Midterm Exam #2, Part A

Exam time limit: 50 minutes. You may use a calculator and both sides of ONE sheet of notes, handwritten only. Closed book; no collaboration. Ignore friction and air resistance in all problems, unless told otherwise.

Part A: For each question, fill in the letter of the one best answer on your bubble answer sheet.

Physical constants: \( g = 9.80 \text{ m/s}^2 \)

Useful conversions: 1 year = 3.156 \times 10^7 \text{ s}

Sun, Earth, & Moon data:

masses

\( M_{\text{Sun}} = 2.00 \times 10^{30} \text{ kg} \)
\( M_{\text{Earth}} = 5.97 \times 10^{24} \text{ kg} \)
\( M_{\text{Moon}} = 7.35 \times 10^{22} \text{ kg} \)

physical radii

\( R_{\text{Sun}} = 6.95 \times 10^8 \text{ m} \)
\( R_{\text{Earth}} = 6.37 \times 10^6 \text{ m} \)
\( R_{\text{Moon}} = 1.74 \times 10^6 \text{ m} \)

orbital distances

\( d_{\text{Earth-Sun}} = 1.50 \times 10^{11} \text{ m} \)
\( d_{\text{Earth-Moon}} = 3.84 \times 10^8 \text{ m} \)

orbital periods

\( T_{\text{Earth}} = 1 \text{ year (exact)} \)
\( T_{\text{Moon}} = 27.3 \text{ days} \)

(2 pts. each) Convert the following quantities into the given units:

1. \( 99 \text{ kW} = \underline{9.9 \times 10^7 \text{ mW}} \)
   A. \( 9.9 \times 10^{-10} \text{ mW} \)
   B. \( 9.9 \times 10^{-7} \text{ mW} \)
   C. \( 9.9 \times 10^{-4} \text{ mW} \)

2. \( 3.3 \times 10^{14} \text{ m}^3 = \underline{3.3 \times 10^2 \text{ km}^3} \)
   A. \( 330 \text{ km}^3 \)
   B. \( 3.3 \times 10^3 \text{ km}^3 \)
   C. \( 3.3 \times 10^4 \text{ km}^3 \)
   D. \( 3.3 \times 10^1 \text{ km}^3 \)
   E. \( 3.3 \times 10^5 \text{ km}^3 \)

3. \( 25 \text{ cm/s} = \underline{0.90 \text{ km/h}} \)
   A. \( 9.0 \times 10^{-5} \text{ km/h} \)
   B. \( 9.0 \times 10^{-2} \text{ km/h} \)
   C. \( 0.90 \text{ km/h} \)

Questions #4-5: A child’s toy airplane flies in uniform circular motion at the end of a massless tether (cord). The plane of the circle is exactly horizontal (parallel to the ground). (Neglect gravity and air resistance.)

4. (1 pt.) The acceleration of the airplane is always...
   A. tangent to the circle, in the direction of the airplane’s velocity
   B. exactly toward the center of the circle
   C. exactly away from the center of the circle
   D. zero

5. (2 pts.) The tether will break if its tension exceeds 95 N. If the length of the tether is 1.5 m, and the airplane has a mass of 0.20 kg, what is the toy airplane’s maximum linear speed?
   A. 11 m/s
   B. 16 m/s
   C. 22 m/s
   D. \( 27 \text{ m/s} \)

\( \Sigma F_{\text{net}} = \frac{m v^2}{r} \) (centripetal force)

\( (F_T)_{\text{max}} = \frac{m v_{\text{max}}^2}{r} \)

\( 95 \text{ N} = \frac{(0.20 \text{ kg}) v_{\text{max}}^2}{1.5 \text{ m}} \) \( \Rightarrow v_{\text{max}} = \frac{26.7 \text{ m/s}}{\sqrt{3}} \)
Questions #6-8: A block of mass \(m\) initially sits at rest on a horizontal surface. The coefficients of friction between the block and the surface are \(\mu_s\) and \(\mu_k\). A person pushes on the block with a horizontal force \(F_p\) in an attempt to dislodge it.

6. (2 pts.) What is the minimum magnitude of \(F_p\) needed for the block to start sliding?
   A. \(\mu_s mg\)  
   B. \(\mu_s mg\)  
   C. \(\frac{mg}{\mu_s}\)  
   D. \(\frac{mg}{\mu_s}\)  
   E. \(\frac{mg}{\mu_s + \mu_k}\)

   \(F_p\) needs to match (and slightly exceed) the maximum magnitude of static friction:
   \[ F_p \geq (F_{fr, s})_{max} = \mu_s \cdot F_N, \]
   and we see \(F_N = m \cdot g\)  
   \[ \Rightarrow F_p \geq \mu_s (m \cdot g) \]

7. (2 pts.) Later, suppose the block is sliding to the right. If the block has a rightward acceleration \(a\), what is the magnitude of \(F_p\)?
   A. \(ma\)  
   B. \(\mu_s ma\)  
   C. \(ma + \mu_k g\)  
   D. \(ma - \mu_k g\)  
   E. \(\frac{ma}{\mu_s}\)

   \[ F_p - F_{fr, k} = m \cdot a \]
   \[ \Rightarrow F_p = m \cdot a + \mu_k \cdot m \cdot g \]

8. (1 pt.) In the previous question, the person exerts a rightward force of \(F_p\) on the crate, and the crate accelerates to the right. At the same time, the crate exerts a leftward force on the person that is...
   A. zero  
   B. weaker than \(F_p\)  
   C. equal to \(F_p\)  
   D. stronger than \(F_p\)

   By Newton's 3rd Law: as person exerts \(F_p\) on crate to right, crate exerts \(F_p\) on person to left. (Regardless of speed or acceleration!)

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Questions #9-11: Consider two spherical masses, \(A\) and \(B\), as shown, released from rest at an initial separation \(r_0\). Mass \(A\) is larger than mass \(B\). (Assume that NO other masses exist in the universe.)

9. (1 pt.) As the two masses fall toward each other, the gravitational force acting on mass \(A\) is _________ the gravitational force acting on mass \(B\), at all times.
   A. stronger than  
   B. equal strength  
   C. weaker than  
   D. None of the above answers is true at all times.

   Newton's 3rd Law: both attractive forces are equal in magnitude (and opposite in direction), even if two masses are unequal!

10. (1 pt.) The two masses will finally collide at a location...
    A. closer to the starting position of mass \(A\)  
    B. closer to the starting position of mass \(B\)  
    C. exactly halfway between their original positions

   Newton's 2nd Law: \(a = \frac{F}{m}\)
   Both forces are equal, but \(m_A > m_B\), so \(a_A < a_B\), and \(A\) will move more slowly than \(B\).

11. (1 pt.) Just before the two masses collide, the speed of mass \(A\) will be _________ the speed of mass \(B\).
    A. faster than  
    B. equal to  
    C. slower than

   \[ v_A < v_B, \] and \(A\) will move more slowly than \(B\).
Questions #12-16: Masses \( m \) and \( 2m \) are attached to ideal, massless springs, \( k \) and \( 2k \), respectively, as shown above. The mass in system A is initially pulled aside to a displacement of \( x = d \), while the mass in system B is initially displaced twice as far. (The surfaces are frictionless. The force vectors \( F_A \) and \( F_B \) are NOT necessarily drawn to scale.)

For the next 3 questions, the two systems are held at rest by applying forces \( F_A \) and \( F_B \), respectively.

12. (1 pt.) For either system, a complete free-body diagram of mass \( m \) would show ___ distinct force vectors acting on \( m \):
   A. 1
   B. 2
   C. 3
   D. 4
   E. 5

13. (2 pts.) Suppose that \( k = 55 \text{ N/m} \), \( m = 1.8 \text{ kg} \), and \( d = 7.5 \text{ cm} \). What is the magnitude of force \( F_A \)?
   A. 2.9 N
   B. 4.1 N
   C. 6.5 N
   D. 8.0 N
   E. 9.2 N

14. (2 pts.) The magnitude of force \( F_A \) is ___ times the magnitude of force \( F_B \):
   A. \( \frac{1}{4} \)
   B. \( \frac{1}{2} \)
   C. 1 (equal)
   D. 2
   E. 4

Now, both forces \( F_A \) and \( F_B \) are removed simultaneously, and both masses are free to move without friction.

15. (1 pt.) Immediately after release, both masses will return to \( x = 0 \) ...
   A. at constant speed
   B. with increasing speed, but with diminishing acceleration
   C. with increasing speed, and with constant acceleration
   D. with increasing speed, and with strengthening acceleration

16. (2 pts.) Immediately after release, the mass's acceleration in system A is ___ times the mass's acceleration in system B:
   A. \( \frac{1}{4} \)
   B. \( \frac{1}{2} \)
   C. 1 (equal)
   D. 2
   E. 4
Midterm Exam #2, Part B

Part B: Show your work on all free-response questions. Be sure to use proper units and significant figures in your final answers. For any multiple-choice questions, circle the letter of the one best answer (unless more than one answer is asked for).

1. A large, heavy crate \((m = 250.0 \text{ kg})\) is suspended on cable \(A\) from a crane. Also, a worker pulls downward on cable \(B\) with \(380. \text{ N}\) of force, to help guide and steady the crate. Both cables are exactly vertical. Assume that both cables are massless and inelastic.

   a. (2 pts.) Using the crate shown at right, create a free-body diagram of \(m\), showing ALL forces acting on it. LABEL ALL force vectors. (You do NOT need to calculate their magnitudes for this diagram.)

   ![Free-body diagram of a crate](image)

   inventory of forces:
   \[
   \begin{align*}
   F_{TA} &= (0, F_{TA}) \\
   F_{TB} &= (0, -F_{TB}) \\
   F_g &= (0, -mg)
   \end{align*}
   \]

   You do NOT need to show your work for part (b).

   \[
   F_{TA} = (250.0 \text{ kg})(9.80 \text{ m/s}^2) + 380. \text{ N} = 2450 \text{ N} + 380. \text{ N} = 2830 \text{ N}.
   \]

   b. (2 pts.) If the crate is at rest, \(F_{TA} = 2830 \text{ N}\).

   c. (5 pts.) Later, while the crate is moving, the tension in cable \(A\) is measured to be 2750 \(\text{ N}\). (The worker is still applying 380 \(\text{ N}\) of downward force on cable \(B\).) Find the magnitude and direction of the crate's acceleration. Show your work completely.

   Newton's 2nd Law: \(\Sigma F_y = m \cdot a_y\)

   \[
   F_{TA} - F_{TB} - mg = m \cdot a_y
   \]

   \[
   2750 \text{ N} - 380. \text{ N} - (250.0 \text{ kg})(9.80 \text{ m/s}^2) = (250.0 \text{ kg}) \cdot a_y
   \]

   \[
   2750 \text{ N} - 380. \text{ N} - 2450 \text{ N} = (250.0 \text{ kg}) \cdot a_y
   \]

   \[
   -80 \text{ N} = (250.0 \text{ kg}) \cdot a_y
   \]

   (note loss of sig figs) \(\Rightarrow a_y = \boxed{-0.32 \text{ m/s}^2}\)

   or, even 1 sig fig: \(\boxed{-0.3 \text{ m/s}^2}\)

   \(a_y\) was chosen to be upward, so \(a_y\) is downward (negative).
2. In the not-too-distant future, astronauts may use Mars’s larger moon, Phobos, as a location for a lunar base and way-station to Mars. Throughout this question, assume that Phobos is a uniform-density, perfectly smooth sphere with radius $1.11 \times 10^4$ m and mass $1.07 \times 10^{16}$ kg. (Ignore the presence of Mars or any other astronomical bodies.)

Two astronauts, Adam and Beverly, are having a friendly argument: Adam bets Bev that he can throw a 145-gram baseball horizontally (tangent to the ground) fast enough to put it into a circular orbit just barely above the surface of Phobos. Bev is skeptical, so she does a quick calculation...

a. (5 pts.) Find the linear speed necessary for the baseball. Show your work. (Thought question: Could a human indeed throw a baseball this fast? Recall: 1 m/s = 2.24 miles/hour)

\[
\begin{align*}
F_g &= \text{(gravitational force)} = \Sigma F_{rad} \\
&= \frac{GM_{ph} \cdot m}{R_{ph}^2} = m \frac{v^2}{R_{ph}} \\
\Rightarrow \quad v &= \sqrt{\frac{GM_{ph}}{R_{ph}}} \\
&= \left[ \left( \frac{6.67 \times 10^{-11} \text{ N m}^2}{\text{kg}^2} \right) \left( 1.07 \times 10^{16} \text{ kg} \right) \right] ^{1/2} / 1.11 \times 10^4 \text{ m} \\
&= 8.02 \text{ m/s} \\
\end{align*}
\]

\[ v = 8.02 \text{ m/s} \]

To prove his point, Adam does it: he throws the baseball at just the right speed, and away it goes in a circular orbit. While Adam stands grinning at Bev, the baseball circles Phobos completely and smacks him right in the helmet. Bev decides that it was worth the extremely long wait.

b. (5 pts.) How much time is needed for the baseball to complete one full orbit of Phobos? Convert your final answer to hours. Show your work. (Hint: Your final answer will be between 1 and 3 hours.)

Uniform circular motion:

\[ v = \frac{2\pi r}{T} \quad \text{period of one cycle} \]

\[ T = \frac{2\pi r}{v} \quad \text{where } r = R_{ph} \]

\[ = \frac{2\pi (1.11 \times 10^4 \text{ m})}{8.02 \text{ m/s}} \quad \text{from part(a)} \]

converting to hours

\[ T = 8.70 \times 10^3 \text{ s} \cdot \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) \]

\[ T = 2.42 \text{ h} \]

\[ \Rightarrow \quad \text{Kepler's 3rd Law:} \]

\[ T^2 = \frac{4\pi^2}{GM_{ph}} r^3 \quad \text{where } r = R_{ph} \]

\[ \Rightarrow \quad T = \left( \frac{4\pi^2 (1.11 \times 10^4 \text{ m})^3}{(6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2)(1.07 \times 10^{16} \text{ kg})} \right)^{1/2} \]

\[ T = 8.70 \times 10^3 \text{ s} \cdot \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) \]

\[ T = 2.42 \text{ h} \]

\[ \text{continued on next page...} \]
2. **continued:**

**Repeat of earlier information:** Assume that Phobos is a uniform-density, perfectly smooth sphere with radius $1.11 \times 10^4$ m and mass $1.07 \times 10^{16}$ kg. (Ignore the presence of Mars or any other astronomical bodies.)

**c.** (1 pt.) If Adam had thrown the baseball horizontally with **slightly greater speed** than the speed calculated in part (a), what would have happened to the baseball's orbit?

- **A.** There would be no change; the baseball would still execute an identical circular orbit just barely above the surface of Phobos, just with a faster speed.
- **B.** The baseball would have ascended to a larger radius, then orbited Phobos in a larger circular orbit at that new radius, high above the surface.
- **C.** The baseball would have orbited Phobos in a large ellipse: ascending for the first half of the orbit, then descending for the second half (striking Adam in the head as it grazes Phobos's surface on its return).

One of the challenging things about working and living on Phobos is the very weak surface gravity.

**d.** (5 pts.) Find the **acceleration due to gravity** on the surface of Phobos, and convert your final answer to Earth "gees." Show your work.

\[
\text{gravitational force} = \text{weight, for any object } m \text{ on surface of Phobos.}
\]

\[
F_g = m \cdot g_{Phobos}
\]

\[
\frac{GM_{Phobos}}{R_{Phobos}^2} = \frac{m}{R_{Phobos}} \cdot g_{Phobos}
\]

\[\Rightarrow g_{Phobos} = \frac{GM_{Phobos}}{R_{Phobos}^2} \]

\[
g_{Phobos} = \left( \frac{6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}}{(1.11 \times 10^4 \text{ m})^2} \right) \times \left( 1.07 \times 10^{16} \text{ kg} \right)
\]

\[
g_{Phobos} = 5.79 \times 10^{-3} \text{ m/s}^2 \left( \frac{1 \text{ gee}}{9.80 \text{ m/s}^2} \right)
\]

\[
g_{Phobos} = 5.91 \times 10^{-4} \text{ gee}
\]

**QR:** For baseball in **circular orbit** just above surface of Phobos,

\[F_g = \Sigma F_{rad} \Rightarrow m \cdot g = m \cdot \frac{v^2}{r} \Rightarrow g = \frac{v^2}{r}
\]

\[g = \frac{(8.02 \times 10^{-5})^2}{1.11 \times 10^4 \text{ m}} = 5.79 \times 10^{-3} \text{ m/s}^2.
\]

(Then convert to gees, as above.)