Measuring $\alpha/\phi_2$ from $B \rightarrow \rho\rho$

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  ... complications due to $\Gamma_\rho \neq 0$
  ... present constraints on $\alpha - \alpha_{\text{eff}}$
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  ... $\propto (1 - f_0)$ and EW penguins
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Introduction

- Want to determine CKM angle $\alpha \equiv \phi_2 \equiv \arg \left[ - (V_{td}V_{tb}^*) / (V_{ud}V_{ub}^*) \right]$ from $S_{+-}$:

$$\frac{\Gamma(B^0_{\text{phys}}(t) \to \rho^+\rho^-) - \Gamma(B^0_{\text{phys}}(t) \to \rho^+\rho^-)}{\Gamma(B^0_{\text{phys}}(t) \to \rho^+\rho^-) + \Gamma(B^0_{\text{phys}}(t) \to \rho^+\rho^-)} = S_{+-} \sin(\Delta m t) - C_{+-} \cos(\Delta m t)$$

If amplitudes with a single weak phase dominate, then $S_{+-} = \sin 2\alpha$

- Summer '03 news: $B \to \rho\rho$ almost purely longitudinally polarized

$$\mathcal{B}(B \to \rho^0\rho^0)/\mathcal{B}(B \to \rho^-\rho^+) < 0.1 \quad (90\% \text{ CL})$$

[compare: $\mathcal{B}(B \to \pi^0\pi^0)/\mathcal{B}(B \to \pi^-\pi^+) \simeq 0.4$]

- $S'_{\rho^+\rho^-}$ may soon give accurate model independent determination of $\alpha$

... concentrate on differences compared to $B \to \pi\pi$
\[ B \to \pi\pi: \text{the problem} \]

- There are tree and penguin amplitudes, just like in \( B \to \psi K_S \)

  **“Tree”** \((b \to u\bar{u}d)\): \[
  A_T = V_{ub}V_{ud}^* A_{u\bar{u}d}
  \]

  **“Penguin”:** \[
  A_P = V_{tb}V_{td}^* P_t + V_{cb}V_{cd}^* P_c + V_{ub}V_{ud}^* P_u
  \]

  unitarity: \[
  \overline{A}_{\pi^+\pi^-} = V_{ub}V_{ud}^*[A_{u\bar{u}d} + P_u - P_t] + V_{cb}V_{cd}^*[P_c - P_t]
  \]

  same as Tree phase not suppressed

Define \( P \) and \( T \) by: \[
\overline{A}_{\pi^+\pi^-} = T_{+-} e^{-i\gamma} + P_{+-} e^{+i\beta}
\]

Two amplitudes with different weak- and possibly different strong phases; their values are not known model independently

- \( B(B \to K^-\pi^+) = (18.2 \pm 0.8) \times 10^{-6} \) to \( B(B \to \pi^-\pi^+) = (4.6 \pm 0.4) \times 10^{-6} \) ratio

  implies \(|P/T| \sim 0.3\), so need \( B \to \pi^0\pi^0 \)

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\[ ZL \rightarrow p.2 \]
Isospin Symmetry
Isospin analysis

\[ \pi \pi: \text{Bose statistics } \Rightarrow I = 0, 2 \]

\[ A_{ij} = T_{ij}e^{+i\gamma} + P_{ij}e^{-i\beta} \quad \]  
\[ \overline{A}_{ij} = T_{ij}e^{-i\gamma} + P_{ij}e^{+i\beta} \]

\[ A_{ij} \text{ [} \overline{A}_{ij} \text{]} \text{ denote } B^+, B^0 \text{ [} B^-, \overline{B}^0 \text{] decays} \]

\[ \tilde{A}^{ij} \equiv e^{2i\gamma} \overline{A}^{ij} \]

\[ \mathcal{B}(B \to \pi^0\pi^0) = (2.0 \pm 0.5) \times 10^{-6}, \text{ so triangles are not squashed} \]

\[ \rho \rho: \text{Mixture of } CP \text{ even/odd } (L = 0, 1, 2), \text{ but since } B \text{ is spin-0, the combined space and spin wave function of the two } \rho \text{'s is symmetric under particle exchange} \]

Bose statistics: isospin of \( \rho \rho \) symmetric under particle exchange \( \Rightarrow I = 1 \) absent

Same holds in transversity basis: isospin analysis applies for each \( \sigma \ (= 0, ||, \perp) \)
Complications due to $\Gamma_\rho \neq 0$

- Even for $\sigma = 0$ the possibility of $I = 1$ is reintroduced by finite $\Gamma_\rho$

Can have antisymmetric dependence on both the two $\rho$ mesons’ masses and on their isospin indices $\Rightarrow I = 1$ ($m_i =$ mass of a pion pair; $B =$ Breit-Wigner)

$$A \sim B(m_1)B(m_2)\frac{1}{2}\left[ f(m_1, m_2) \rho^+(m_1)\rho^-(m_2) + f(m_2, m_1) \rho^+(m_2)\rho^-(m_1) \right]$$

$$= B(m_1)B(m_2)\frac{1}{4}\left\{ \left[ f(m_1, m_2) + f(m_2, m_1) \right] \left[ \rho^+(m_1)\rho^-(m_2) + \rho^+(m_2)\rho^-(m_1) \right]_{I=0,2} ight. \\
+ \left. \left[ f(m_1, m_2) - f(m_2, m_1) \right] \left[ \rho^+(m_1)\rho^-(m_2) - \rho^+(m_2)\rho^-(m_1) \right]_{I=1} \right\}$$

If $\Gamma_\rho$ vanished, then $m_1 = m_2$ and $I = 1$ part is absent

E.g., no symmetry in factorization: $f(m_{\rho^-}, m_{\rho^+}) \sim f_\rho(m_{\rho^+}) F^{B \rightarrow \rho}(m_{\rho^-})$

- Could not rule out $O(\Gamma_\rho/m_\rho)$ contributions; no interference $\Rightarrow O(\Gamma_\rho^2/m_\rho^2)$ effects

How would they show up...?
Constraining $I = 1$

- Leading $I = 1$ term can be parameterized as [e.g., from $B_i H_j^{kl}(\rho^i_k \partial^2 \rho^j_l - \rho^i_l \partial^2 \rho^j_k)$]

$$\left[ c \frac{m_1 - m_2}{m_\rho} \right]^2 |B_\rho(m_1^2)B_\rho(m_2^2)|^2$$

Unfortunately, subleading $I = \text{even}$ contribution (cross-term) can have same form

$$\left[ a + b \frac{(m_1 - m_2)^2}{m_\rho^2} \right]^2 |B_\rho(m_1^2)B_\rho(m_2^2)|^2$$

Expect $a, b, c$ of the same order, so $ab/c^2 = O(1)$

- To constrain them, either:
  - Add new term to fit and check for stability of the $a^2$ term, for which the isospin analysis should be carried out ($I = 1$ absent for $\rho^0 \rho^0$)
  - Decrease the widths of the $\rho$ bands or impose a cut on $|m_1 - m_2|$ to eliminate possible $I = 1$ term
**Bounds on \( \delta (= \alpha - \alpha_{\text{eff}}) \)**

- Until the \( \mathcal{B}[B^0 \rightarrow (\rho^0 \rho^0)_\sigma] \) and \( \mathcal{B}[\overline{B}^0 \rightarrow (\rho^0 \rho^0)_\sigma] \) tagged rates are separately measured, one can bound \( \delta_{\sigma} \) using Babar & Belle data:

  \[
  \mathcal{B}_{+-} = \frac{1}{2} (|A_{+-}|^2 + |\bar{A}_{+-}|^2) = (27 \pm 9) \times 10^{-6}, \quad (f_0)_{+-} = 0.99^{+0.01}_{-0.07} \pm 0.03 \\
  \mathcal{B}_{+0} = \frac{1}{2} (|A_{+0}|^2 + |\bar{A}_{-0}|^2) = (26 \pm 6) \times 10^{-6}, \quad (f_0)_{+0} = 0.97^{+0.03}_{-0.07} \pm 0.04 \\
  \mathcal{B}_{00} = \frac{1}{2} (|A_{00}|^2 + |\bar{A}_{00}|^2) = (0.6^{+0.8}_{-0.6}) \times 10^{-6}, \quad [\mathcal{B}_{00} < 2.1 \times 10^{-6} \text{ (90\% CL)}] \\
  \]

  The first two measured, and upper bound on \( \mathcal{B}_{00} \) constrains \( \mathcal{B}_{00}^0 \ll \mathcal{B}_{+-}^0, \mathcal{B}_{+0}^0 \)

- Can bound \( \delta_0 \) the same way as in \( B \rightarrow \pi \pi \) [Grossman-Quinn / Gronau-London-Sinha-Sinha]:

  \[
  \cos 2\delta_0 \geq 1 - \frac{2\mathcal{B}_{00}^0}{\mathcal{B}_{+0}^0} + \frac{(\mathcal{B}_{+-}^0 - 2\mathcal{B}_{+0}^0 + 2\mathcal{B}_{00}^0)^2}{4\mathcal{B}_{+-}^0\mathcal{B}_{+0}^0} + \ldots
  \]

  The bound also depends on experimental constraints on \( C_{+-} \) and \( C_{00} \)

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*ZL — p. 6*
Resulting constraints

- Present data implies: $\cos 2\delta_0 > 0.83$ or $|\delta_0| < 17^\circ$ (90% CL)

\[ \text{Took } B_{+-} = B_{0-}^0 \text{ and } B_{+0} = B_{+0}^0 \text{ for simplicity} \]

[Fits done using CKMfitter package]
Presently allowed range of $CP$ asymmetries

- Small $\mathcal{B}_{00}/\mathcal{B}_{+0}$ also bounds direct CPV:

$$|C_{+-}^\sigma| < 2 \sqrt{\frac{\mathcal{B}_{00}}{\mathcal{B}_{+0}^{\sigma}}} - \left( \frac{\mathcal{B}_{00}^{\sigma}}{\mathcal{B}_{+0}^{\sigma}} \right)^2 \Rightarrow |C_{+-}^0| < 0.53 \ (90\% \ CL)$$

![Diagram of $C_{+-}$ vs $S_{+-}$ with the allowed range shaded in blue]
Corrections...
Corrections proportional to $1 - f_0$

- If $S_{+-}$ not measured in longitudinal mode alone, use $S_{+-} = \sum_\sigma f_\sigma S^\sigma_{+-}$ to bound

  $$|S^0_{+-} - S_{+-}| \leq (1 - f_0) (1 + |S^0_{+-}|)$$

Expect the error in estimating $S^0_{+-}$ to be smaller — to zeroth order in $|P^\sigma_{+-} / T^\sigma_{+-}|$
we have $S^\parallel_{+-} = -S^\perp_{+-} = S^0_{+-}$, so

$$S^0_{+-} - S_{+-} = (1 - f_0 - f_\parallel + f_\perp) S^0_{+-} + O[(1 - f_0) |P_{+-} / T_{+-}|]$$

- Non-resonant $B \rightarrow 4\pi$ decays and other resonances that decay to $4\pi$ could have opposite $CP$ than the dominant longitudinal mode

Contamination due to such contributions effectively included in the fit error of $1 - f_0$
In $B \to \pi\pi$ isospin analysis, neglecting EWP: one more observable than unknown. Including EWP: 2 new unknowns, but in $B \to \rho\rho$ yet one more observable, $S_{\rho^0\rho^0}$. Insufficient: constrains a combination of $|P_{ew}|$ and $\arg(P_{ew})$, but does not fix $\Delta 2\delta$. For now, consistent to neglect them: $\mathcal{A}_{\mp 0} = \frac{|\bar{A}_{-0}|^2 - |A_{+0}|^2}{|A_{-0}|^2 + |A_{+0}|^2} = -0.09 \pm 0.16$. Isospin violation due to $\rho - \omega - \phi$ mixing expected to be small.
Conclusions
Summary

- Present measurements of the various $B \to \rho \rho$ rates already give significant limits on the uncertainty in the extraction of $\alpha$ from the $CP$ asymmetry in $B \to \rho^+ \rho^-$.

- With higher precision, need to parameterize the data to allow for impact of possible $I = 1$ contributions that can affect results at the $O(\Gamma^2_\rho/m^2_\rho)$ level.

- $S^\prime_{\rho^+ \rho^-}$ may give best model independent determination of $\alpha$ for some time to come.

- Limit on theory error of $\alpha$ seems to be at the $5^\circ$ level (data may tell us it’s larger).