Take-Home Midterm Exam #3, Part A

NO exam time limit. Calculator required. All books and notes are allowed, and you may obtain help from others. Complete all of Part A AND Part B.

For multiple-choice questions, circle the letter of the one best answer (unless more than one answer is asked for). For fill-in-the-blank and multiple-choice questions, you do NOT need to show your work.

Show your work on all free-response questions. Be sure to use proper units and significant figures in your final answers.

Ignore friction and air resistance in all problems, unless told otherwise.

Physical constants: It’s an open-book test, so you can look them up in your textbook!

Useful conversions: It’s an open-book test, so you can look them up in your textbook!

1. (4 pts.) Convert the following quantities into the given units. Fill in the blanks. (You do NOT need to show your work.) Use scientific notation where appropriate (very large or very small values), and express all final values to appropriate significant figures.

   a. 885 GW = __________________________ kW

   b. 5.89 × 10⁻⁷ m = __________________________ nm

   c. 60.5 g·cm² = __________________________ kg·m²

   d. 4 × 10²¹ g/m³ = __________________________ kg/cm³ (this is roughly the density of atomic nuclei and of neutron stars)

2. Two ice skaters (m₁ = 85 kg and m₂ = 55 kg) in a pairs-skating tournament are both initially holding on to each other while traveling to the right at 4.0 m/s. Skater #1 then throws skater #2 to the right in a graceful toss. Immediately afterward, skater #2 is traveling to the right at 9.0 m/s. (Ignore friction.)

   a. (2 pts.) What is the final velocity of skater #1? Is it to the right or left? __________________________

   b. (2 pts.) What was the change in total kinetic energy of the skaters due to the throw? __________________________

   Was net kinetic energy gained or lost?

   c. (2 pts.) If the throw took 0.50 s, what was the magnitude of the impulsive force on skater #2? __________________________

   d. (1 pt.) The impulsive force acting on skater #1 was __________________________ the force on skater #2.

      A. less than               B. the same as               C. greater than

   e. (1 pt.) Find the final velocity (after the throw) of the center-of-mass of the two skaters: __________________________
3. A hardcover physics textbook (length $\ell = 28$ cm, width $w = 22$ cm, thickness $h = 4.5$ cm) is sealed shut (gasp!) so that it can be rotated about any desired axis. It forms a rectangular solid of uniform density and total mass 1.6 kg.

All three of the spin axes shown above pass through the book’s center-of-mass, and all three axes run parallel/perpendicular to the sides of the book.

a. (1 pt.) Which one of the above three rotation axes creates the largest moment of inertia?
   A. A   B. B   C. C   D. two or more are tied for the largest value

b. (1 pt.) Which one of the above three rotation axes creates the smallest moment of inertia?
   A. A   B. B   C. C   D. two or more are tied for the smallest value

c. (2 pts.) Find the book’s moment of inertia about axis B: ____________________
   Express your final answer in MKS units.

d. (1 pt.) How would moving the axis closer to one corner of the book change your answer to part (c)?
   A. increase moment of inertia   B. decrease moment of inertia   C. no change

**Hint for part (d) & Bonus:**
The parallel-axis theorem states:

$$I_z = I_{c.m.} + M d^2$$

where:

- $I_{c.m.}$ = moment of inertia about some axis that passes through the object’s center-of-mass
- $I_z$ = moment of inertia about a different axis, parallel to that of $I_{c.m.}$, but offset by distance $d$
BONUS (1 pt.): **Draw a rotation axis** through the book below that creates the *greatest possible moment of inertia* for the object. (Your axis MUST touch the book, but it need not pass through the center-of-mass.)

![Book Diagram]

**e.** (2 pts.) Find the book’s **density**, in MKS units: ________________________

**f.** (1 pt.) As long as the book remains sealed closed, would it **float** or **sink** in (fresh) water?

A. float  B. sink  C. neither; the book is neutrally buoyant
4. A particular blender has five different angular-speed settings for its blades:

A. Stir \( \omega_A = 4000 \text{ rpm} \) (assume 2 sig. figs.)
B. Chop \( \omega_B = \sqrt{2} \omega_A \)
C. Mix \( \omega_C = \sqrt{3} \omega_A \)
D. Purée \( \omega_D = 2 \omega_A \)
E. Liquefy \( \omega_E = 3 \omega_A \)

Assume that the metal blade assembly (which looks like a small propeller or fan) has a constant moment of inertia.

a. (1 pt.) Which setting gives the blades 2 times the frequency of the “Stir” setting?
B. Chop
D. Purée
F. none of the above

b. (1 pt.) Which setting gives the blades 2 times the angular momentum of the “Stir” setting?
B. Chop
D. Purée
F. none of the above

c. (1 pt.) Which setting gives the blades 3 times the rotational kinetic energy of the “Stir” setting?
C. Mix
E. Liquefy
F. none of the above

d. (2 pts.) Find the period of the blades on the “Stir” setting: ______________________

e. (3 pts.) The entire blade assembly has a moment of inertia \( I = 4.1 \times 10^{-5} \text{ kg} \cdot \text{m}^2 \). If speeding up from “Stir” to “Liquefy” requires 0.55 s, find the average net torque acting on the blades during that time (assuming constant angular acceleration): ______________________

f. (2 pts.) How many revolutions do the blades make during the acceleration in part (e)? __________________

5. A particular apparatus allows you to launch rolling objects at the bottom of a ramp so that they always start with the same initial linear velocity \( v_0 \), regardless of the mass, radius, or moment of inertia of the objects.

Each object rolls (without slipping) up the ramp until it momentarily comes to rest at some height \( H \) that varies from object to object.

a. Suppose you have a solid sphere and a cylinder, both with the same mass \( M \) and the same radius \( R \). (Each has uniform density inside, although they are filled with different densities.)

(i). (1 pt.) Which object will reach a greater final height, \( H \)?
A. sphere
B. cylinder
C. same height for both

(ii). (3 pts.) Find the ratio of the final heights, \( H \), for the two objects (either \( H_s/H_c \) or \( H_c/H_s \)): __________________________
You may give your final answer as a simplified pure rational number, or as a decimal (to 3 sig. figs.).

b. (1 pt.) Suppose you have two solid spheres, one with a larger radius \( 2R \), and other with smaller radius \( R \). Their masses are both the same, \( M \). (Each has uniform density inside, although they are filled with different densities.) Which sphere will reach a greater final height, \( H \)?
A. larger sphere (\( 2R \))
B. smaller sphere (\( R \))
C. same height for both
1. Two identical masses, \( m \), can glide along a frictionless surface, as shown above. The left-hand mass is pushed against a spring-plunger with spring constant \( k \), and the plunger is compressed through a distance \( 2d \). Conversely, the right-hand mass is pushed against a spring-plunger with stronger spring constant \( 2k \), but it is only compressed through a shorter distance \( d \). Both masses are released from rest at the same time. When they eventually collide, they stick together.

For both parts of this question,
- Show all steps of your derivation.
- Simplify each final answer to its most compact algebraic form.

a. (5 pts.) Find an expression for the final velocity of the stuck-together pair of masses after collision. Is it to the right or to the left? Express your final answer ONLY in terms of \( m, k, d \), and any necessary mathematical constants.

b. (4 pts.) Find the fraction of initial mechanical energy that is lost in the collision. You may give your final answer as a simplified pure mathematical expression; as a decimal (to 3 sig. figs.); or as a percentage (to 3 sig. figs.).
2. (6 pts.) A horizontal platform (3.00 m long, mass $m_C = 60.0$ kg) is suspended at its ends by two vertical ropes that supply upward tensions $F_L$ and $F_R$. Two window washers are standing on the platform:

- worker $A$ ($m_A = 100.0$ kg) at 0.80 m from the left end of the platform, and
- worker $B$ ($m_B = 75.0$ kg) at 1.80 m from the left end of the platform.

Everything is at rest, in equilibrium. Find the magnitudes of both tensions, $F_L$ and $F_R$. Show your work completely.