Two infinite vertical planes of charge are parallel to each other and are separated by a distance $d = 4 \text{ m}$. Find the electric field to the left of the planes, to the right of the planes, and between the planes (a) when each plane has a uniform surface charge density $\sigma = +3 \text{ C/m}^2$ and (b) when the left plane has a uniform surface charge density $\sigma = +3 \text{ C/m}^2$ and that of the right plane is $\sigma = -3 \text{ C/m}^2$. Draw the electric field lines for each case.

\[ \text{a) } \sigma_A = +3 \text{ C/m}^2 \quad \sigma_B = +3 \text{ C/m}^2 \]

\[ \vec{E}_{\text{left}} = \left( \vec{a} \cdot 2 \pi k \sigma_A - 2 \pi k \sigma_B \right) \hat{x} = \left( \frac{2 \pi \times 9 \times 10^9 \text{ N} \cdot \text{m}^2}{\epsilon_0} \left( 3 \text{ C/m}^2 + 3 \text{ C/m}^2 \right) \right) \hat{x} = \frac{-3.4 \times 10^5 \text{ N/C} \hat{x}}{\epsilon_0} \]

\[ \vec{E}_{\text{middle}} = -2 \pi k \left( \sigma_A - \sigma_B \right) \hat{x} = 0 \]

\[ \vec{E}_{\text{right}} = 2 \pi k \left( \sigma_A + \sigma_B \right) \hat{x} = 3.4 \times 10^5 \text{ N/C} \hat{x} \]

\[ \text{b) } \sigma_A = +3 \text{ C/m}^2 \quad \sigma_B = -3 \text{ C/m}^2 \]

\[ \vec{E}_{\text{left}} = -2 \pi k \left( \sigma_A + \sigma_B \right) \hat{x} = 0 \]

\[ \vec{E}_{\text{right}} = 2 \pi k \left( \sigma_A + \sigma_B \right) \hat{x} = 3.4 \times 10^5 \text{ N/C} \hat{x} \]

\[ \vec{E}_{\text{middle}} = 2 \pi k \left( \sigma_B - \sigma_A \right) \hat{x} = -3.4 \times 10^5 \text{ N/C} \hat{x} \]

An electric field is $\vec{E} = 300 \text{ N/C} \hat{x}$ for $x > 0$ and $\vec{E} = -300 \text{ N/C} \hat{x}$ for $x < 0$. A cylinder of length 20 cm and radius 4 cm has its center at the origin and its axis along the $x$ axis such that one end is at $x = +10 \text{ cm}$ and the other is at $x = -10 \text{ cm}$. (a) What is the flux through each end? (b) What is the flux through the curved surface of the cylinder? (c) What is the net outward flux through the entire cylindrical surface? (d) What is the net charge inside the cylinder?

\[ \text{a) Flux left end} = \text{Flux right end} = \vec{E} \pi r^2 = 300 \text{ N/C} \cdot \pi \cdot (0.04 \text{ m})^2 = 1.51 \text{ Nm}^2/\text{C} \]

\[ \text{b) Flux thru barrel} = 0 \cdot \hat{n} \]

\[ \text{c) } \phi E \cdot dA = 2 \times 1.51 \text{ Nm}^2/\text{C} = 3.02 \text{ Nm}^2/\text{C} \]

\[ \text{d) } Q_{\text{net}} = \varepsilon_0 \int \phi E \cdot dA = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2 \times 3.02 \text{ Nm}^2/\text{C} = 2.67 \times 10^{-11} \text{ C} \]
32. A sphere of radius 6 cm carries a uniform volume charge density \( \rho = 450 \text{nC/m}^3 \). (a) What is the total charge of the sphere? Find the electric field at (b) \( r = 2 \) cm, (c) \( r = 5.9 \) cm, (d) \( r = 6.1 \) cm, and (e) \( r = 10 \) cm. Compare your answers with Problem 31.

\[
a) \quad Q_{\text{tot}} = \int \rho \, dV = \rho \int dV = \rho \cdot 4\pi \frac{4}{3} \pi (0.06 \text{ m})^3 = 0.41 \text{nC}
\]

b) Gaussian Surface: Sphere \( r = 0.02 \text{ m} \) centered at origin
\[
\Phi = E_r \cdot 4\pi r^2 = \frac{1}{\varepsilon_0} \rho \cdot \frac{4}{3} \pi r^{-3} \Rightarrow E_r = \frac{\rho r}{3\varepsilon_0}
\]

\[
c) \quad \text{Use } r = 0.059 \text{ m sphere: } \Phi = \frac{450 \text{nC/m}^3 \cdot 0.059 \text{ m} \cdot r}{3 \times 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2} = 1003 \text{nC}
\]

d) \quad \text{Use } r = 0.061 \text{ m sphere: } \Phi = \frac{450 \text{nC/m}^3 \cdot 0.061 \text{ m} \cdot r}{3 \times 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2} = 992 \text{nC}
\]

e) \quad \text{Use } r = 0.1 \text{ m sphere: } E_r = \frac{9 \times 10^9 \text{Nm}^2}{(0.1 \text{ m})^2} = 369 \text{nC}
\]

40. A point charge of +5 nC is located at the origin. This charge is surrounded by a spherically symmetric negative charge distribution with volume density \( \rho(r) = C e^{-r/a} \).

(a) Find the constant \( C \) in terms of \( a \) if the total charge of the system is zero. (b) What is the electric field at \( r = a \)?

\[
a) \quad +5 \text{nC} + \int_0^a \rho \, dV = 0 \Rightarrow C \int_0^\infty e^{-r/a} 4\pi r^2 \, dr = -5 \text{nC} = 4\pi C \int_0^\infty e^{-r/a} r^2 \, dr
\]

\[
4\pi C \cdot 2a^3 = -5 \text{nC} \Rightarrow C = \frac{-5 \text{nC}}{8\pi a^3} \quad \text{from integral table}
\]

b) \quad Q(r \leq a) = 4\pi C \int_0^r e^{-\tau/a} \tau^2 \, d\tau = -4\pi C a^3 e^{-r/a} \frac{r^2}{a^2} + \frac{2r}{a} \bigg|_{\tau=0}^{r=a}

\[
E_r(r=a) = \frac{1}{4\pi \varepsilon_0} \frac{Q_{\text{in}}}{{a}^2} = \frac{1}{a^2} \left( +5 \text{nC} - 0.4 \text{nC} \right) = \frac{9 \times 10^9 \text{Nm}^2}{a^2} \cdot 4.6 \text{nC} = \frac{4.14 \times 10^4 \text{Nm}^2}{a^2} \]
Consider two infinitely long, concentric cylindrical shells. The inner shell has a radius $R_1$ and carries a uniform surface charge density of $\sigma_1$, and the outer shell has a radius $R_2$ and carries a uniform surface charge density of $\sigma_2$. (a) Use Gauss's law to find the electric field in the regions $r < R_1$, $R_1 < r < R_2$, and $r > R_2$. (b) What is the ratio of the surface charge densities $\sigma_2/\sigma_1$ and their relative signs if the electric field is zero at $r > R_2$? What would the electric field between the shells be in this case? (c) Sketch the electric field lines for the situation in (b) if $\sigma_1$ is positive.

\[
\begin{align*}
\text{a) } & \quad r < R_1 \quad E_r \cdot 2\pi rl = \frac{1}{\varepsilon_0} Q \sin = 0 \quad E_r (r < R_1) = 0 \\
\text{ } & \quad R_1 < r < R_2 \quad E_r \cdot 2\pi rl = \frac{1}{\varepsilon_0} Q \sin = \frac{1}{\varepsilon_0} \sigma_1 \cdot 2\pi R_1 l \Rightarrow E_r (R_1 < r < R_2) = \frac{\sigma_1}{\varepsilon_0 r} \\
\text{ } & \quad r > R_2 \quad E_r \cdot 2\pi rl = \frac{1}{\varepsilon_0} Q \sin = \frac{1}{\varepsilon_0} 2\pi l (\sigma_1 R_1 + \sigma_2 R_2) \Rightarrow E_r (r > R_2) = \frac{\sigma_1 R_1 + \sigma_2 R_2}{\varepsilon_0 r} \\
\text{b) for } & \quad E_r (r > R_2) = 0 \quad \sigma_1 R_1 + \sigma_2 R_2 = 0 \\
\text{ } & \quad \Rightarrow \sigma_2 R_2 = -\sigma_1 R_1 \\
\text{ } & \quad \Rightarrow \frac{\sigma_2}{\sigma_1} = -\frac{R_1}{R_2} \\
\text{E}(r < R_2) \text{ would be the same (as answer to part a)}
\end{align*}
\]

(c) See sketch above.

A charge of 6 nC is placed uniformly on a square sheet of nonconducting material of side 20 cm in the yz plane. (a) What is the surface charge density $\sigma$? (b) What is the magnitude of the electric field just to the right and just to the left of the sheet? (c) The same charge is placed on a square conducting slab of side 20 cm and thickness 1 mm. What is the surface charge density $\sigma$? (Assume that the charge distributes itself uniformly on the large square surfaces.) (d) What is the magnitude of the electric field just to the right and just to the left of each face of the slab?

\[
\begin{align*}
\text{a) } & \quad \sigma = \frac{Q}{A} = \frac{6 \text{ nC}}{(0.2 \text{ m})^2} = 150 \text{ nC/m}^2 \\
\text{b) } & \quad \overrightarrow{E}_L = -2\pi k \sigma \hat{k} = -2\pi \cdot 9 \times 10^9 \text{ N/m}^2 \hat{k} \cdot 150 \text{ nC/m}^2 \hat{k} = -8.48 \times 10^8 \text{ N/C} \hat{k} \\
\text{c) } & \quad \sigma = \frac{1}{2} \frac{Q}{A} = 75 \text{ nC/m}^2 \\
\text{d) Same as b (i.e. } & \quad \overrightarrow{E}_L = -8.48 \times 10^8 \text{ N/C} \hat{k} \\
\text{E} & \quad \text{inside conduct (look at answer to 22-20a))}
\end{align*}
\]