, quantities determined for zone or interface (mass, radius, fluxes, temperature, etc)
- Seven free parameters for convection incl. mixing length
    - No theoretical guidance!!!
- Not included: magnetic fields, rotation, stellar evolution

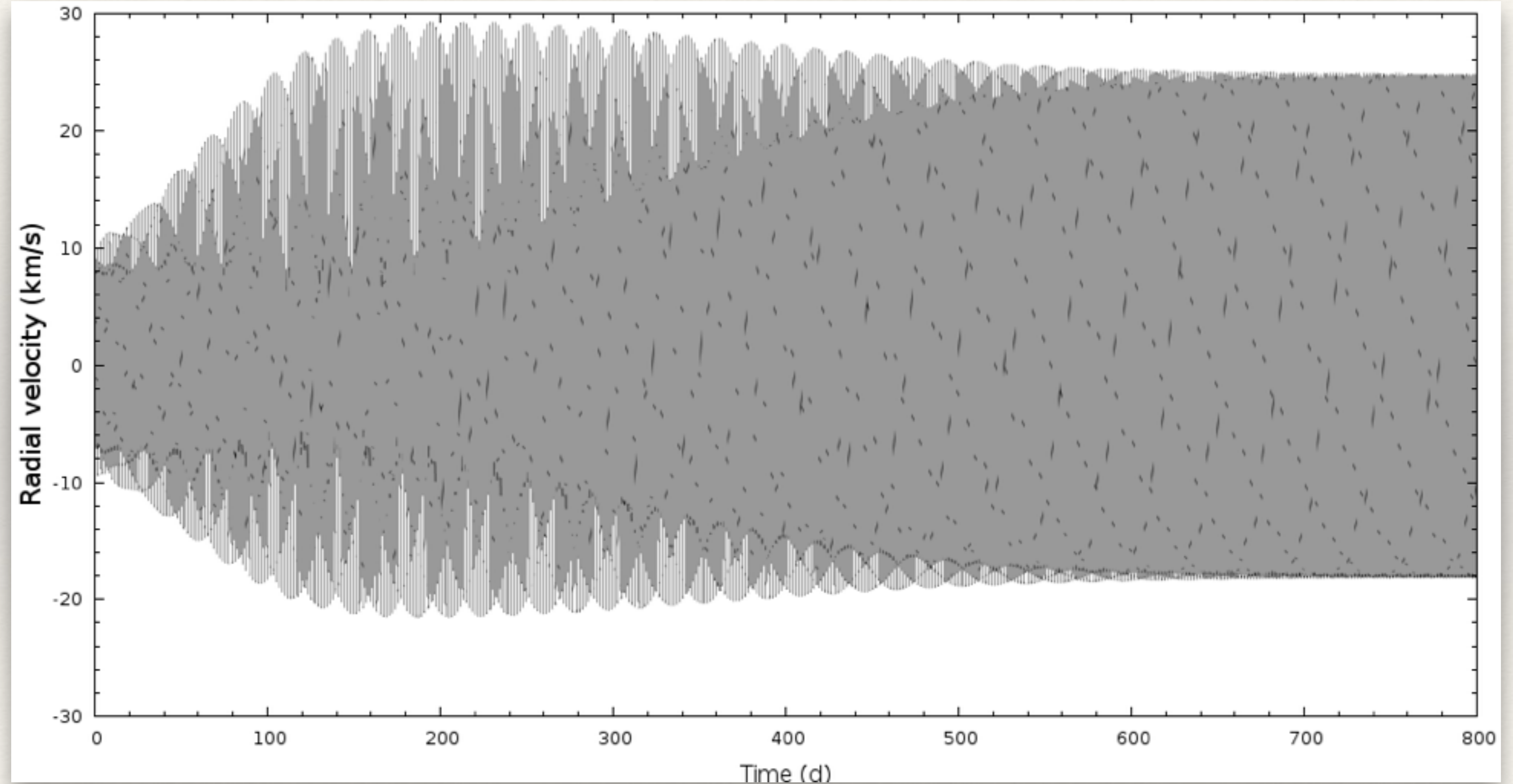
Turbulent convective models

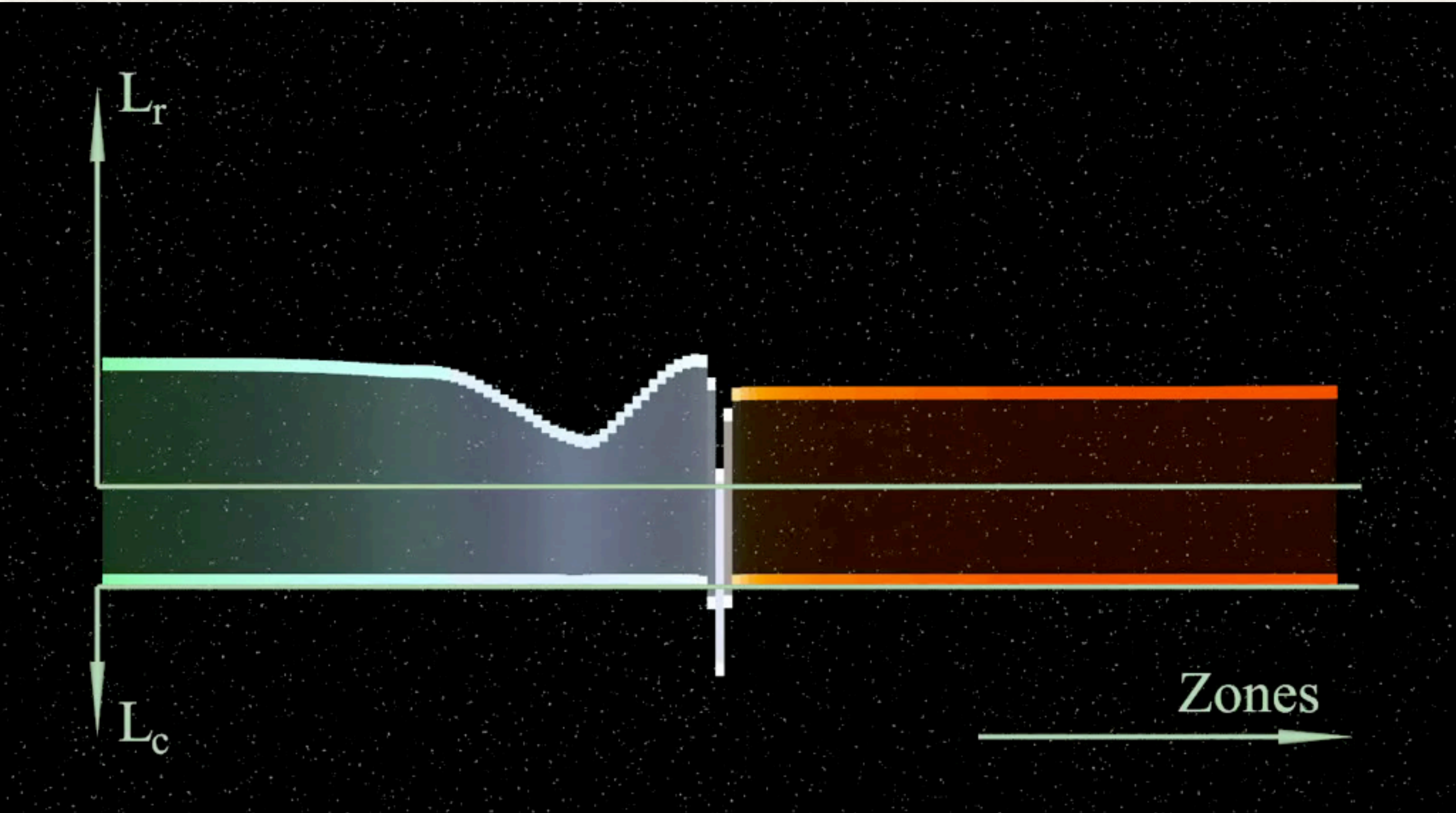
- Start: construction of the equilibrium model
  - Mass, T_{eff}, luminosity, chemical composition (X, Y, Z)
  - Integration of static equations ($d/dt = 0$)
  - Iteration until convergence
- Linear stability analysis: eigenfrequencies and growth rates (but not excitation!)

Turbulent convective models

- The non-linear code
  - Initial perturbation in some eigenmodes (FM, O1-2)
  - Forward integration of full set of equations in time
- Small growth rates: limit cycles are often reached very slowly…
  - Can be relaxed into -> stability analysis of limit cycle

Evolution of an RR Lyrae model

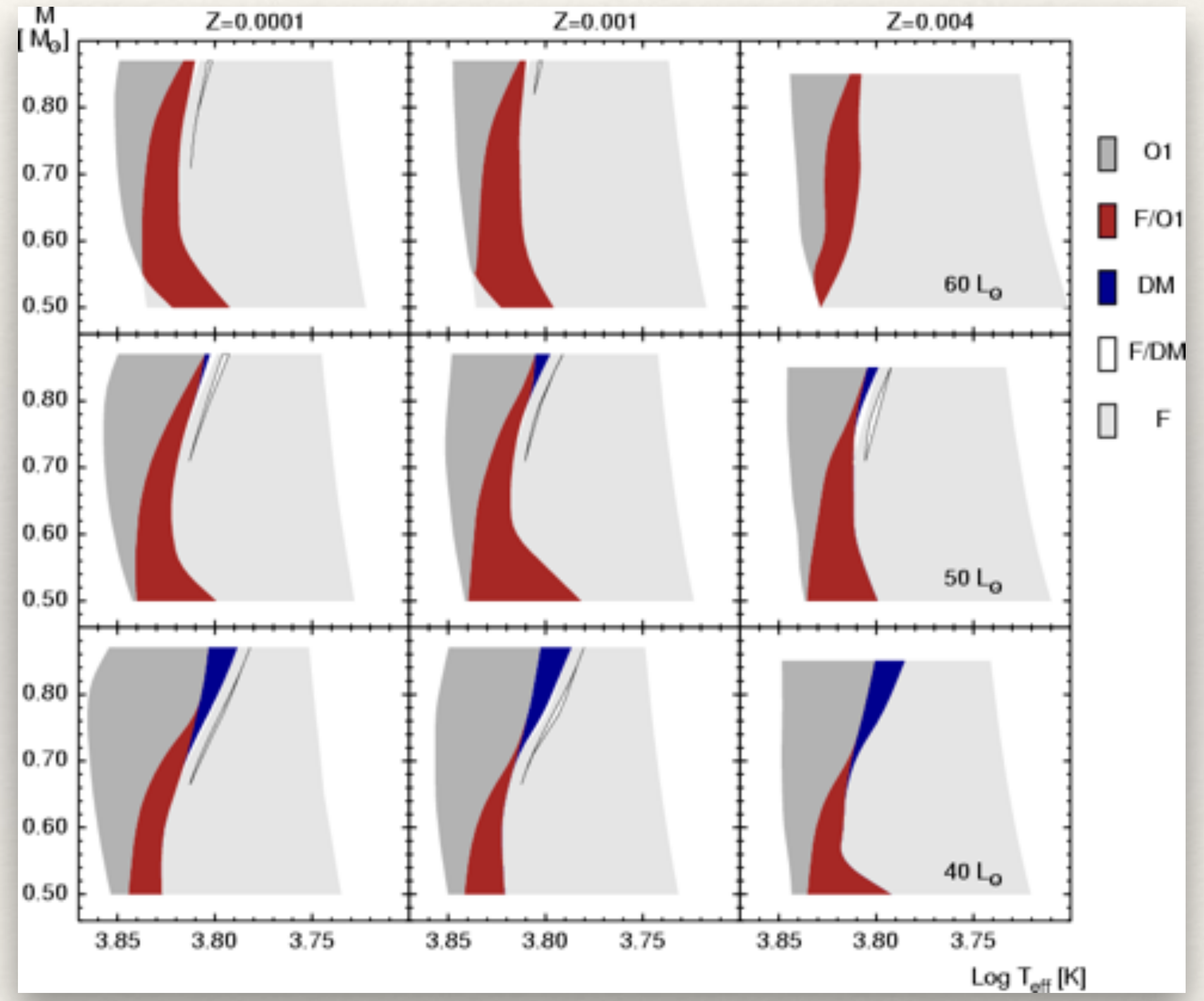




courtesy of Zoltán Kolláth

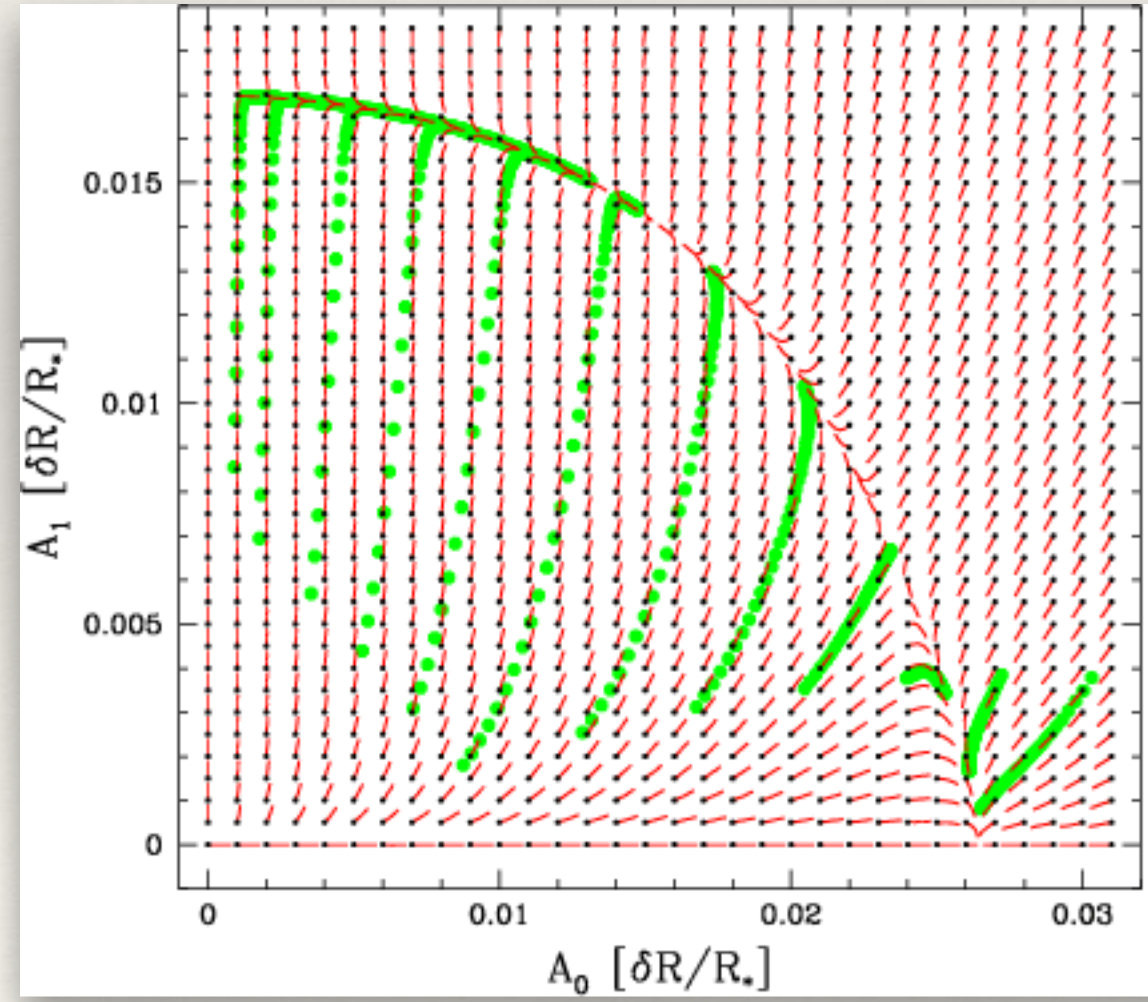
First results from TC models

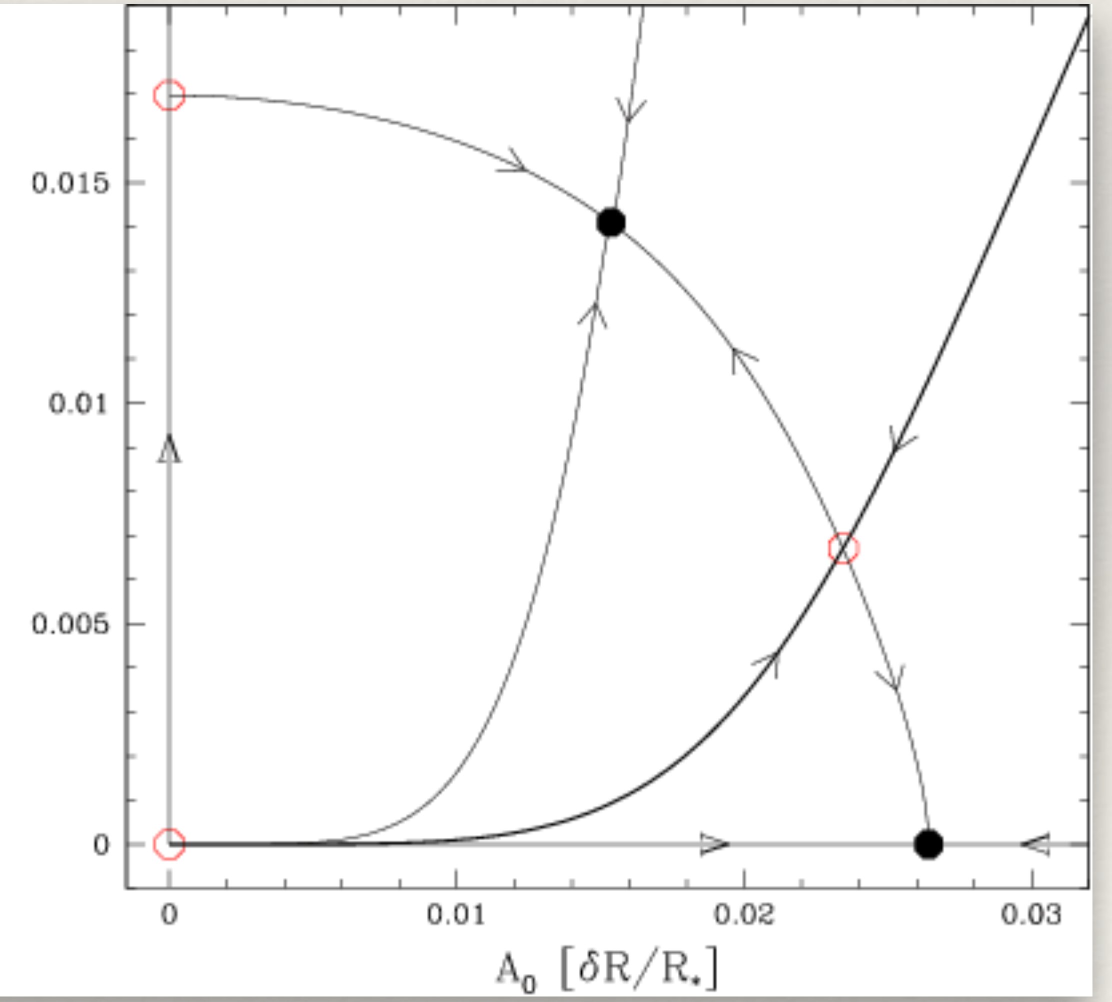
- Double-mode pulsations
  - beat Cepheids (Kolláth et al. 1998)
  - RR Lyrae: narrow parameter range BUT wider than resonant-only models
- (Fidelity still disputed)



Mode selection and switching

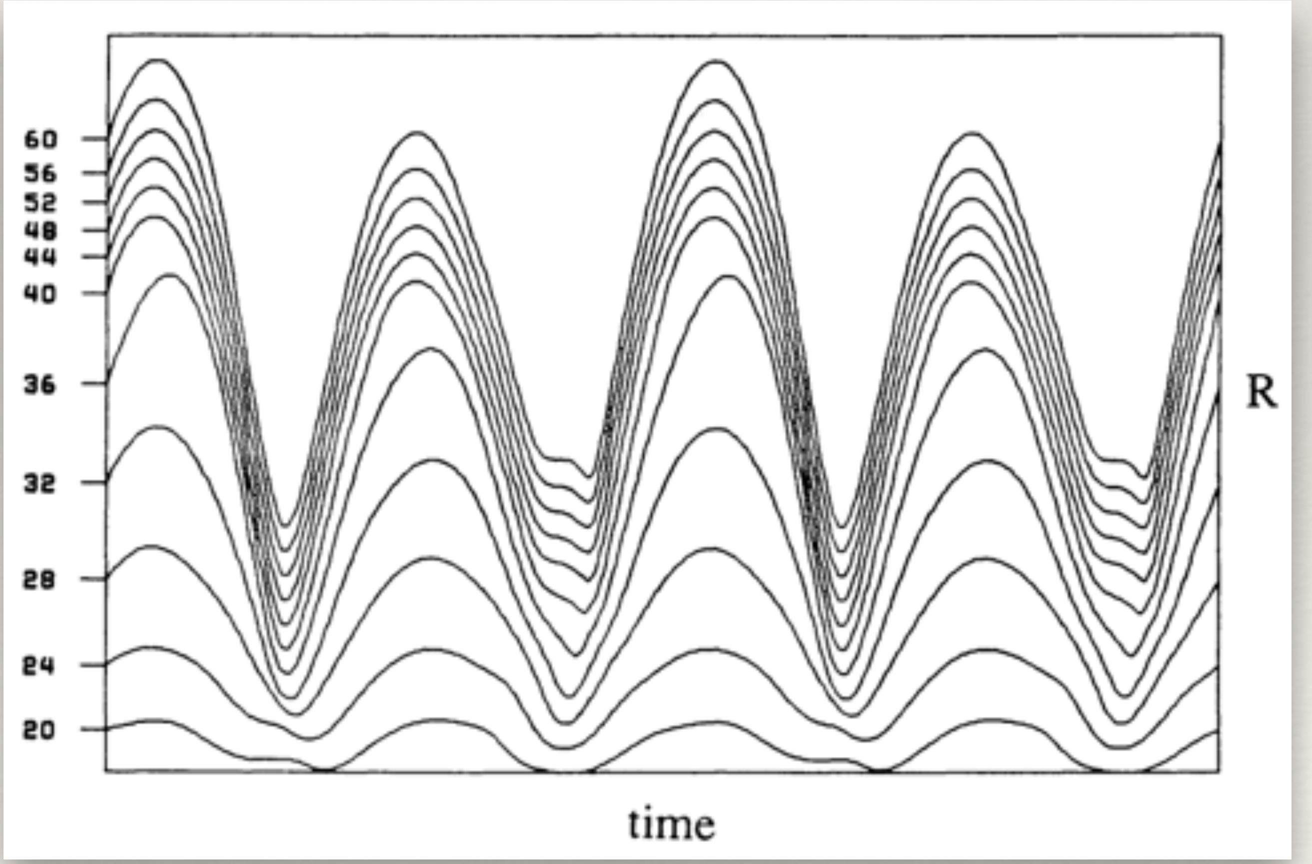
- Green: amplitude evolution of models - flow
  - Empty: unstable fixed point, filled: stable fixed point





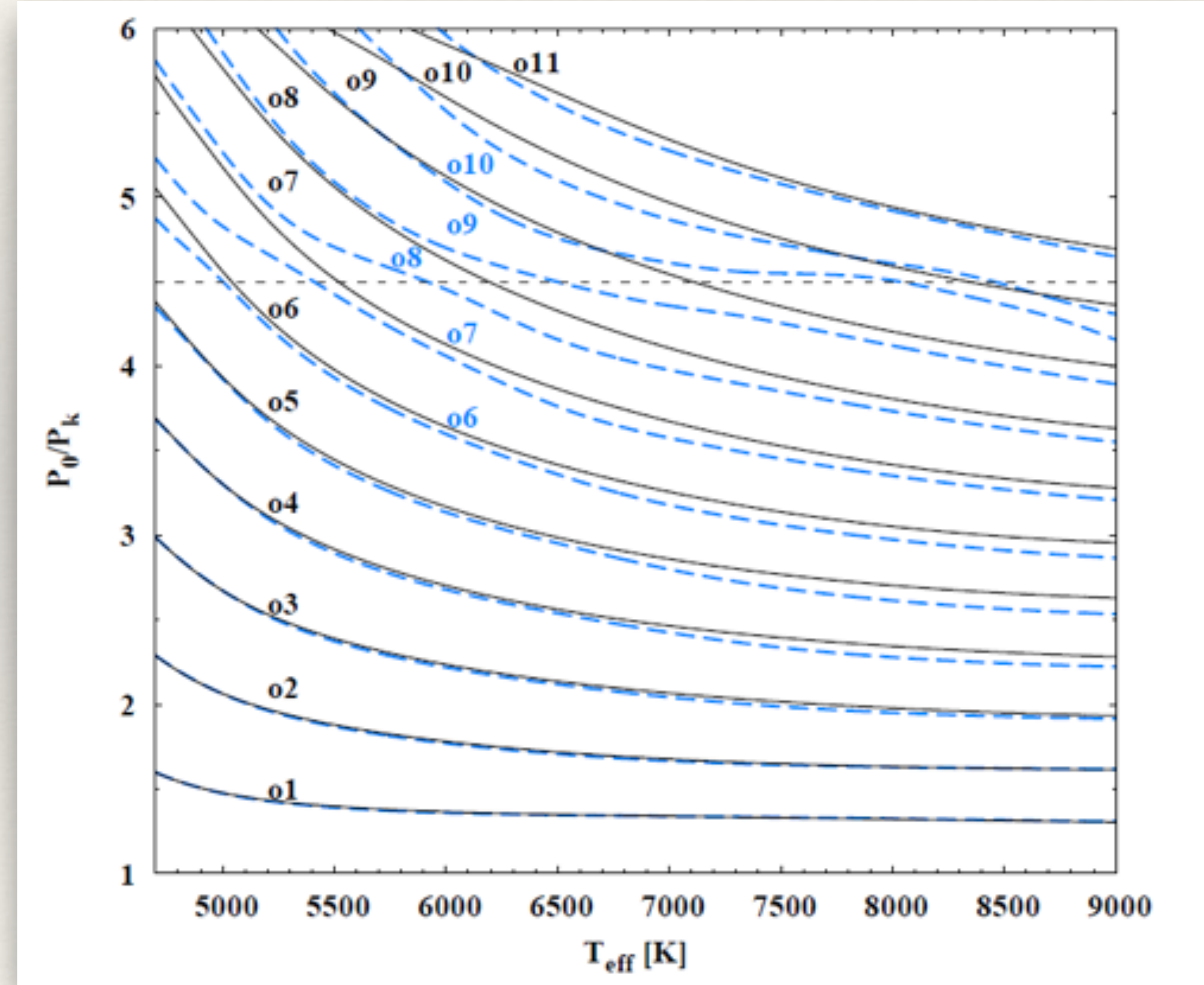
Mode interactions

- Mode resonances in Cepheid models
  - Destabilizes the limit cycle - period-doubling bifurcation (Moskalik & Buchler 1990)
- Chaos can also occur in some Type II Cepheid models
- Chaos observed in some semiregular stars



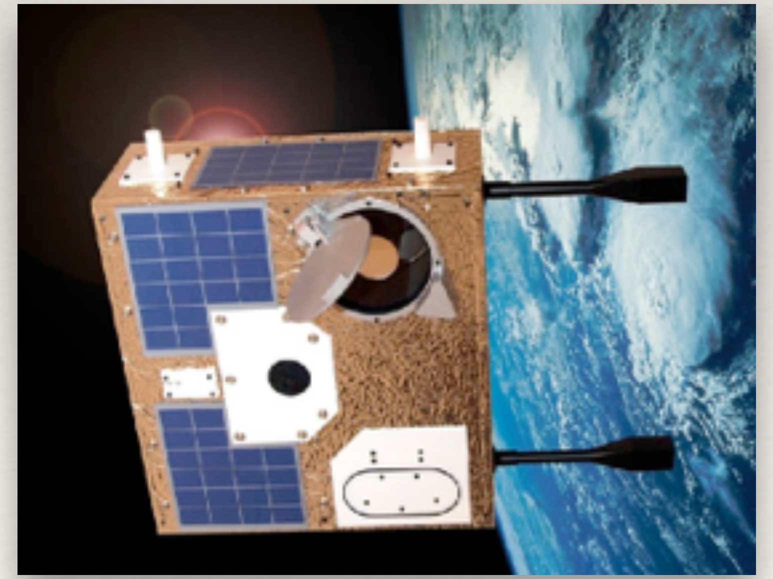
Strange modes

- Modes trapped close to the surface
- Can become excited
- (Very low amplitudes, no standalone detection so far)
- Periods may differ from the adiabatic case



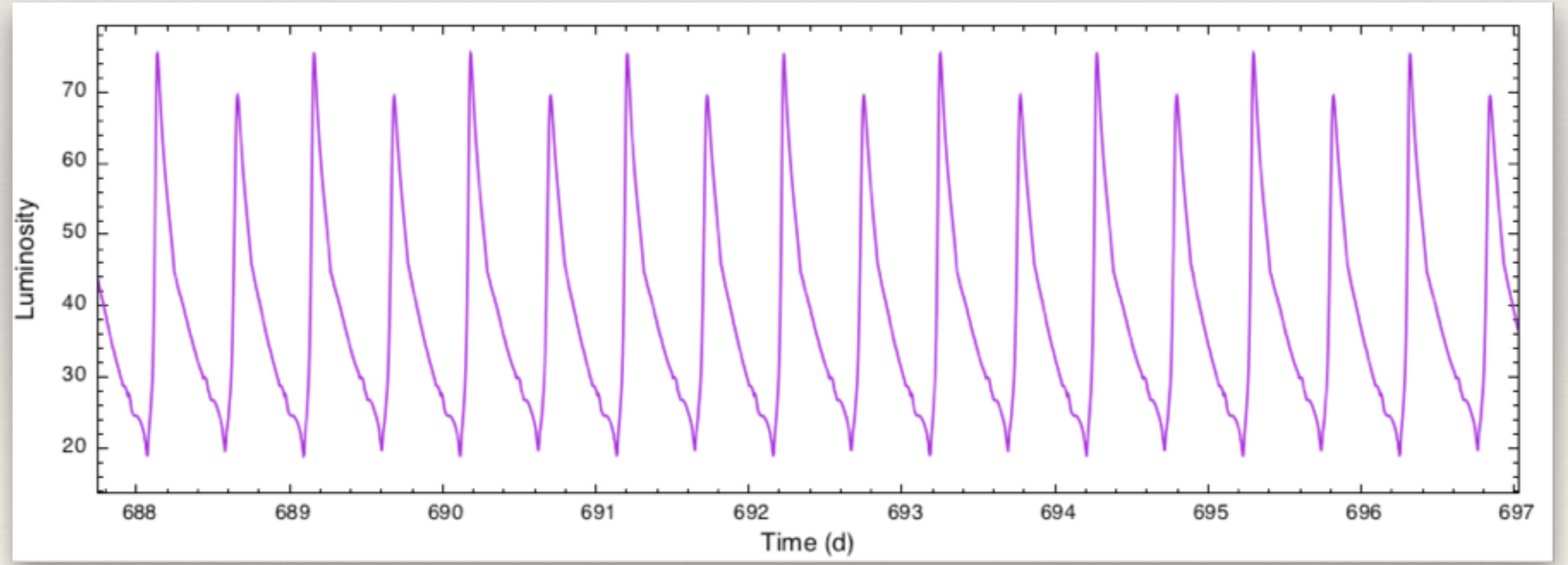
What's next?

New developments



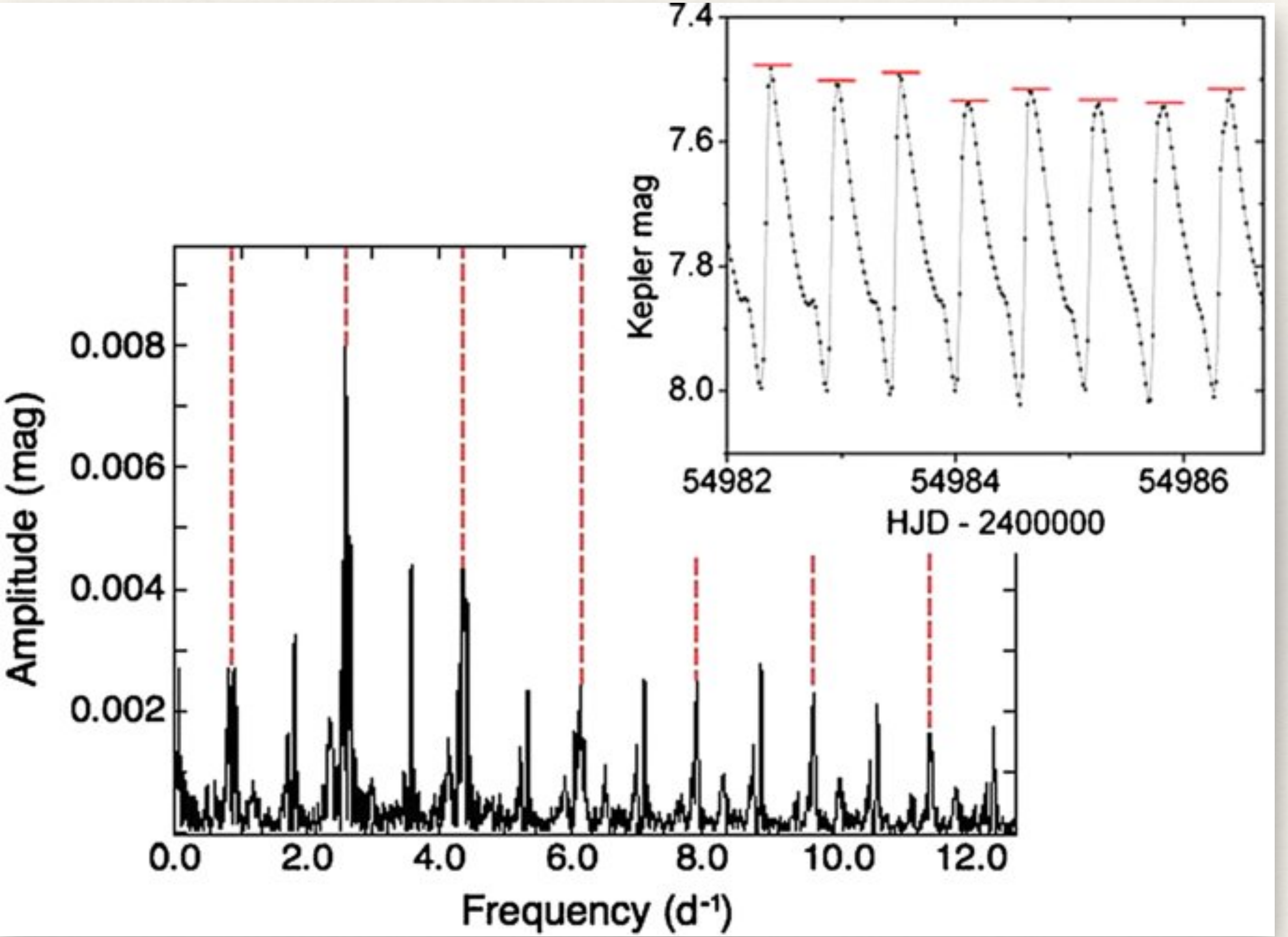
Period doubling in RR Lyrae models

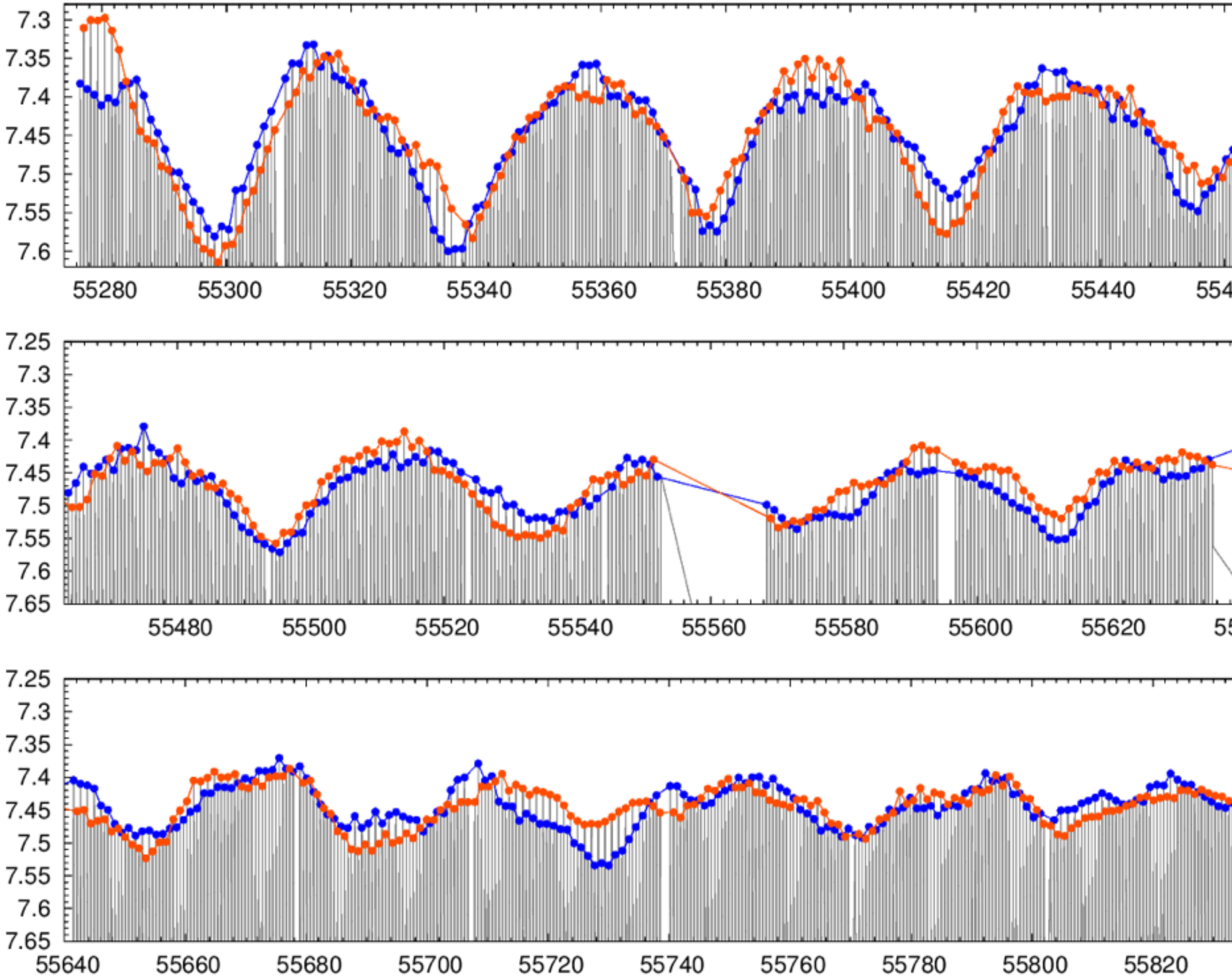
- My first grad student assignment
- Nothing similar in observations - shelved



Within weeks…

- First observations of Kepler:

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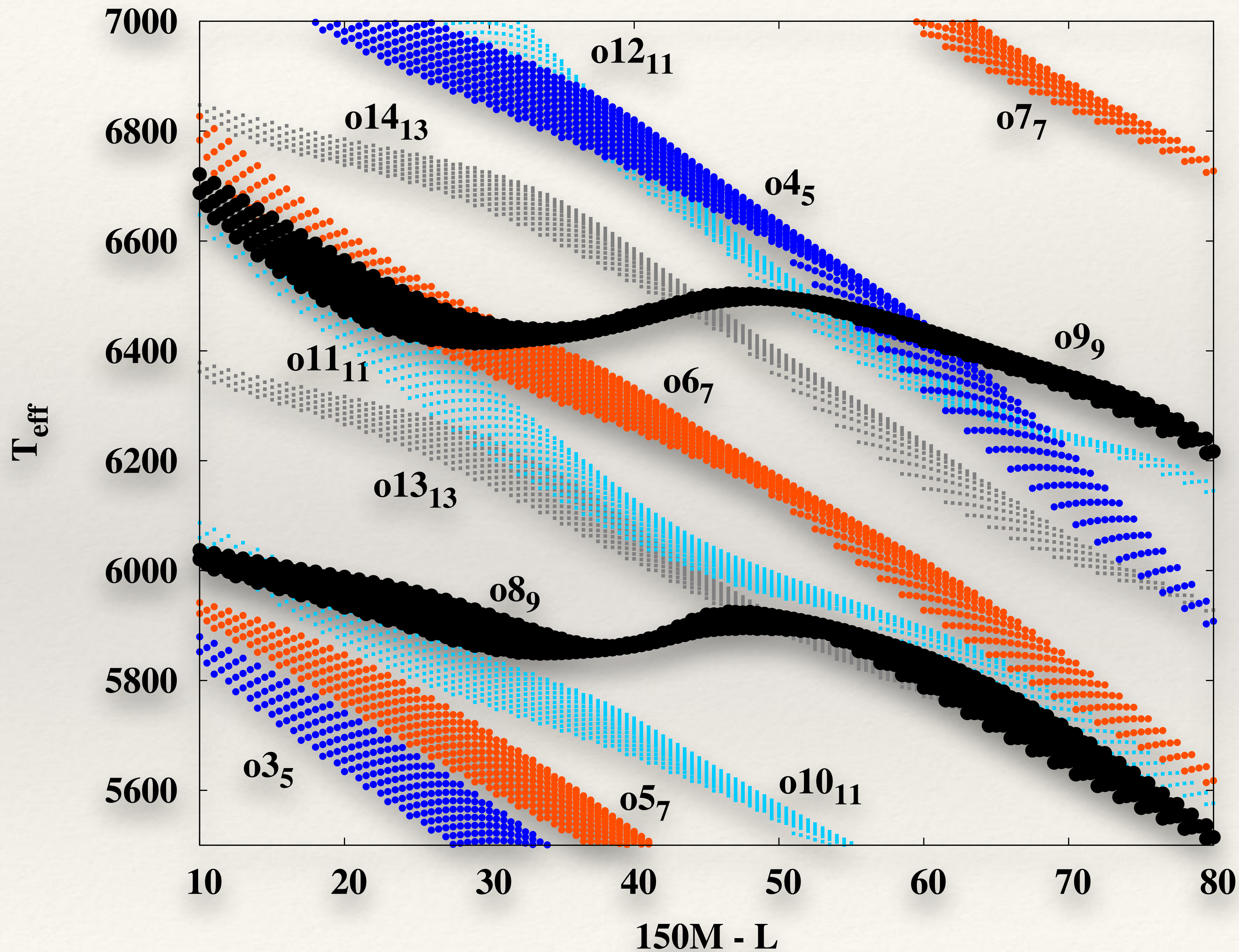
“Scientists baffled”

- Earlier model studies: no signs of PD
- RR Lyrae stars are weakly non-adiabatic, no low-order resonances —> extended model survey
- One good candidate: 9th radial overtone, 9:2 resonance

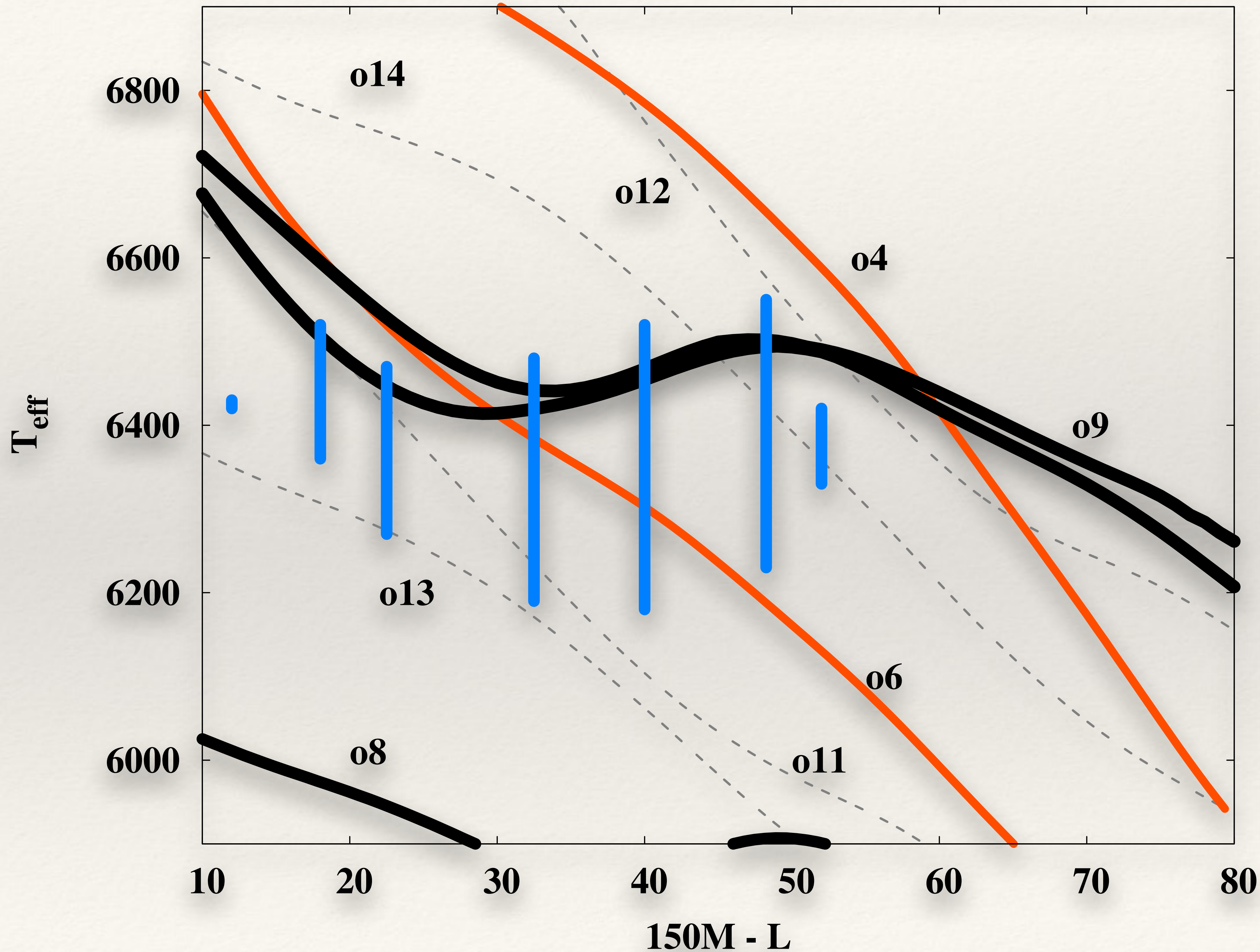
Diagnostic diagram

- Vicinity of linear mode resonances
- oN_m: P_0/P_N:m/2
- Which is at work?



Non-linear model survey

- PD models follow 9:2 resonance with O9
- Note nonlinear period shift



Yay!

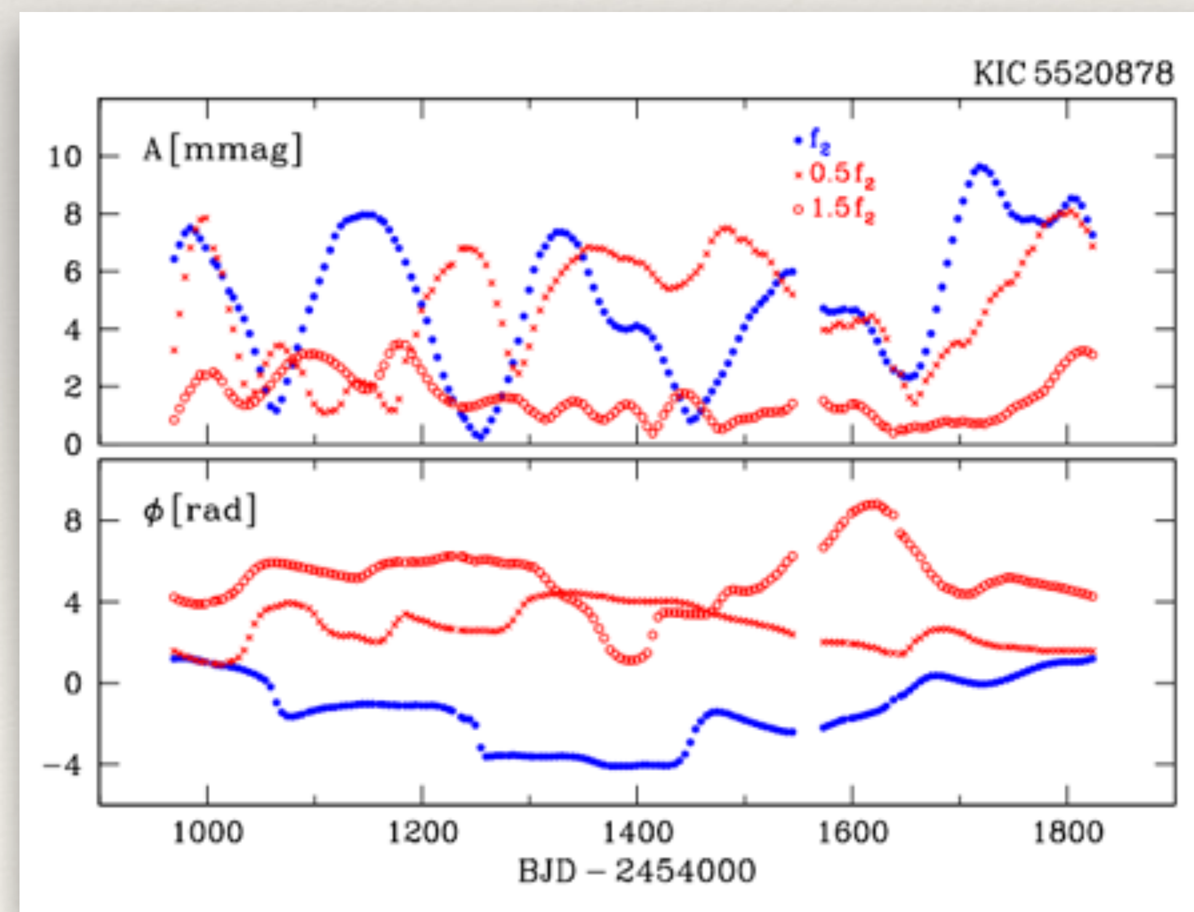
Not so fast…

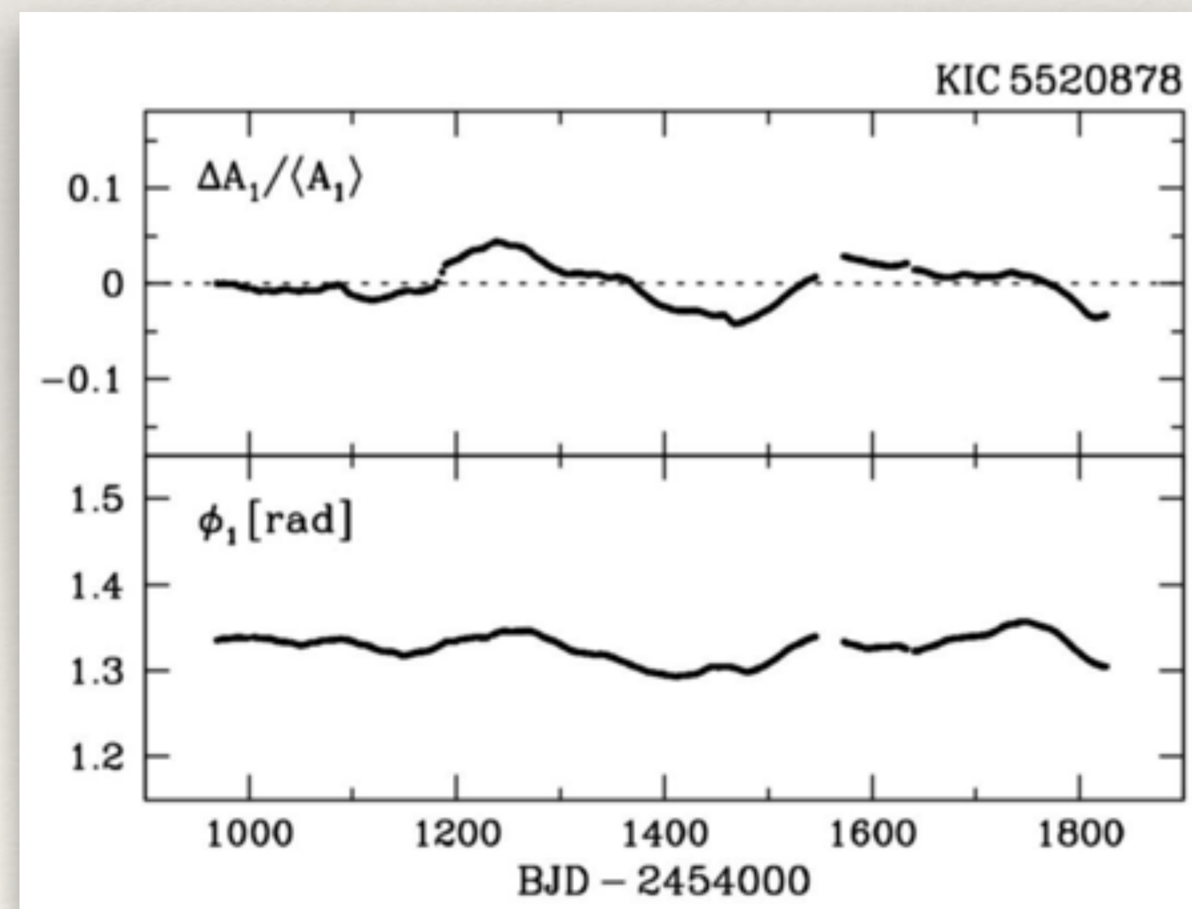
- PD is not stable in stars
- Stars have other additional modes
- Blazhko effect!

- Some PD models displayed other effects too
- Chaotic solutions - where did they come from?

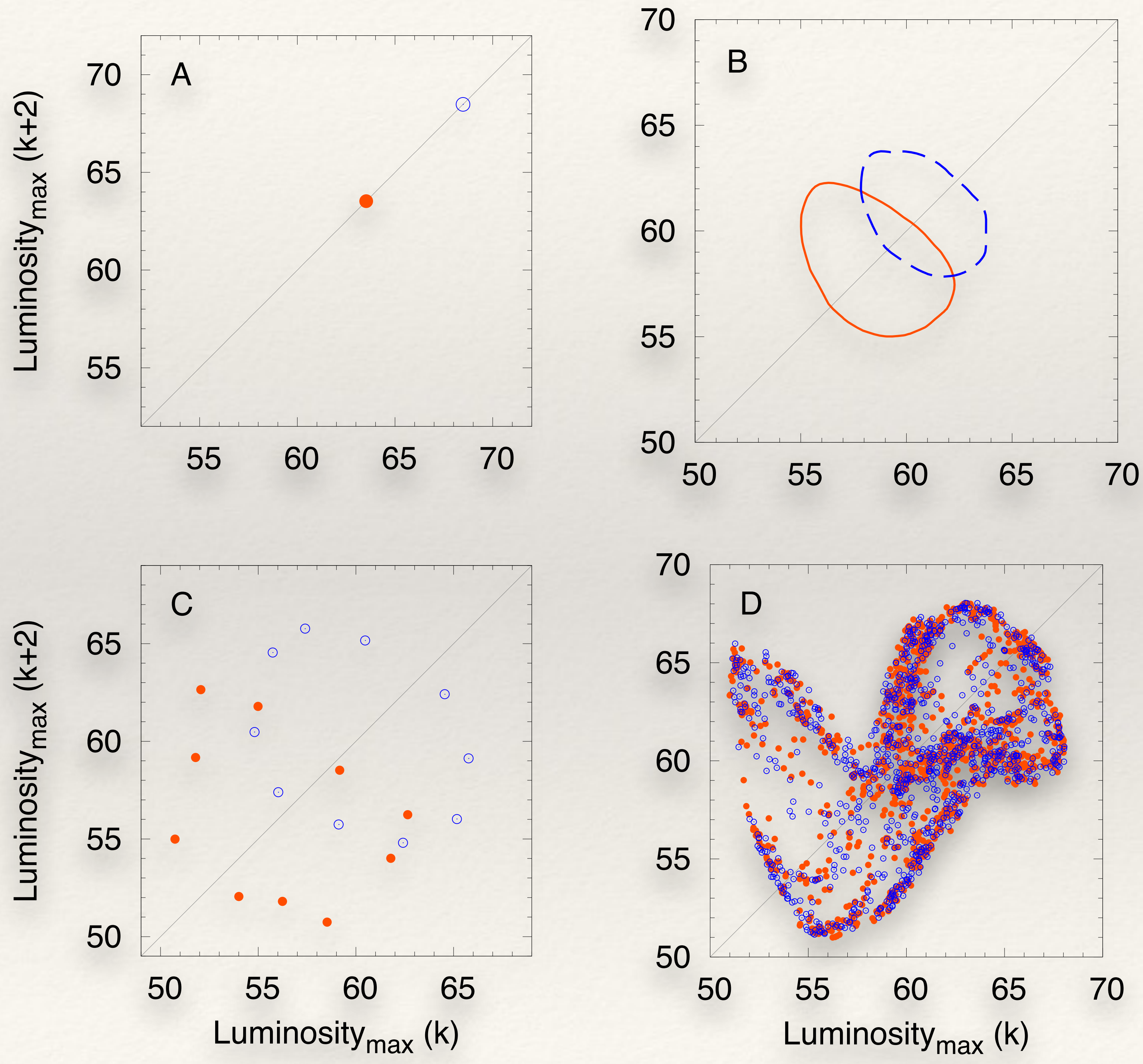
Additional modes

- Various low-amp. modes (A_1 / A ~ 50-1000)
- Patterns arise (similar modes excited in stars?)
  - Mode amplitudes and frequencies change over time

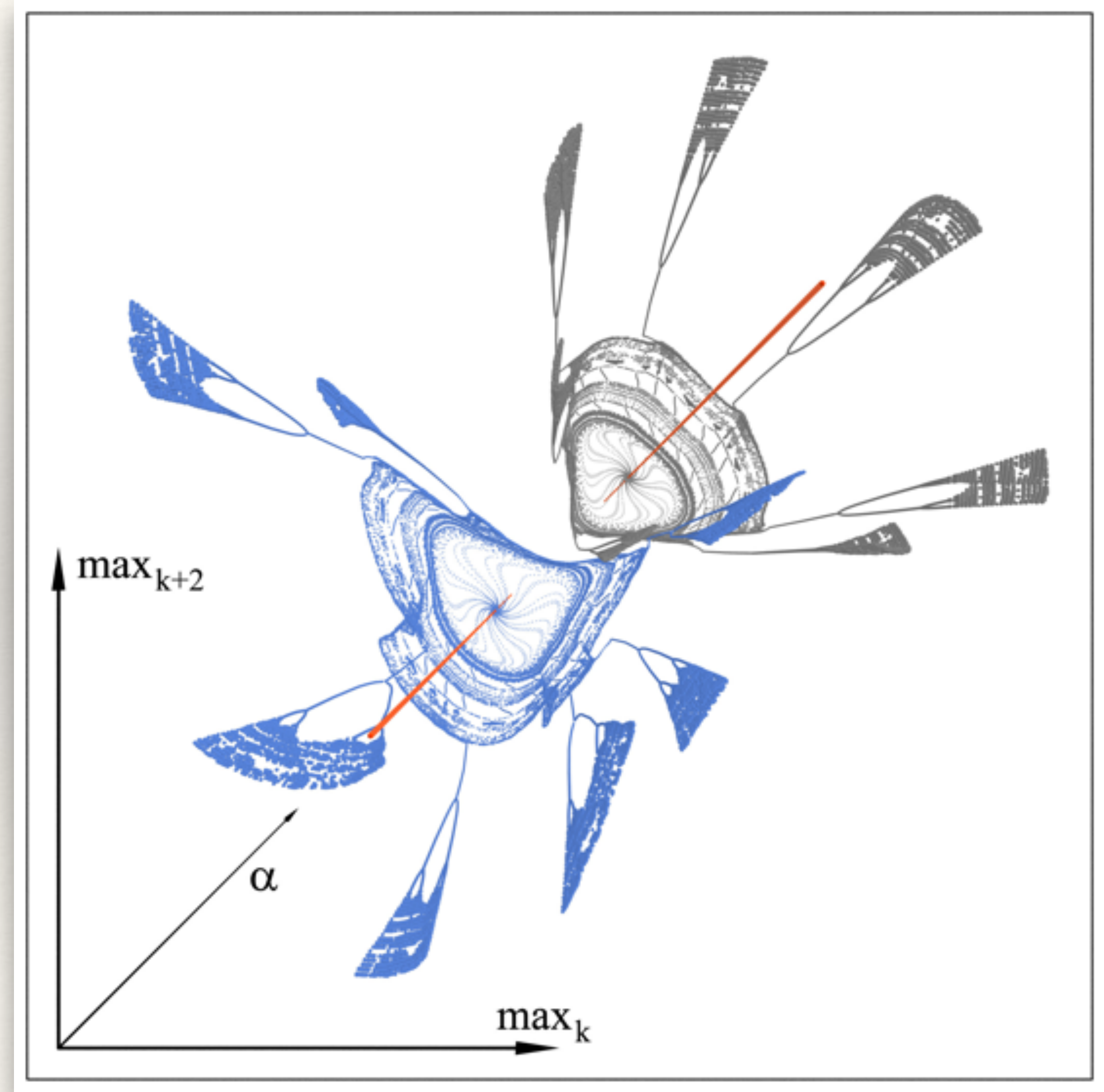




Return maps



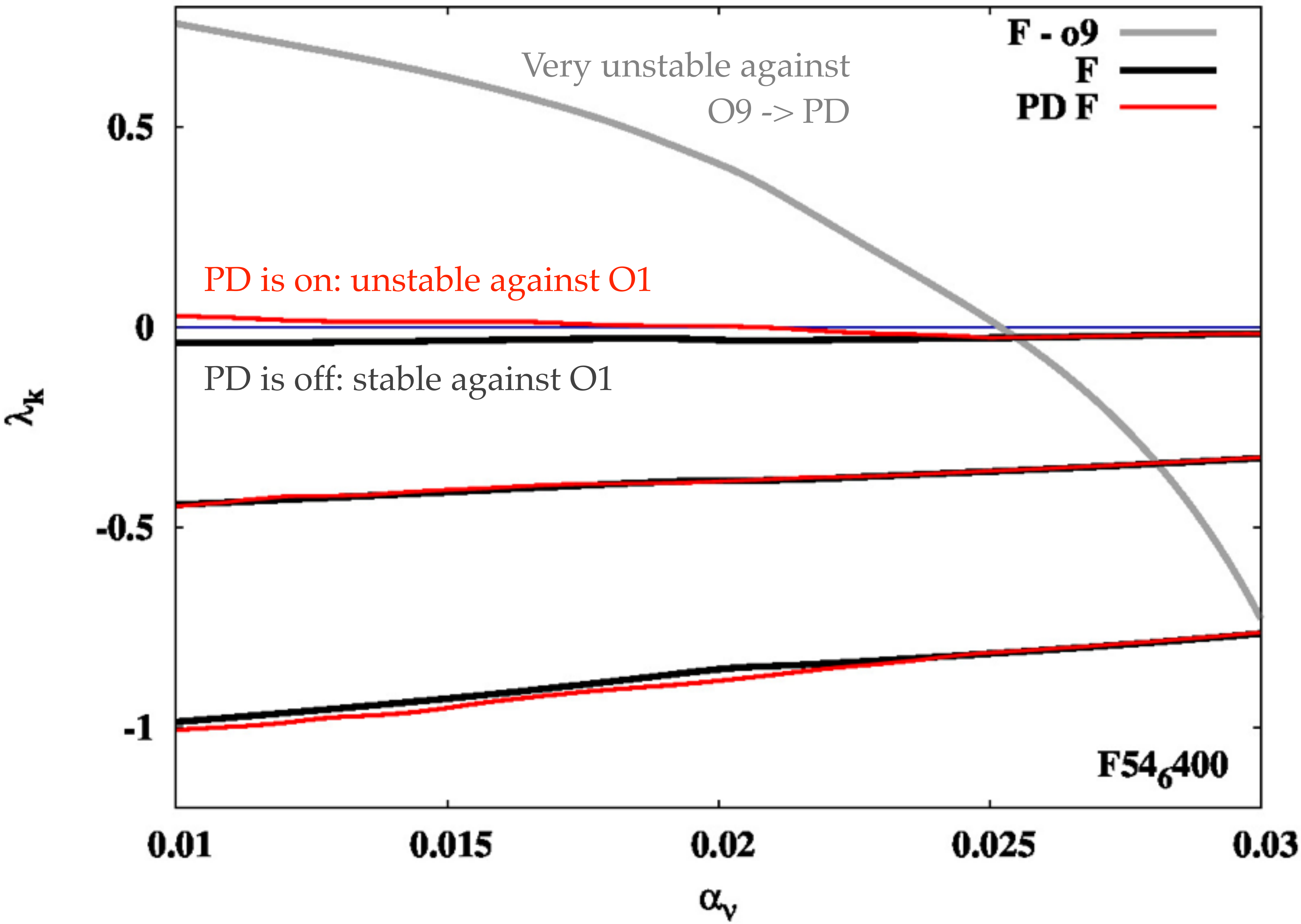
Chaotic solutions!



More details:
Emese's talk tomorrow

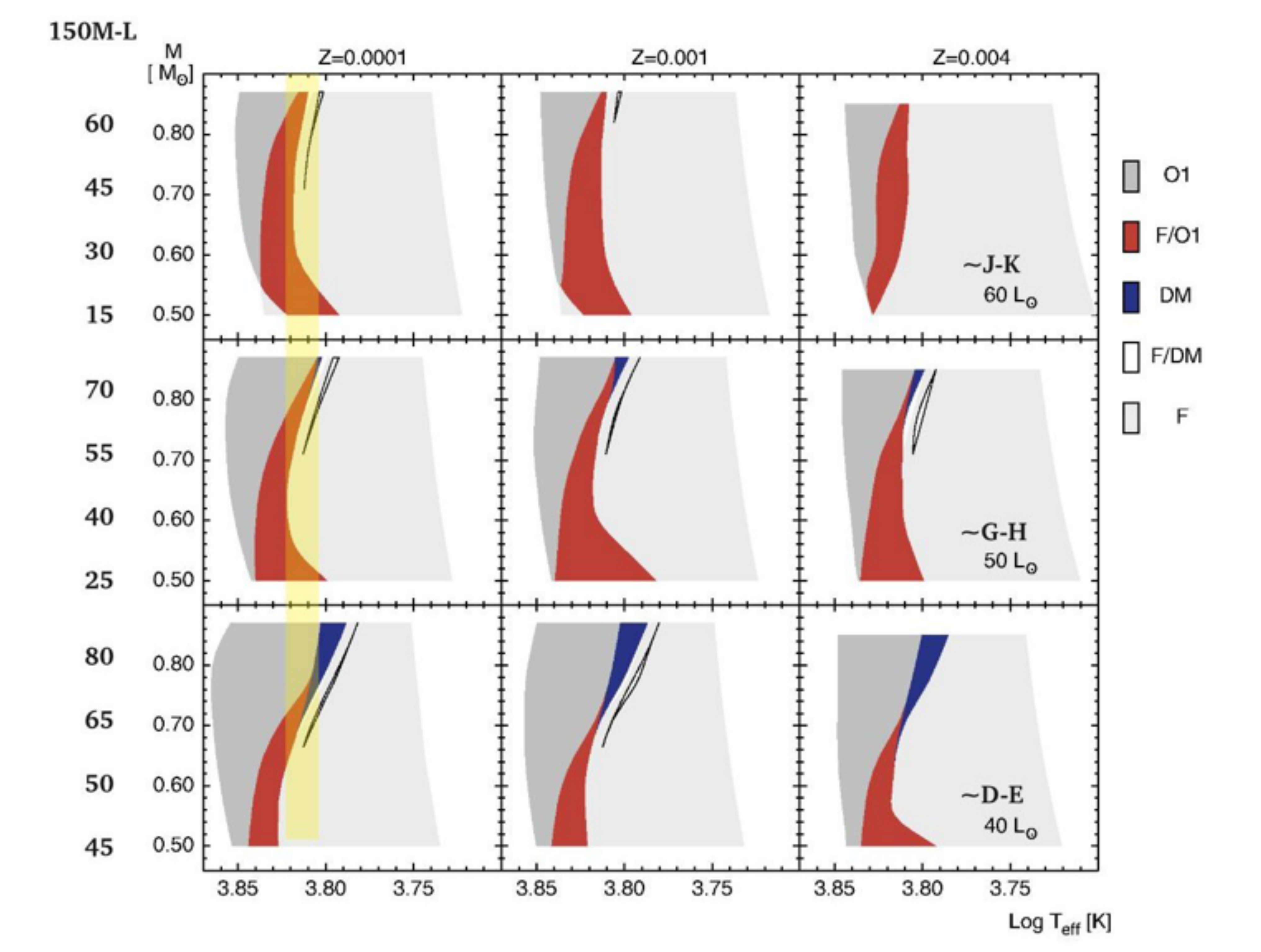
Origin of chaotic models

- First overtone is normally damped
- Need lower viscosity / mixing length
- Resonance with O9 leads to PD -> new limit cycle
- NEW limit cycle unstable against O1
- At work: 3 modes (FM + O1 + O9)
- Observable: period-doubled FM + low-amp. O1

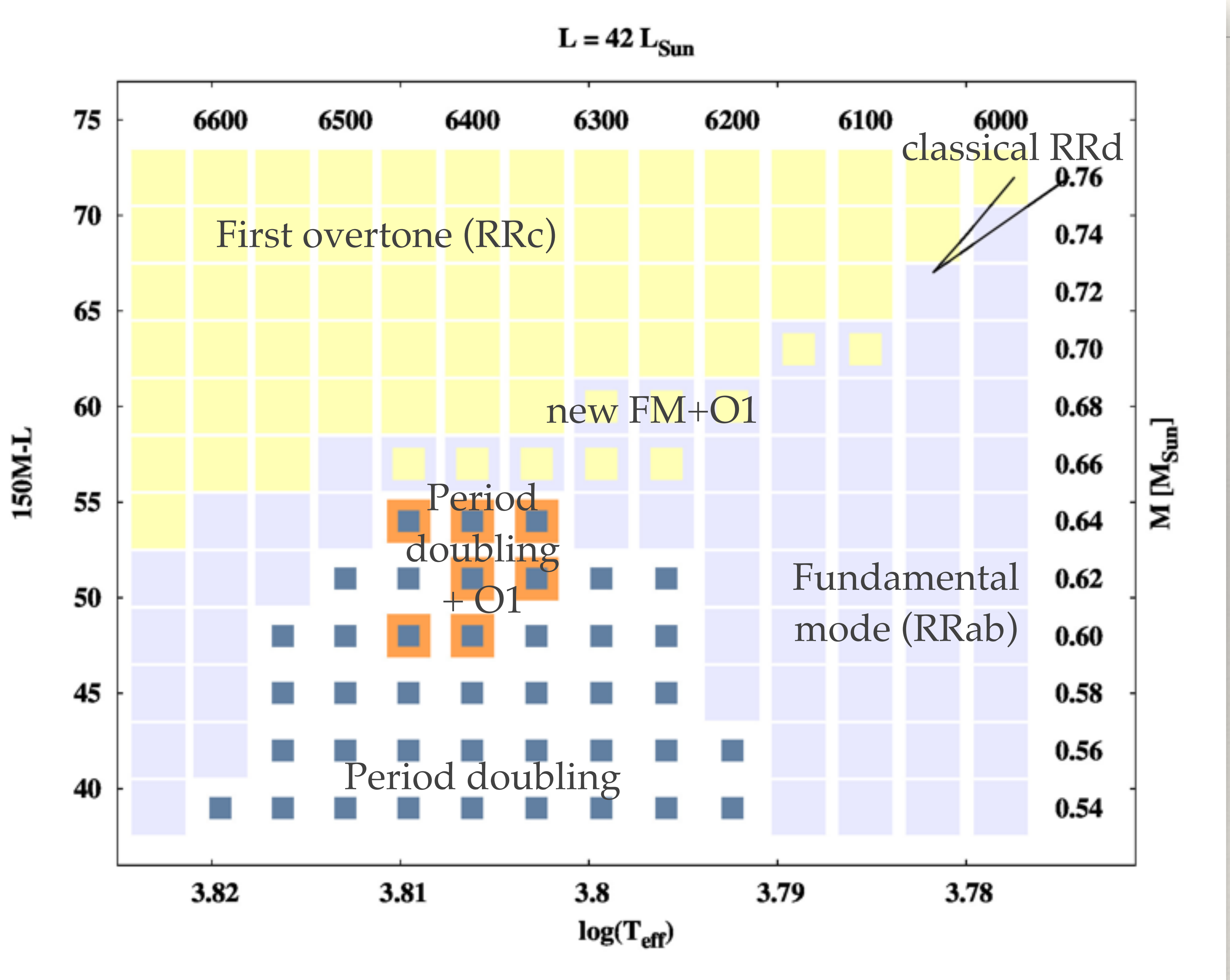


Very unstable against
O9 -> PD
F - o9
F
PD F
PD is on: unstable against O1
PD is off: stable against O1
0.5
0
-0.5
-1
λK
F546400
0.01
0.015
0.02
0.025
0.03
αν

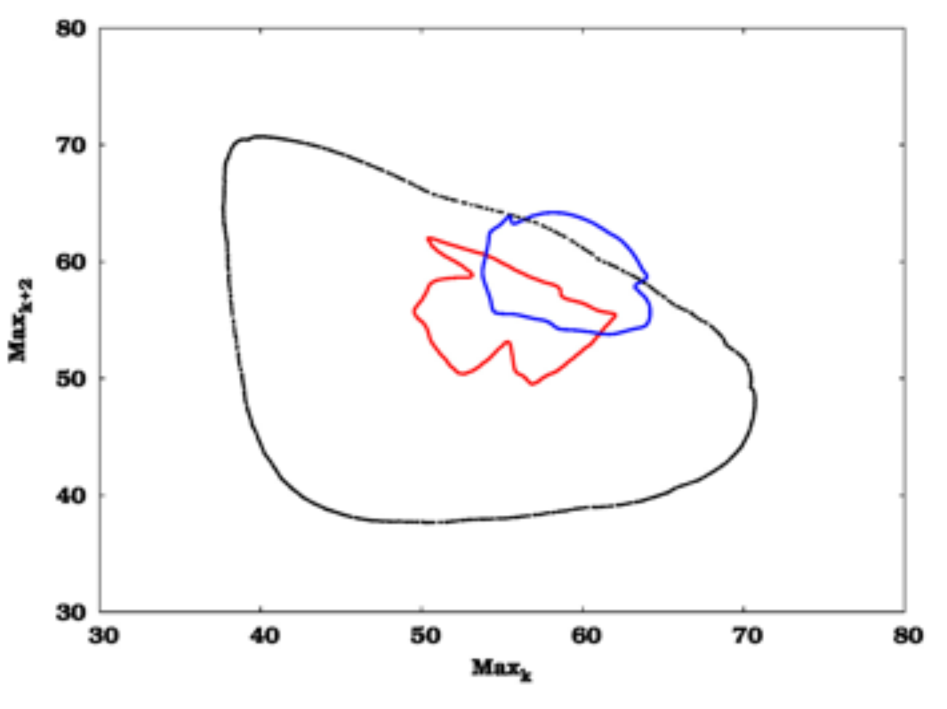
Instability strip



Instability strip - zoomed



New RR Lyrae states



- Need to map the instability strip
- Issues:
  - need dense grid
  - multiple stable states for same model(!) exist
  - dependence on convective parameters
  - (need to write all down)

Only partial success

- Period doubling + low-amplitude O1 observed in a few stars
- However…
- Most stars show f2 - might be O2?
- No time variability
- Still no Blazhko effect!

BL Her (type II Cep) stars

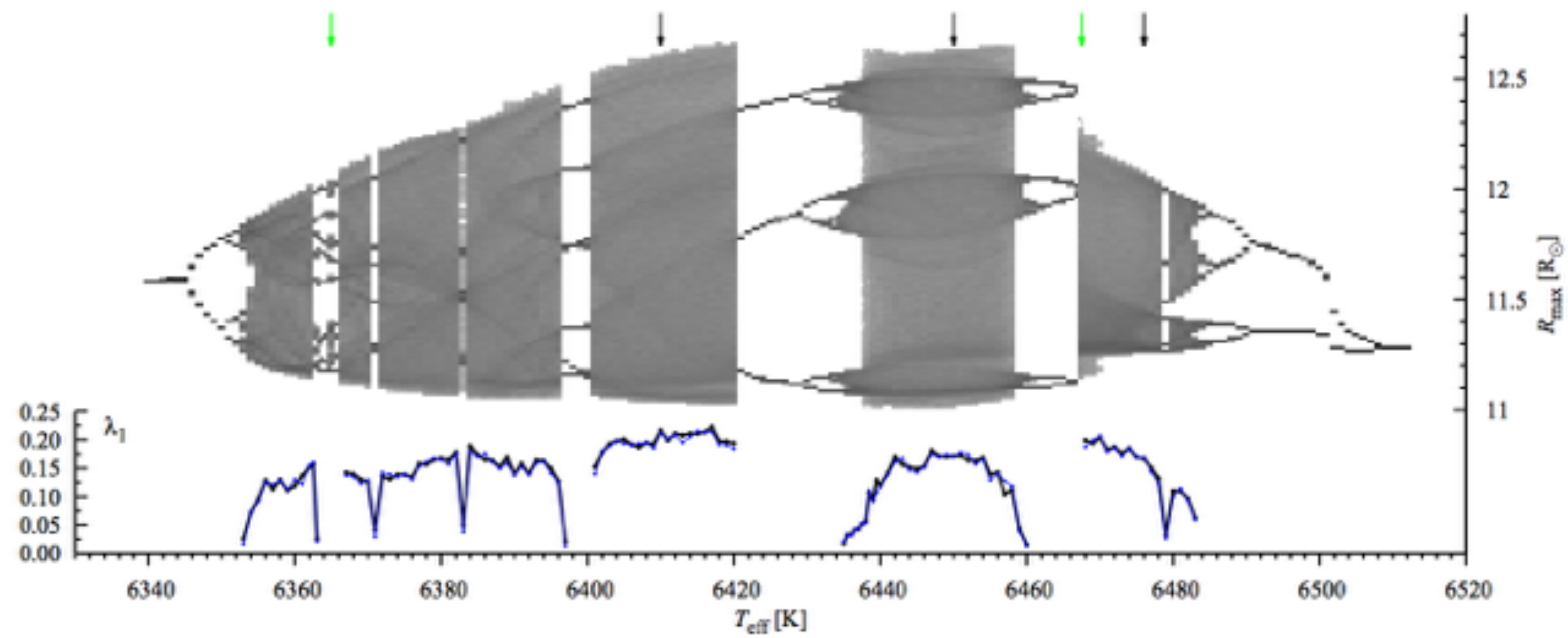


Figure 8. Bifurcation diagram for hydrodynamic BL Her models constructed with values of maximum radii over 9 000 pulsation cycles. Arrows in the top section point the location of particular chaotic (black arrows) and periodic (green arrows) models discussed in Sections 4.3 and 4.6, respectively. In the bottom we plot the largest Lyapunov exponents for the chaotic models – see Section 5 for details.

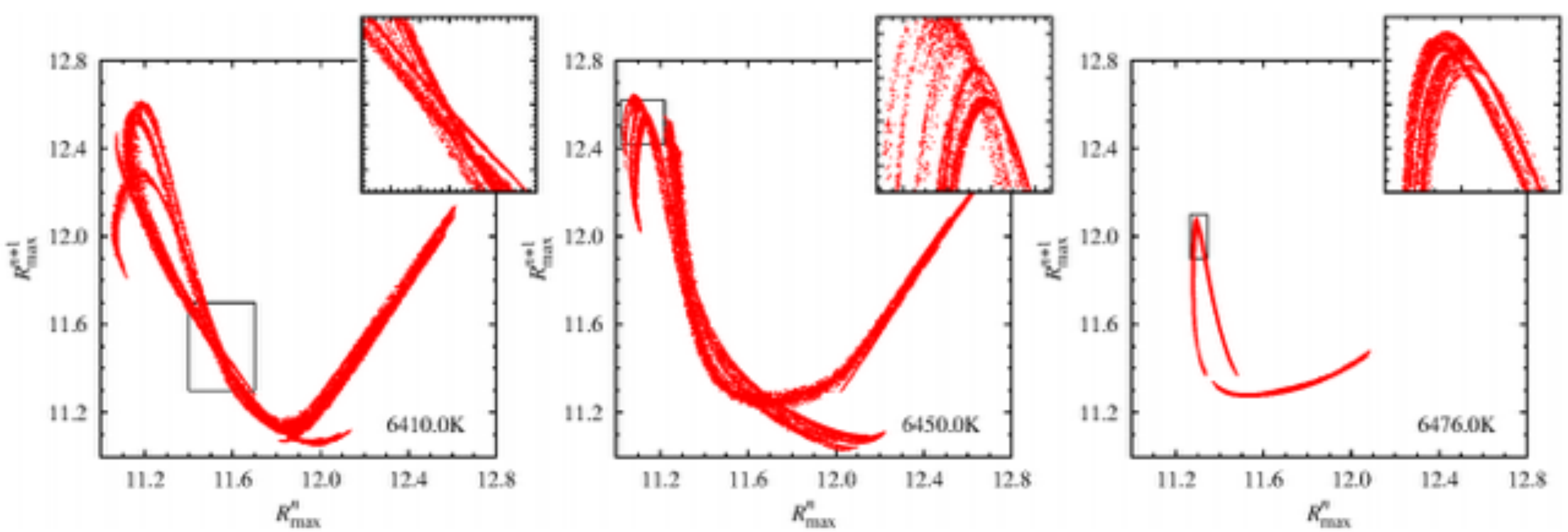


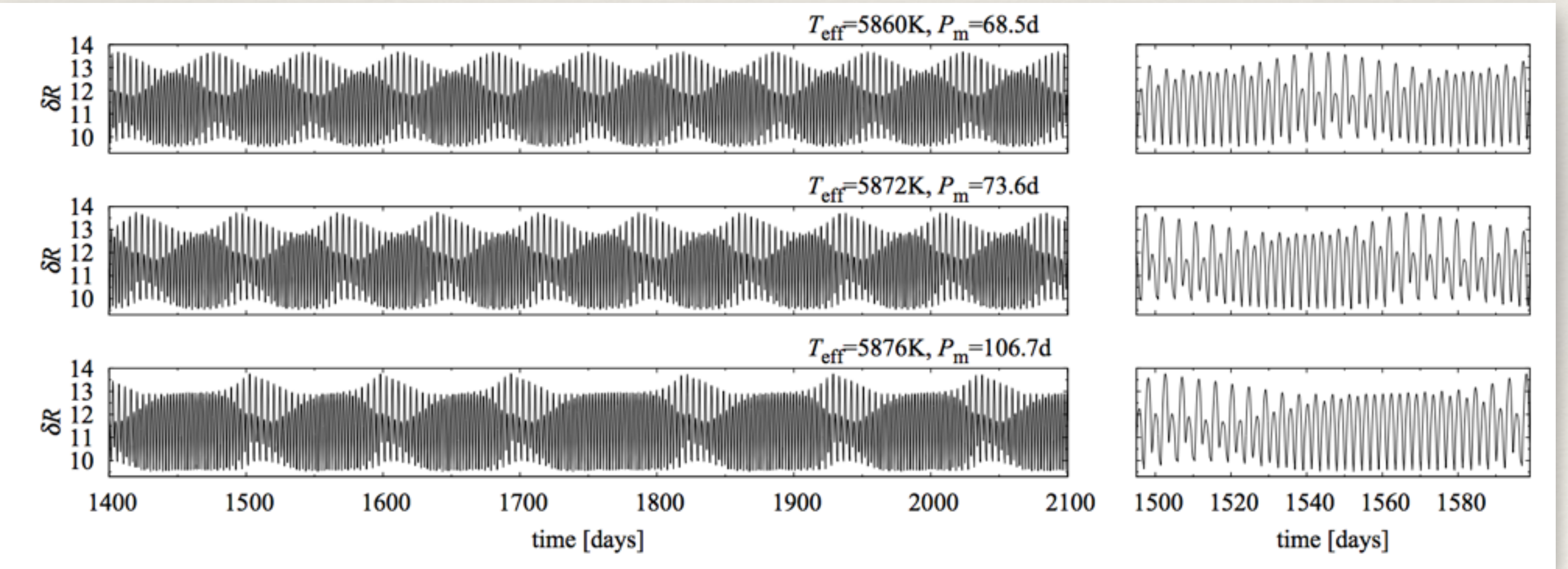
Figure 9. First return maps for three chaotic models (marked with arrows in Fig. 8). Data for the last 46 000 cycles (out of 50 000 computed) are plotted. *[This is low resolution version of the Figure]*

Smolec & Moskalik 2014

BL Her (type II Cep) models

- Variable period doubling + low-level amplitude variation (~Blazhko?)

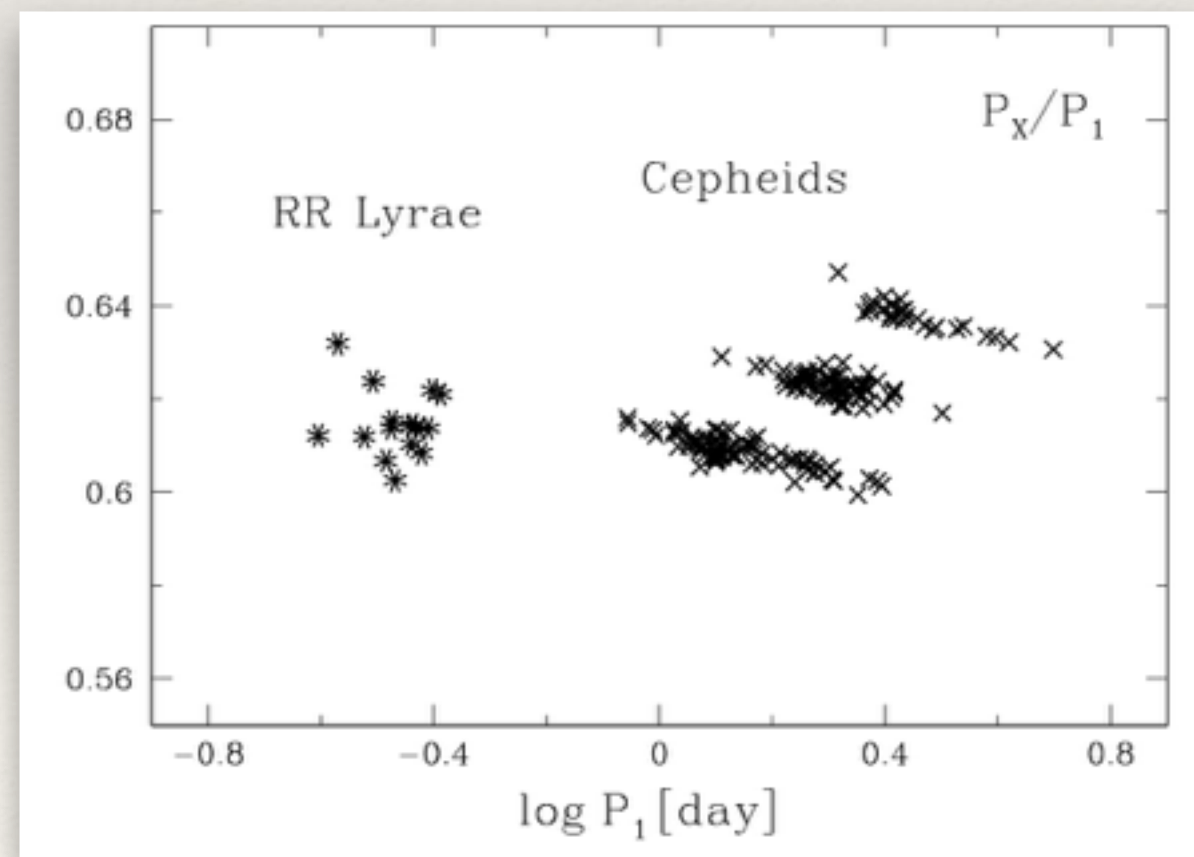
Smolec & Moskalik 2012

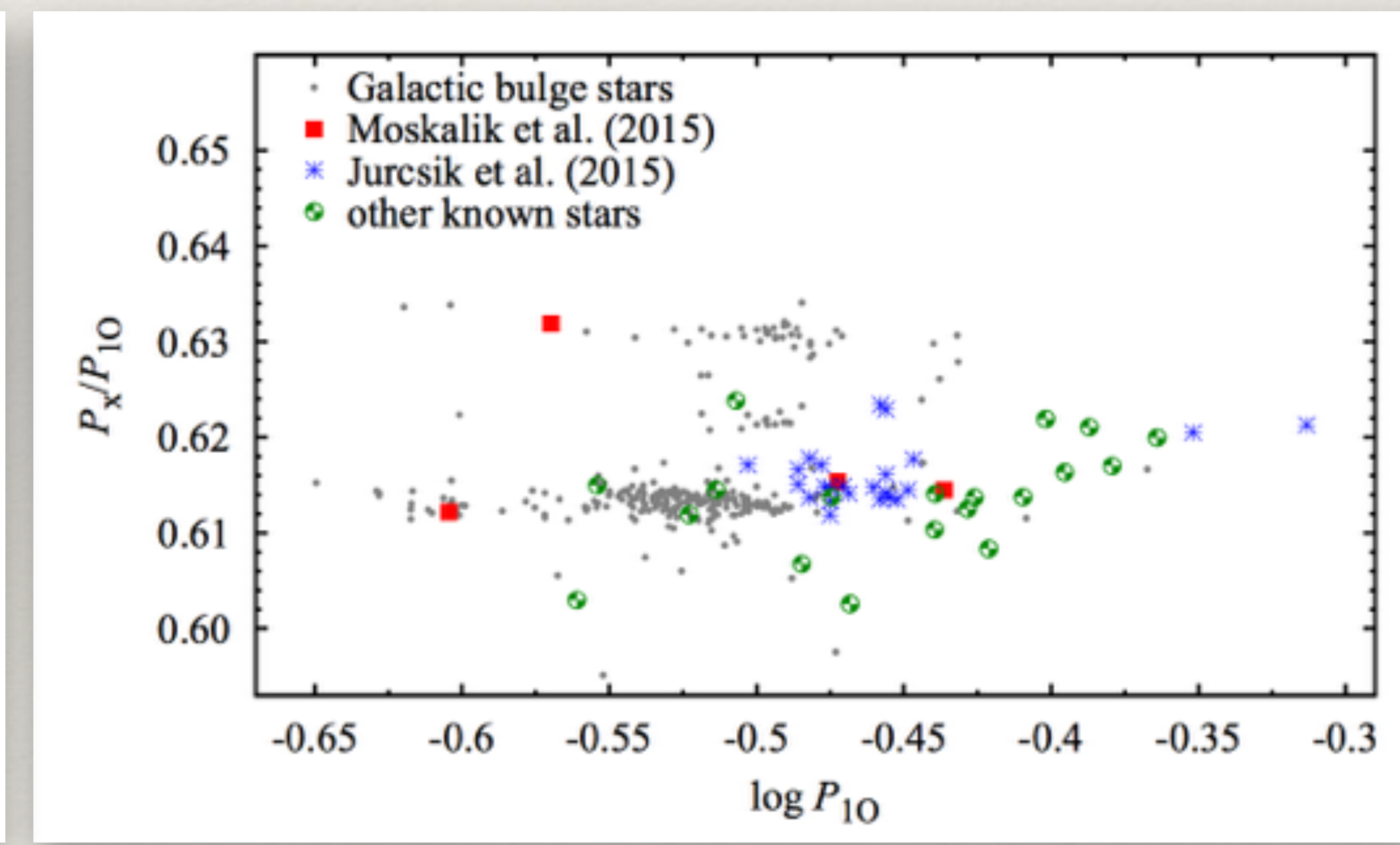


What about O1 stars?

- Various additional modes, peculiar patterns
- Golden ratio?… some might be…
- No model survey out yet

Moskalik 2014, Netzel et al. 2015





What about the Blazhko effect?
Emese's talk tomorrow

Alternative ways

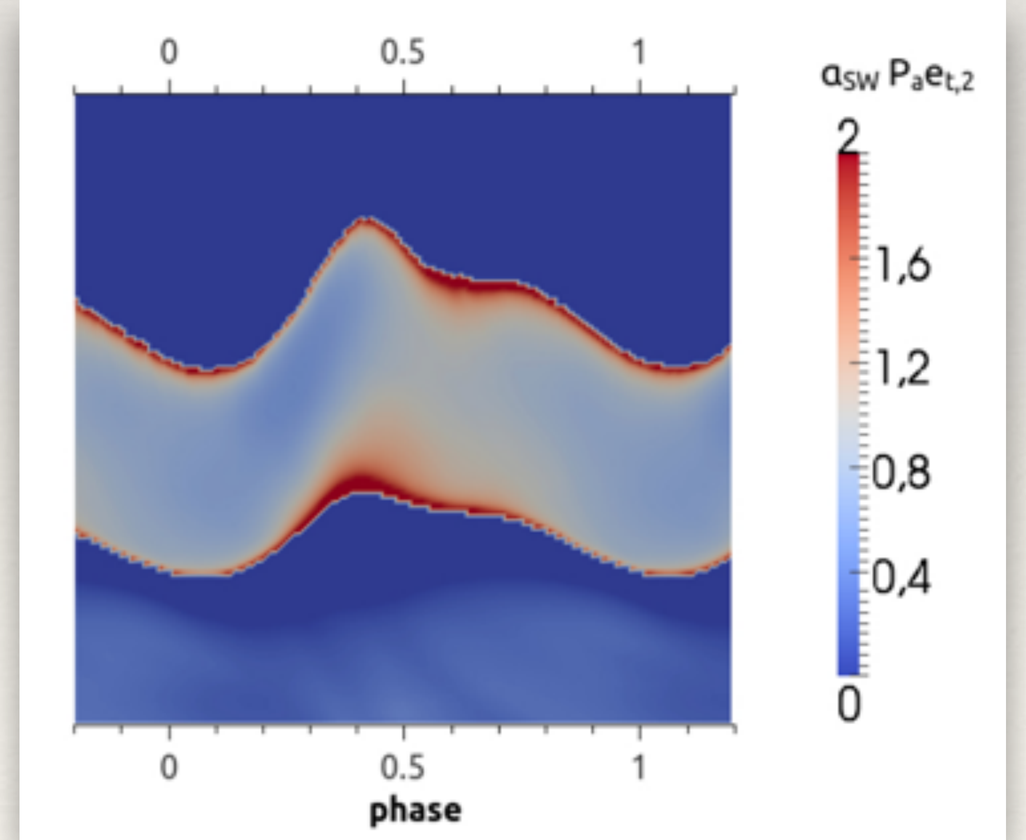
- Analogs, toy models
- Amplitude equations: eliminate fast var. (pulsation)
    - Coefficients: need input from HD models!
    - 9:2 resonance capable of modulation - is it physical?
    - Buchler & Kolláth (2011) - only Blazhko model without direct contradictions…

Alternative ways

- Respect the physics! We KNOW stuff from the 1D models
- "If the overtone frequency drops sufficiently to reach the fundamental frequency… " - **NOPE**
- " The waveform "bump" is generated simply by combining the modes, similar to the resonance mechanism in bump Cepheids." - **NOPE**
- "the first overtone is the only mode that is driven strongly into the nonlinear regime, while the fundamental remains very nearly sinusoidal" - **NOPE**
- "Molnar et al. (2012) claims to observe the first overtone in RR Lyr spectra; however, this work does not consider that the modal frequencies may shift substantially (probably downward)" - **DUDE, NO.**

Alternative ways

- 2D/3D models mature
- Exposing problems with 1D models
  - convective parameters may vary with depth AND phase



Alternative ways

- Reproduction of (one-color) light curves is easy
- We know so much more
  - Color data, radial velocity, line variations…
  - K2, TESS will observe hundreds of RR Lyrae stars
- Ad-hoc prescriptions for observations (modulation, PD, low-amp. p- and g-modes…) may not a good idea

The way forward

- Models: still lot of potential in 1D models
  - fast, lot of dynamics to explore
  - Prepare for 2D/3D models
- W Vir, RV Tau models
- Double-mode models

The way forward

- Data keep flowing in
  - K2, TESS, Plato: exquisite details, great challenges
  - Large numbers: rare cases will pop up (e.g. BEP stars)
  - Gaia: parallaxes!!!