Faraday's Law

\[ \Phi_B \equiv \int \vec{B} \cdot d\vec{S} \]

\[ \varepsilon = -\frac{d \Phi_B}{dt} \]
Faraday’s Law of Induction

Recall the definition of magnetic flux is

\[ \Phi_B = \int \vec{B} \cdot d\vec{A} \]

Faraday’s Law is the induced EMF in a closed loop equal the negative of the time derivative of magnetic flux change in the loop,

\[ \mathcal{E} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A} = -\frac{d\Phi_B}{dt} \]

Constant B field, no induced EMF in loop

changing B field, causes induced EMF in loop
In a physics laboratory experiment, a coil with 200 turns enclosing an area of 122cm$^2$ is rotated in a time interval of 0.04s from a position where its plane is perpendicular to the earth's magnetic field to one where its plane is parallel to the field. The earth's magnetic field at the lab location is 6x10$^{-5}$T.

What is the total magnetic flux through the coil before it is rotated? What is the total magnetic flux through the coil after it is rotated? What is the average emf induced in the coil?

\[
\Phi_i = N \int \vec{B} \cdot d\vec{A} = NB \cos 0^\circ \int dA = NBA \\
= 200(6 \times 10^{-5} T)(0.0122m^2) = 1.46 \times 10^{-4} Tm^2
\]

\[
\Phi_f = NB \cos 90A = 0
\]

\[
\bar{\varepsilon} = -\frac{(\Phi_f - \Phi_i)}{\Delta t} = \frac{7.31 \times 10^{-7} Tm^2}{0.04s} = 3.66 \times 10^{-3} V
\]
Slide Wire Generator; work and power

Last lecture we found
\[ \mathcal{E} = BLv \]
we now ask what is
power dissipated, \( P_d \)
and power applied, \( P_a \)
assume total resistance is \( R \), then \( I = \frac{\mathcal{E}}{R} \) and
\[ P_d = I^2R = B^2L^2v^2/R \]

The power applied, \( P_a = Fv \), is the rate of work to the bar to the right against the force, \( F \), which is pointed to the left. \( F = ILB = (\mathcal{E}/R)LB = B^2L^2v/R \). Thus \( P_a = Fv = B^2L^2v^2/R = P_d \) and we find the power put into the moving the bar equals the heat dissipated in the circuit.

⇒ Power needed to move bar = Power dissipated in circuit
History of the Discovery of Electromagnetic Induction

In 1825, J. Colladon (who first measured sound speed), had a coil connected to a galvanometer and moved a large magnet nearby, BUT since his magnetic field would affect his galvanometer, he moved the meter to the next room. So when he moved the magnet and then went to the other room to check on the meter, the effect disappeared.

In 1930, Joseph Henry of Princeton (invented telegraph) observed induction, “conversion of magnetism to electricity”, but did not publish and did not recognize the vast importance of this observation. Later however, he observed and published evidence for self induction.

In 1831, Faraday observed and published evidence of electromagnetic induction and also invented the homopolar generator that converts continuous DC.

Note the unit of capacitance is “Farad” after Faraday and the unit of inductance is “Henry” after Henry.
**Lenz’s Law**

The direction of any magnetic induction effect is such as to oppose the cause of the effect.

⇒ Convenient method to determine \( I \) direction

Example if an external magnetic field on a loop is **increasing**, the induced current creates a field **opposite** that reduces the net field.

Example if an external magnetic field on a loop is **decreasing**, the induced current creates a field **parallel** to the that tends to **increase** the net field.
Lenz’s Law; consider moving magnet towards a wire loop.

- No current and no field
- Induced field is opposite
- Induced field is opposite

Stationary magnet

Moving N side of magnet into loop

Moving S side of magnet into loop

Repulsive force, like pushing two S poles together.
Lenz’s Law says direction creates field that opposes change in magnetic flux.

If we pull bar to right, the net magnetic flux in rectangle increases into screen, hence the $I$ direction must induce opposite $B$ field which is out of screen and is correct in drawing.

Suppose Lenz’s law were reversed, then $I$ would be reversed and $F$ would go right and the bar would be accelerated to the right, w/o need of external positive work and heat would be dissipated at the same time. The is violates cons. of Energy, so Lenz's law is correct.
Motional Electromotive Force

\[ \varepsilon = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A} = -\frac{d\Phi_B}{dt} \]

In Faraday’s Law, we can induce EMF in the loop when the magnetic flux, \( \Phi_B \), changes as a function of time. There are two Cases when \( \Phi_B \) is changing:

1) Change the magnetic field (non-constant over time)
2) Change or move the loop in a constant magnetic field

The slide wire generator is an example of #2 and the induction of EMF via moving parts of the loop is called, motional EMF.
Suppose we move a conducting bar in a constant B field, then a force $F = q \vec{v} \times \vec{B}$ moves $+$ charge up and $-$ charge down. The charge distribution produces an electric field and EMF, $E$, between a & b. This continues until equilibrium is reached.

$$
\vec{E} = \frac{\vec{F}}{q} = \frac{q \vec{v} \times \vec{B}}{q} = \vec{v} \times \vec{B} \\
e = \int_{a}^{b} \vec{E} \cdot d\vec{l} = vBl
$$

In effect the bar motional EMF is an equivalent to a battery EMF.
If the rod is on the U shaped conductor, the charges don’t build up at the ends but move around the U shaped portion. They produce an electric field in the U shaped circuit. The wire acts as a source of EMF - just like a battery. Called *motional electromotive force.*

\[ \varepsilon = \int_{a}^{b} \vec{E} \cdot d\vec{l} = vBl \]
Direct Current Homopolar Generator invented by Faraday

Rotate a metal disk in a constant perpendicular magnetic field. The charges in the disk when moving receive a radial force. The causes current to flow from center to point b.

\[ \varepsilon = \int_0^R \omega Br \, dr = \frac{1}{2} \omega BR^2 \]

Faraday’s original apparatus had the disk fixed with a meter and then he spun a magnet under the disk.

Speed of small radial segment of length \( dr \) at radius \( r \) is \( v = \omega r \)

Emf induced across this segment is \( d\varepsilon = vB \, dr = \omega Br \, dr \)
Faraday's Law (continued)

What causes current to flow in wire?
Answer: an $E$ field in the wire.

A changing magnetic flux not only causes an EMF around a loop but an induced electric field.
Can write Faraday’s Law:

$$\varepsilon = \oint E \cdot dl = -\frac{d}{dt} \int B \cdot dA = -\frac{d\Phi_B}{dt}$$

Remember for a long straight wire of length $l$, $V = EI$.

Note: For electric fields from static charges, the EMF from a closed path is always zero. Not true here.
There are two sources for electric fields!
Induced Electric Fields

Suppose we have electromagnetic
that has an increasing magnetic field
Using Faraday’s Law we predict,

\[ \oint E \cdot d\ell = -\frac{d}{dt} \oint B \cdot dA = -\frac{d\Phi_B}{dt} \]

If we take a circular path inside and
centered on the magnet center axis,
the electric field will be tangent to the
circle. (E field lines are circles.) NOTE
such an E field can never be made by
static charges

E field lines will look like an onion slice (w/o smell!!)

NOTE there are no wire loops, the E fields can appear w/o loops
If we place a loop there, a current would flow in the loop
Induced Electric Fields; example

If we have a solenoid coil with changing current there will be circular electric fields created outside the solenoid. It looks very much like the mag. field around a current carrying wire, but it is an E field and there are no wires or loops.

\[ E = \frac{d}{dt} \left( I \cdot \frac{dI}{dt} \right) \]

Note the E fields are predicted by Faraday eqn.

\[ \oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A} = -\frac{d\Phi_B}{dt} \]
Eddy Currents

Changing magnetic fields in metal induce eddy currents.

Example: Energy loss in transformers.
To reduce use laminations.

But eddy currents often useful.