The sun radiates energy at the rate of approximately $4 \times 10^{26}$ W. Assume that this energy is produced by a reaction whose net result is the fusion of 4 H nuclei to form 1 He nucleus, with the release of 25 MeV for each He nucleus formed. Calculate the sun's loss of mass per day.

$$E_{\text{day}} = P_0 \cdot T_{\text{day}} = \Delta M c^2 \implies \Delta m = \frac{P_0 \cdot T_{\text{day}}}{c^2}$$

$$\Delta M = \frac{4 \times 10^{26} \text{ J/s} \times 24 \text{ hr/day} \times 3600 \text{ s/hr}}{9 \times 10^{16} \text{ m}^2/\text{s}^2}$$

$$\Delta M = \frac{3.46 \times 10^{39}}{9 \times 10^{16} \text{ m}^2/\text{s}^2} = 3.8 \times 10^{14} \text{ kg}$$
Two events in $S$ are separated by a distance $D = x_2 - x_1$ and a time $T = t_2 - t_1$. (a) Use the Lorentz transformation to show that in frame $S'$, which is moving with speed $v$ relative to $S$, the time separation is $t_2' - t_1' = \gamma(T - vD/c^2)$. 

(b) Show that the events can be simultaneous in frame $S'$ only if $D$ is greater than $cT$. (c) If one of the events is the cause of the other, the separation $D$ must be less than $cT$, since $D/c$ is the smallest time that a signal can take to travel from $x_1$ to $x_2$ in frame $S$. Show that if $D$ is less than $cT$, $t_2'$ is greater than $t_1'$ in all reference frames. This shows that if the cause precedes the effect in one frame, it must precede it in all reference frames.

(d) Suppose that a signal could be sent with speed $c' > c$ so that in frame $S$ the cause precedes the effect by the time $T = D/c'$. Show that there is then a reference frame moving with speed $v$ less than $c$ in which the effect precedes the cause.

\[ \begin{align*}
\text{a)} \quad t_2' - t_1' &= \gamma(t_2 - t_1 - \frac{v}{c^2} (x_2 - x_1)) = \gamma(T - \frac{v}{c^2} D) \\
\text{b)} \quad \text{for} \quad t_2' - t_1' &= 0 \Rightarrow T - \frac{v}{c^2} D = 0 \Rightarrow D = \frac{c}{\sqrt{v^2 - 1}} \cdot cT \\
& \quad \text{if} \quad v < c \text{ always} \quad D > cT \\
\text{c)} \quad t_2' - t_1' &= \gamma(T - \frac{v}{c^2} D) = \gamma T(1 - \frac{v}{c} \cdot \frac{D}{cT}) \\
& \quad \text{if} \quad \frac{D}{cT} < 1 \quad t_2' - t_1' > 0 \quad \text{for} \quad \frac{v}{c} \text{ always} < 1 \\
\text{d)} \quad t_2' - t_1' &= \gamma(T - \frac{v}{c^2} D) = \gamma T(1 - \frac{v}{c} \cdot \frac{D}{cT}) \\
& \quad \text{if} \quad T = \frac{D}{c^2}, \quad c' > c \quad t_2' - t_1' = \gamma T(1 - \frac{v}{c} \cdot \frac{D}{c}) \\
& \quad t_2' - t_1' < 0 \quad (\text{i.e.} \ t_2' \text{ precedes} \ t_1') \\
& \quad \text{for} \quad \frac{v}{c} \frac{c'}{c} > 1 \Rightarrow \gamma > \left(\frac{c}{c'}\right) \cdot c
\end{align*} \]
A spaceship, at rest in a certain reference frame $S$, is given a speed increment of $0.50c$ (call this boost 1). Relative to its new rest frame, the spaceship is given a further $0.50c$ increment 10 seconds later (as measured in its new rest frame; call this boost 2). This process is continued indefinitely, at 10-s intervals, as measured in the rest frame of the ship. (Assume that the boost itself takes a very short time compared to 10 s.)

(a) Using a spreadsheet program, calculate and graph the velocity of the spaceship in reference frame $S$ as a function of the boost number for boost 1 to boost 10. 
(b) Graph the gamma factor the same way. 
(c) How many boosts does it take until the velocity of the ship in $S$ is greater than $0.999c$? 
(d) How far has the spaceship moved after 5 boosts, as measured in reference frame $S$? What is the average speed of the spaceship (between boost 1 and boost 5) as measured in $S$?

\[ u_x = \frac{V + u_x}{1 + \frac{V u_x}{c^2}} \]

<table>
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<tr>
<th>Step</th>
<th>$u_x$</th>
<th>$u_y = \frac{V + u_x}{1 + \frac{V u_x}{c^2}}$</th>
<th>$V_{av}$</th>
<th>$\gamma = \frac{1}{\sqrt{1 - u_y^2/c^2}}$</th>
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<td>1</td>
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<td>10</td>
<td></td>
<td>0.9999c</td>
<td>0.9997c</td>
<td>70.7</td>
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</table>

\[ D = \sum_{k=1}^{5} V_{av} \cdot \Delta t \]

\[ = c \Delta t \left( 0.25 + 0.65 + 0.865 + 0.95 + 0.98 \right) \]

\[ = 3 \cdot 10^9 \text{m} \cdot 3.69 \]

\[ = 1.11 \times 10^{10} \text{ m} \]
The $K^0$ particle has a mass of 497.7 MeV/c$^2$. It decays into a $\pi^-$ and $\pi^+$, each with mass 139.6 MeV/c$^2$. Following the decay of a $K^0$, one of the pions is at rest in the laboratory. Determine the kinetic energy of the other pion and of the $K^0$ prior to the decay.

\[
\ln K^0 \text{ rest frame: } \quad \begin{array}{c}
\pi^- \ \leftrightarrow \ K^0 \rightarrow \pi^+
\end{array}
\]

\[
E_{\pi^-}^{cm} = E_{\pi^+}^{cm} = E_{K^0}^{cm}, \quad E_{\pi^+}^{cm} + E_{\pi^-}^{cm} = 2E_{\pi}^{cm} = m_{K^0}c^2
\]

\[
\Rightarrow E_{\pi}^{cm} = \frac{m_{K^0}}{2} m_{\pi} c^2 = \frac{1}{2} m_{K^0} c^2
\]

\[
\frac{1}{(\delta_{\pi}^{cm})^2} = 1 - (\frac{V_{\pi}^{cm}}{c})^2 = \frac{4m_{\pi}^2}{m_{K^0}^2} = V_{\pi}^{cm} = c \sqrt{1 - \frac{4m_{\pi}^2}{m_{K^0}^2}}
\]

In lab $\pi^-$ is at rest. Thus, transformation to the lab has $\Lambda = V_{\pi}^{cm} \Rightarrow \nu_{\pi}^{lab} = \frac{V_{\pi}^{cm} + V_{\pi}^{cm}}{1 + \frac{V_{\pi}^{cm}}{c} V_{\pi}^{cm}}$

For the $K^0$: $\nu_{K^0}^{cm} = 0 \Rightarrow \nu_{K^0}^{lab} = \nu_{\pi}^{cm}$

\[
\nu_{K^0}^{lab} = \sqrt{1 - \frac{4m_{\pi}^2}{m_{K^0}^2}} \times c = 0, 828c
\]

For the $\pi^+$: $\nu_{\pi^+}^{cm} = \nu_{\pi}^{cm} \Rightarrow \nu_{\pi^+}^{lab} = \frac{2V_{\pi}^{cm}}{1 + (\frac{V_{\pi}^{cm}}{c})^2}$

\[
\nu_{\pi^+}^{lab} = 0, 982c
\]

\[
\gamma_{\pi^+}^{lab} = 5, 29; \quad \gamma_{\pi^+}^{lab} = 1, 783
\]

\[
K_{\pi}^{lab} = (\gamma_{\pi}^{lab}) m_{\pi} c^2 = 608 \text{ MeV} \quad ; \quad K_{K^0}^{lab} = (\gamma_{K^0}^{lab}) m_{K^0} c^2 = 389 \text{ MeV}
\]
An antiproton has the same rest energy as a proton. It is created in the reaction $p + p \rightarrow p + p + p + \bar{p}$. In an experiment, protons at rest in the laboratory are bombarded with protons of kinetic energy $K_L$, which must be great enough so that kinetic energy equal to $2mc^2$ can be converted into the rest energy of the two particles. In the frame of the laboratory, the total kinetic energy cannot be converted into rest energy because of conservation of momentum. However, in the zero-momentum reference frame in which the two initial protons are moving toward each other with equal speed $u$, the total kinetic energy can be converted into rest energy.

(a) Find the speed of each proton $u$ so that the total kinetic energy in the zero-momentum frame is $2mc^2$. (b) Transform to the laboratory's frame in which one proton is at rest, and find the speed $u'$ of the other proton. (c) Show that the kinetic energy of the moving proton in the laboratory’s frame is $K_L = 6mc^2$.

\[ E = K_{beam} + m_pc^2 + K_{tgt} + m_pc^2 = 4m_pc^2 \]

Initial \[ K_{beam} = K_{tgt} = K_p^c \]

Final \[ 2K_p^c + 2m_pc^2 = 4m_pc^2 \]

\[ K_p^c = m_pc^2 \]

\[ \Rightarrow \frac{1}{(\frac{V_p^c}{c})^2} = 1 - \left(\frac{V_p^c}{c}\right)^2 = \frac{1}{4} \]

\[ \Rightarrow \left(\frac{V_p^c}{c}\right) = \frac{3}{4} \]

\[ \Rightarrow V_p^c = \frac{\sqrt{3}}{2} c \]

(0.87c)

\[ a) \]

\[ \Rightarrow \frac{1}{(\frac{V_p^c}{c})^2} = 1 - \left(\frac{V_p^c}{c}\right)^2 = \frac{1}{4} \]

\[ \Rightarrow \left(\frac{V_p^c}{c}\right) = \frac{3}{4} \]

\[ \Rightarrow V_p^c = \frac{\sqrt{3}}{2} c \]

(0.87c)

\[ b) \] Transform to lab \[ V_{lab} = 0 \]

\[ V_{beam} = \frac{V_{lab} + V_{beam}}{1 + \frac{V_{beam}c}{V_{lab}}^2} \]

\[ V_{beam} = V_{p}^c \]

\[ V_{beam} = \frac{4\sqrt{3}}{5} c = 0.99 c \]

\[ c) \]

\[ K_{beam}^c = (\frac{1}{2})m_pc^2 = (\frac{1}{\left(\frac{1}{\left(\frac{\sqrt{3}}{2} c\right)^2} - 1\right)}) m_pc^2 = 6m_pc^2 \]