Students in a physics lab are trying to determine the value of Planck's constant $h$, using a photoelectric apparatus similar to the one shown in Figure 34-2. For a light source, the students use a helium–neon laser with tunable wavelength. The data that the students obtain for the maximum electron kinetic energy are

$$f = \frac{c}{\lambda} = \frac{5.51 \times 10^{14} \text{ Hz}}{544 \text{ nm}} = \frac{5.05 \times 10^{14} \text{ Hz}}{594 \text{ nm}} = \frac{4.97 \times 10^{14} \text{ Hz}}{604 \text{ nm}} = \frac{4.90 \times 10^{14} \text{ Hz}}{612 \text{ nm}} = \frac{4.74 \times 10^{14} \text{ Hz}}{633 \text{ nm}}$$

$K_{\text{max}}$ 0.360 eV 0.199 eV 0.156 eV 0.117 eV 0.062 eV

(a) Using a spreadsheet program or graphing calculator, convert the wavelengths to frequencies and plot $K_{\text{max}}$ versus frequency. (b) Use the graph to estimate the value of Planck's constant implied by the students' data. (Note: You may wish to use the linear regression function of your spreadsheet program or graphing calculator.) (c) Compare your result with the accepted value for Planck's constant.

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{frequency (}10^{13}\text{Hz)} & 55.1 & 50.5 & 49.7 & 49.0 & 47.4 \\
\text{\hspace{1cm} } & & & & & \\
\text{K}_{\text{max}} \text{ (eV)} & 0.360 & 0.199 & 0.156 & 0.117 & 0.062 \\
\hline
\end{array}
\]

Determining Planck's Constant

\[y = 0.0387x - 1.7664\]

\[\text{Thanks to Sarah Cook}\]

b) $h = \text{slope} = 3.87 \times 10^{-15} \text{ eVs}$

c) $\frac{\Delta h}{h} \approx \frac{4.136 \times 10^{-15} \text{ eVs}}{3.87 \times 10^{-15} \text{ eVs}} \\ = 0.06 \hspace{1cm} \text{(within 6%)}$
An X-ray photon of wavelength 6 pm that collides with an electron is scattered by an angle of 90°. (a) What is the change in wavelength of the photon? (b) What is the kinetic energy of the scattered electron?

\[ \lambda_2 - \lambda_1 = \frac{h}{m_e c} (1 - \cos \theta) = \lambda_c (1 - \cos \theta) \]

\[ \lambda_2 = \lambda_1 + \lambda_c (1 - \cos \theta) \]

\[ \lambda_c = 2.43 \text{ pm} \]

\[ \theta = 90° \cos \theta = 1 \]

\[ \lambda_2 = \lambda_1 + \lambda_c = 6 \text{ pm} + 2.43 \text{ pm} = 8.43 \text{ pm} \]

\[ KE_e = E_{\nu_1} - E_{\nu_2} = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2} = 1.24 \times 10^6 \text{ eV pm} \left( \frac{1}{6 \text{ pm}} - \frac{1}{8.43 \text{ pm}} \right) \]

\[ = 59.6 \times 10^3 \text{ eV} = 59.6 \text{ keV} \]

The distance between Li\(^+\) and Cl\(^+\) ions in a LiCl crystal is 0.257 nm. Find the energy of electrons that have a wavelength equal to this spacing.

\[ \lambda = \frac{h}{p} \implies p = \frac{h}{\lambda} \]

\[ E = \frac{p^2}{2m_e} = \frac{h^2}{2m_e \lambda^2} = \frac{(hc)^2}{2m_e c^2 \lambda^2} \]

\[ E = \frac{(hc)^2}{2m_e c \lambda} = \frac{(1.240 \times 10^3 \text{ eV nm})^2}{2 \times 0.511 \times 10^6 \text{ eV} \times (0.257 \text{ nm})^2} \]

\[ = \frac{1.54 \times 10^4 \text{ eV} \cdot \text{nm}}{1.02 \times 10^4 \text{ eV} \times 0.066 \text{ nm}^2} = 22.8 \text{ eV} \]
• What is the de Broglie wavelength of a neutron with speed $10^6 \text{ m/s}$?

\[
\lambda = \frac{h}{mv} = \frac{h}{mc^2} \cdot \frac{c}{v} = \frac{1.24 \times 10^6 \text{ eV pm}}{940 \times 10^8 \text{ eV}} \cdot \frac{3 \times 10^8 \text{ m/s}}{1 \times 10^6 \text{ m/s}} = 0.396 \text{ pm}
\]

• A neutron has a kinetic energy of 10 MeV. What size object is necessary to observe neutron diffraction effects? Is there anything in nature of this size that could serve as a target to demonstrate the wave nature of 10-MeV neutrons?

\[
\lambda = \frac{h}{P} = \frac{h}{\sqrt{2m_nE}} = \frac{hc}{\sqrt{2m_n c^2 E}} = \frac{1.24 \times 10^{-12} \text{ MeV m}}{\sqrt{2 \times 940 \text{ MeV} \cdot 10 \text{ MeV}}} = \frac{1.24 \times 10^{-12} \text{ m}}{137 \text{ MeV}} = 9 \times 10^{-15} \text{ m}
\]

Size of an atomic nucleus

When 10 MeV neutrons scatter from atomic nuclei, wave effects are apparent.
A particle in a one-dimensional box is in the first excited state \((n = 2)\). (a) Sketch \(\psi^2(x)\) versus \(x\) for this state. (b) What is the expectation value \(\langle x \rangle\) for this state? (c) What is the probability of finding the particle in some small region \(dx\) centered at \(x = L/2\)? (d) Are your answers for Part (b) and Part (c) contradictory? If not, explain.

\[
\langle x \rangle = \frac{2}{L} \int_0^L x \sin^2 \frac{2\pi x}{L} dx
\]

Let \(\Theta = \frac{2\pi x}{L}\) : \(x = \frac{x}{2\pi} \Theta\), \(dx = \frac{1}{2\pi} d\Theta\)

\[
\langle x \rangle = \frac{2}{L} \left(\frac{L}{2\pi} \right)^2 \int_0^{2\pi} \sin^2 \Theta d\Theta = \frac{L}{2\pi} \left(\frac{\Theta^2}{4} - \frac{\Theta \sin 2\Theta}{4} - \frac{\cos 2\Theta}{8}\right) \bigg|_0^{2\pi} = \frac{L}{2}\pi - \frac{L}{2}
\]

(c) \(P(x = \frac{1}{2}) dx = \psi^2(x = \frac{1}{2}) dx = \frac{2}{L} \sin^2 \frac{2\pi}{4} dx = 0\)

(d) It's ok; \(\frac{1}{2}\) is the average over all measurements, not any one measurement.

A one-dimensional box is on the \(x\)-axis in the region of \(0 \leq x \leq L\). A particle in this box is in its ground state. Calculate the probability that the particle will be found in the region (a) \(0 < x < \frac{1}{2}L\), (b) \(0 < x < L/3\), and (c) \(0 < x < 3L/4\).

\[
\text{Prob}(A \rightarrow B) = \int_A^B \psi^2(x) dx = \frac{3}{L} \int_{\pi/4}^{\pi/2} \sin^2 \frac{2\pi x}{L} dx
\]

Let \(\Theta = \frac{x}{2\pi}\) : \(x = \frac{x}{2\pi} \Theta\), \(dx = \frac{1}{2\pi} d\Theta\)

\[
\text{Prob}(A \rightarrow B) = \frac{2}{L} \cdot \frac{1}{2\pi} \int_{\pi/4}^{\pi/2} \sin^2 \Theta d\Theta = \frac{2}{2\pi} \left(\frac{\Theta}{2} - \frac{\sin 2\Theta}{4}\right) \bigg|_{\pi/4}^{\pi/2}
\]

(a) \(\text{Prob}(0 \rightarrow \frac{1}{2}) = \frac{2}{\pi} \left(\frac{\Theta}{2} - \frac{\sin 2\Theta}{4}\right) \bigg|_{0}^{\pi/2} = \frac{\Theta}{\pi} - \frac{\sin 2\Theta}{2\pi} \bigg|_{0}^{\pi/2} = \frac{1}{2}\)

(b) \(\text{Prob}(0 \rightarrow \frac{1}{3}) = \frac{\Theta}{\pi} - \frac{\sin 2\Theta}{2\pi} \bigg|_{0}^{\pi/3} = \frac{1}{3} - \frac{\sqrt{3}}{4\pi} = 0.195\)

(c) \(\text{Prob}(0 \rightarrow \frac{3\pi}{4}) = \frac{\Theta}{\pi} - \frac{\sin 2\Theta}{2\pi} \bigg|_{0}^{3\pi/4} = \frac{3}{4} + \frac{1}{2\pi} = 0.91\)
The classical probability distribution function for a particle in a box of length $L$ is given by $P(x) = 1/L$. Use this to show that $\langle x \rangle = L/2$ and $\langle x^2 \rangle = L^2/3$ for a classical particle in the box described in Problem 60.

$$\rho(x) = \frac{1}{L} \quad \langle x \rangle = \frac{1}{L} \int_0^L x \rho(x) \, dx = \frac{1}{L} \int_0^L x \, dx = \frac{1}{L} \frac{x^2}{2} \bigg|_0^L = \frac{1}{L} \frac{L^2}{2} = \frac{L}{2}$$

$$\langle x^2 \rangle = \frac{1}{L} \int_0^L x^2 \rho(x) \, dx = \frac{1}{L} \int_0^L x^2 \, dx = \frac{1}{L} \frac{x^3}{3} \bigg|_0^L = \frac{1}{L} \frac{L^3}{3} = \frac{L^2}{3}$$

When light of wavelength $\lambda_1$ is incident on the cathode of a photoelectric tube, the maximum kinetic energy of the emitted electrons is 1.8 eV. If the wavelength is reduced to $\lambda_1/2$, the maximum kinetic energy of the emitted electrons is 5.5 eV. Find the work function $\phi$ of the cathode material.

$$KE_1 = \frac{hc}{\lambda_1} - \phi \quad KE_1 = 1.8 \text{ eV}$$

$$KE_2 = \frac{hc}{\lambda_2} - \phi = \frac{2hc}{\lambda_1} - \phi$$

$$KE_2 - 2KE_1 = + \phi$$

$$5.5 \text{ eV} - 2 \times 1.8 \text{ eV} = \phi$$

$$\phi = 5.5 \text{ eV} - 3.6 \text{ eV} = 1.9 \text{ eV}$$