Lecture 8  -- Physics 272
Electric Currents
Resistance & Resistivity
Electric Current

NO longer electrostatics! Can have E in conductor and a flow of charge.

Consider free electron motion without any external electric field. The net motion (blue) is random and the average displacement is zero. Electron moves from P1 → P2

Consider electron motion with an external electric field. The net motion (red) has a drift and the average displacement is opposite to the electric direction. Electron moves P1 → P2’. The net drift velocity \( v_d \) is \( \sim 10^{-4} \) m/s. The drift velocity is very small!
Apparent Paradox: Light turns on immediately when you flip the switch despite the tiny drift velocity of charge carriers ($\sim 10^{-4}$ m/s)

Think of water in a garden hose. If the hose is full the water will flow out of the end immediately.
**Current in Wire; charge carriers**

Current, \( I \), is the rate of charge flow through cross section of wire or charge per unit time:

\[
I = \frac{dQ}{dt}
\]

units of \( C/s \) = 1 Ampere

Current can be formed with positive charges moving in positive direction or with negative charges moving in the opposite direction.

Electron flow is opposite to the direction of current flow. This is a historical convention due to Benjamin Franklin.

House outlets are fused at 15Amps
Whole house circuit breaker is around 200Amps

(b) Y&F fig 25.2
Current Density, \( J \), is the current flow per unit area (amp/m\(^2\))

\[
J = \frac{I}{A} = \frac{1}{A} \frac{dQ}{dt}
\]

If + charges, \( q \), have velocity \( v_d \) and a volume density, \( n \) (#/volume). Then in a time \( dt \), a volume, \( A v_d \, dt \), is swept out and the differential amount of charge is

\[
dQ = q \, n \, A \, v_d \, dt
\]

We can write current as,

\[
I = \frac{dQ}{dt} = nqv_d A
\]
Example

A wire is made of copper and has a radius 0.815 mm. Calculate the drift velocity assuming 1 free electron per atom for $I = 1\text{A}$.

\[ I = \frac{dQ}{dt} = nqv_d A \]

\[ v_d = \frac{I}{nqA} \]

\[ n = n_a = \frac{\rho N_A}{M} = \frac{(8.93\text{g/cm}^3)(6.02\times10^{23}\text{atoms/mole})}{63.5\text{g/mole}} \]

\[ = 8.47\times10^{28}\text{atoms/m}^3 \]

$n_a$ = number density of copper atoms
\[ \rho = \text{density of copper} \]
\[ N_A = \text{Avogadro’s number} \]
\[ M = \text{atomic mass of Cu} \]

\[ v_d = \frac{I}{nqA} = \frac{1\text{A}}{(8.47\times10^{28}\text{m}^{-3})(1.6\times10^{-19}\text{C})\pi(8.15\times10^{-4}\text{m})^2} \]

\[ = 3.54\times10^{-5}\text{m/s} \]

Very slow. 7.8 hours to travel 1 m!

Why does the light come on so quickly when switch is thrown?
Current Density, $J$

Since the current is, \[ I = \frac{dQ}{dt} = nqv_d A \]

We can write the current density as, \[ J = \frac{I}{A} = nqv_d \]

Technically, current density is a vector quantity since velocity is a vector quantity, \[ \vec{J} = nq\vec{v}_d \]
Review:

Conductivity – high for good conductors.

**Ohm’s Law:** \( J = \sigma E \)

Observables:

\[
\begin{aligned}
V &= EL \\
I &= JA \\
\end{aligned}
\]

\[
\begin{aligned}
I/A &= \sigma V/L \\
I &= V/(L/\sigma A) \\
\end{aligned}
\]

\[
R = \text{Resistance} \\
\rho = 1/\sigma \\
\]

\[
\begin{aligned}
I &= V/R \\
R &= \frac{L}{\sigma A} \\
\end{aligned}
\]
This is just like plumbing!

I is like flow rate of water
V is like pressure
R is how hard it is for water to flow in a pipe

\[ R = \frac{L}{\sigma A} \]

To make R big, make L long or A small
To make R small, make L short or A big
Clicker problem I

Same current through both resistors

Compare voltages across resistors

\[ V = IR \propto \frac{L}{A} \]

A) \( V_1 > V_2 \)
B) \( V_1 = V_2 \)
C) \( V_1 < V_2 \)

\[ A_2 = 4 \ A_1 \Rightarrow V_2 = 1/4 \ V_1 \]

Clicker problem II

\[ V_2 = 2V_1 \]

L₂ = 2L₁

A) \( V_1 > V_2 \)
B) \( V_1 = V_2 \)
C) \( V_1 < V_2 \)
12) The SAME amount of current I passes through three different resistors. R₂ has twice the cross-sectional area and the same length as R₁, and R₃ is three times as long as R₁ but has the same cross-sectional area as R₁.

In which case is the CURRENT DENSITY through the resistor the smallest?

A) B) C)

\[ J = \frac{I}{A} \quad \Rightarrow \quad J_1 = J_3 = 2J_2 \]

Same Current \[ J \propto \frac{1}{A} \]
Let's review:

**CLICKER QUESTION**

Suppose we have a wire, with cross section area, $A$, length $L$, and a charge $dQ$ flows through the wire in a time $dt$, what is the Current Density, $J$, defined as??

\[
\begin{align*}
  a) & \quad \frac{dQ}{dt} \\
  b) & \quad \frac{1}{A} \frac{dQ}{dt} \\
  c) & \quad A \frac{dQ}{dt} \\
  d) & \quad \frac{1}{LA} \frac{dQ}{dt} \\
  e) & \quad \text{none of these}
\end{align*}
\]
Y&F Problem 25.1 (Example, not a clicker question)

A current of 3.60 A flows through an automobile headlight. How many coulombs of charge flow through the headlight in a time of 2.60 hrs?

\[ I = \frac{dQ}{dt} \]
\[ dQ = I \, dt \]
\[ Q = \int I \, dt \quad \text{I constant} \]
\[ Q = It = (3.6 \, \text{A})(2.6 \, \text{hr})(3600 \, \text{s/hr}) \quad (\text{A=C/s}) \]
\[ = 3.37 \times 10^4 \, \text{C} \]
Resistivity $\rho$

The current density at a point in a material depends on the material and on $E$. For some materials, $J$ is proportional to $E$ at a given temperature. These materials (metals for example) are ohmic and are said to obey “Ohm’s Law”.

Other materials (e.g. semiconductors) are non Ohmic.

Define resistivity as the ratio of electric field to current density ($V \cdot m / Amp$). The symbol for resistivity is the Greek letter $\rho$.

$$\rho = \frac{|E|}{|J|} \quad \text{OR} \quad \vec{E} = \rho \; \vec{J}$$

For ohmic materials, $\rho$ at a given temperature is nearly constant.

New notation for $V/Amp$ is unit, Ohm, represented by Greek letter, $\Omega$. Units of $\rho$ are $\Omega \cdot m$. Insulators have large values of $\rho$.

For Glass, $\rho > 10^{10} \Omega \cdot m$. 
Resistivity $\rho$

The inverse of resistivity is called conductivity. Conductors have large values of conductivity or very small values of $\rho$.

For copper $\rho = 2.44 \times 10^{-8} \, \Omega \cdot m$.

Temperature dependence of $\rho$:
For ohmic materials:

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)]$$

$\rho_0 =$ resistivity at room temp (20° C)
$\rho(T) =$ resistivity at $T$
$\alpha =$ temperature coefficient of resistivity

Other materials (non-ohmic) more complicated.
What is the definition of resistivity??

a) \( \frac{\text{Voltage}}{\text{Current}} \)

b) \( \frac{\text{Amps}}{\text{Area}} \)

c) \( \frac{\text{current density}}{\text{electric field}} \)

d) Voltage Current

e) \( \frac{\text{electric field}}{\text{current density}} \)
Consider a uniform, straight section of wire of length $L$ and cross section $A$ and with current $I$.

$$\vec{E} = \rho \vec{J}$$

Multiply by length $L$

$$|\vec{E}|L = V = \rho |\vec{J}| L = \rho \frac{I}{A}$$

$$L = \left( \rho \frac{L}{A} \right) I = R I$$

We define resistance, $R$ as

$$R = \rho \frac{L}{A}$$

And we have for ohmic materials Ohm’s Law:

$$V = I R$$

Resistance increases with bigger $L$ and decreases with bigger $A$

$V$, $R$, $I$ easier to measure than $E$, $\rho$, and $J$. Units of $R$: $\Omega$
Resistance \( R \)

We define resistance, \( R \) as

\[
R = \rho \frac{L}{A}
\]

And we have for ohmic materials Ohm’s Law:

\[
V = I \cdot R
\]

Units of \( R \): \( \Omega \)

Temperature dependence (ohmic materials):

\[
R(T) = R_0 [1 + \alpha(T - T_0)]
\]
Example: A 80.0 m Cu wire 1.0 mm in diameter is joined to a 49.0 m iron wire of the same diameter. The current in each is 2.0 A.
a.) Find V in each wire.

\[
R_{Cu} = \frac{\rho_{Cu} L_{Cu}}{A} = \frac{(1.7 \times 10^{-8} \, \Omega \, m)(80.0 \, m)}{\pi(0.5 \times 10^{-3} \, m)^2} = 1.73 \, \Omega
\]

\[
R_{Fe} = \frac{\rho_{Fe} L_{Fe}}{A} = \frac{(10 \times 10^{-8} \, \Omega \, m)(49.0 \, m)}{\pi(0.5 \times 10^{-3} \, m)^2} = 6.24 \, \Omega
\]

\[
V_{Cu} = IR_{Cu} = (2.0A)(1.73\Omega) = 3.46V
\]

\[
V_{Fe} = IR_{Fe} = 12.5V
\]

b.) Find E in each wire.

\[
V = EL
\]

\[
E_{Cu} = \frac{V_{Cu}}{L_{Cu}} = 0.43 \frac{V}{m}
\]

\[
E_{Fe} = \frac{V_{Fe}}{L_{Fe}} = 0.255 \frac{V}{m}
\]
Two cylindrical resistors are made from the same material, and they are equal in length. The first resistor has diameter $d$, and the second resistor has diameter $2d$.

2) Compare the resistance of the two cylinders.

- $R_1 > R_2$
- $R_1 = R_2$
- $R_1 < R_2$

3) If the same current flows through both resistors, compare the average velocities of the electrons in the two resistors:

- $v_1 > v_2$
- $v_1 = v_2$
- $v_1 < v_2$
Clicker problem

Two cylindrical resistors, $R_1$ and $R_2$, are made of the same material. $R_2$ has twice the length of $R_1$ but half the radius of $R_1$.

These resistors are then connected to a battery $V$ as shown:

What is the relation between $I_1$, the current flowing in $R_1$, and $I_2$, the current flowing in $R_2$?

(a) $I_1 < I_2$          (b) $I_1 = I_2$          (c) $I_1 > I_2$
Clicker problem

- Two cylindrical resistors, $R_1$ and $R_2$, are made of identical material. $R_2$ has twice the length of $R_1$ but half the radius of $R_1$.
  - These resistors are then connected to a battery $V$ as shown:

  ![Diagram](image)

  - What is the relation between $I_1$, the current flowing in $R_1$, and $I_2$, the current flowing in $R_2$?

    (a) $I_1 < I_2$  
    (b) $I_1 = I_2$  
    (c) $I_1 > I_2$

- The resistivity of both resistors is the same ($\rho$).
- Therefore the resistances are related as:

  \[ R_2 = \rho \frac{L_2}{A_2} = \rho \frac{2L_1}{(A_1 / 4)} = 8 \rho \frac{L_1}{A_1} = 8R_1 \]

- The resistors have the same voltage across them; therefore

  \[ I_2 = \frac{V}{R_2} = \frac{V}{8R_1} = \frac{1}{8} I_1 \]