Physics 272A - Midterm II, April 5, 2013, 5 Problems

Problem 1: 20 points—you MUST show all of your work!

As shown in the figure, ions of mass $m$ and charge $q$ are accelerated through a potential difference $V$. They then enter a uniform magnetic field $B$ that is perpendicular to their velocity $v$ and are deflected as shown. A detector then measures where they complete their semi-circular trip of radius $R$.

(a) First derive an expression for the velocity $v$ that the ions have as they first enter the uniform magnetic field $B$ in terms of their mass $m$, charge $q$, and the accelerating potential $V$.

$$\frac{1}{2} m v^2 = q V \Rightarrow v = \sqrt{\frac{2qV}{m}}$$

(b) The ion now enters the magnetic field. Determine the radius $R$ of the ions circular trajectory in terms of the ions mass $m$, charge $q$, velocity $v$, and the magnetic field $B$.

$$F = q v B = \frac{m v^2}{R} \Rightarrow q B = \frac{m v}{R} \Rightarrow R = \frac{m v}{q B}$$

(c) Using your results from (a) and (b) calculate the charge to mass ratio $q/m$ of the ion from measurements of $v$ and $R$, and the stated spectrograph settings of $B$ and $V$.

$$R = \frac{m v}{q B} \Rightarrow \frac{q}{m} = \frac{v}{BR}$$

$$v = \sqrt{\frac{2qV}{m}} \quad \text{use this here} \quad \Rightarrow \frac{q}{m} = \frac{1}{BR^2} \frac{2qV}{m}$$

$$\frac{q}{m} = \frac{2V}{BR^2}$$
Problem 2: 20 points – Show all work!

A cylindrical conductor with radius \( R \) carries a current \( I \) as shown in the figure. The current-density \( J_0 \) is uniformly distributed over the cross-sectional area of the conductor.

(a) Determine the total current \( I_0 \) flowing in the wire and determine the magnitude of \( \mathbf{B} \) at the surface of the wire

\[
I_0 = J_0 \pi R^2 = JA = \int_0^R J_0 \, dr \quad dA = 2\pi r \, dr
\]

Ampere's law

\[
\oint B \cdot dl = \mu_0 I_{\text{enclosed}} \quad \mu_0 2\pi R = \mu_0 I_0 \quad B = \frac{\mu_0 I_0}{2\pi R} = \frac{\mu_0 I_0 R}{2}
\]

(b) Suppose now that the current-density, while being axially symmetric, now has a radial dependence \( r \) given by \( J(r) = J_0(r/a) \) determine the total current now flowing in the wire and determine the magnitude of \( \mathbf{B} \) at the surface of the wire. Hint: recall that \( dA = 2\pi r \, dr \) in polar coordinates.

\[
I = \int_0^R J_0 \frac{r}{a} \, 2\pi r \, dr = \frac{J_0}{a} \int_0^R \frac{r}{a} \, 2\pi r \, dr = \frac{2\pi J_0}{3a} R^3 = \frac{I_0 R}{3a} \quad \text{using } I = \mu_0 \text{Hm} \quad (a)
\]

\[
B \cdot (2\pi R) = \mu_0 \frac{2\pi J_0 R^3}{3a} \quad B = \frac{\mu_0 J_0 R^2}{3a}
\]

(c) Using your result from (b) determine the magnitude of \( B(r) \) at the point \( r = R/2 \) Hint: what is the enclosed current in this case?

\[
I_{\text{enc}} = \int_0^{R/2} J_0 \frac{r}{a} \, 2\pi r \, dr \quad \text{from } (b) \quad B = \frac{\mu_0 J_0 R^3}{3a} \quad B = \frac{I_0 R}{12a}
\]

\[
B \left( \frac{2\pi R}{2} \right) = \mu_0 \frac{2\pi J_0 R^3}{12a} = \frac{\mu_0 J_0 R^2}{12a} = \frac{\mu_0 I_0}{12\pi a}
\]

(d) Using your results from (b) and (c) sketch the magnitude of the magnetic field \( B(r) \) in the region from \( r = 0 \) to \( r = R \), and continue on out to \( r = 2R \) outside the wire.

In general \( r < R \)

\[
B = \frac{\mu_0 I_0}{3a} \frac{r^2}{R^2}
\]

\[
\text{part (a)}
\]

\[
\text{part (c)}
\]

\[
\text{part (a)}
\]
Problem 3: 20 points—Show all work!

The loop shown is being pulled to the right at constant speed \( v \). A constant current \( I \) flows in the long wire in the direction shown.

(a) Calculate the magnitude of the net \( emf \) \( \mathcal{E} \) induced in the loop. Do this two ways: (i) by using Faraday's law of induction, and (ii) by looking at the \( emf \) induced in each segment of the loop due to its motion.

\[
\phi = \frac{\mu_0}{2\pi} \int_{r}^{r+a} b \, dx = \frac{\mu_0 b r}{2\pi} \ln \left( \frac{r+a}{r} \right) \quad r = ut
\]

\[
\frac{d\phi}{dt} = \frac{\mu_0 b}{2\pi} \left( \frac{1}{r^2} \left( -\frac{dv}{dt} \right) \right) \quad \mathcal{E} = \frac{d\phi}{dt} = \frac{\mu_0 b v}{2\pi r} \frac{r^2}{r+a}
\]

(b) Find the direction (CW or CCW) of the current induced in the loop. Do this two ways: (i) Using Lenz's law and (ii) using the magnetic force on charges in the loop.

\[
\mathcal{E}_1 = B_1 b v = \frac{\mu_0 I}{2\pi r} b v \quad \mathcal{E}_2 = B_2 b v = \frac{\mu_0 I}{2\pi (r+a)} b v \quad \mathcal{E}_1 - \mathcal{E}_2 = \frac{\mu_0 I b v r}{2\pi r (r+a)}
\]

(c) Check you answer for the \( emf \) in part(a) in the following special cases to see whether it is physically possible: The loop is stationary; (ii) the loop is very thin, so \( a \to 0 \); (iii) the loop gets very far from the wire.

(i) Stationary \( \Rightarrow \frac{d\phi}{dt} = \mathcal{E} = 0 \)

(ii) \( a \to 0 \Rightarrow a \mu_0 A \to 0 \quad \phi = B/A \to 0 \quad \mathcal{E} \to 0 \)

(iii) \( r \to \infty \quad B \to 0 \quad \mathcal{E} \to 0 \)
Problem 4: 20 points

In the circuit shown, the switch $S$ is closed at time $t = 0$ with no charge initially on the capacitor.

(a) Find the reading on each ammeter and voltmeter just after $S$ is closed.

(b) Find the reading of each meter after a long time has elapsed.

(c) Find the maximum charge on the capacitor

\[ Q = CV = 12 \times 10^{-6} \text{F} \times 16 \text{V} \]
\[ = 192 \mu \text{C} \]

(d) Draw a qualitative graph of the reading of voltmeter $V_2$ as a function of time.
Problem 5: 20 points

An L-R-C series circuit has \( R = 300 \Omega \) as shown. At the frequency of the source \( V_s \), the inductor has reactance \( X_L = 900 \Omega \) and the capacitor has a reactance \( X_C = 500 \Omega \). The amplitude of the voltage across the inductor \( V_L \) is 450V.

(a) What is the amplitude of the voltage \( V_R \) across the resistor \( R \)?

\[
V_R = iR \quad \text{but} \quad V_L = (jxL)i = 450 \ V = (900j)i
\]

\[
i = \frac{450}{900} = 0.5
\]

\[
V_R = iR = 0.5 \times 300 = 150 \ V
\]

(b) What is the amplitude of the voltage \( V_C \) across the capacitor?

\[
V_C = -jxC i = -j\left(\frac{1}{j\omega C}\right)i = -j(500)(0.5) = -250 \ V
\]

(c) What is the voltage amplitude of the voltage source \( V_s \)?

\[
V = V_R + V_L + V_C = \sqrt{(150)^2 + (450 - 250)^2} = 250 \ V \quad (345) \ \text{triangle}
\]

\[
\tan \phi = \frac{X_L - X_C}{R} = \frac{200}{150} \quad \phi = 53.13^\circ
\]

\[
\text{Power factor!!}
\]

(d) What is the average rate at which the voltage source is delivering electrical energy into the circuit?

\[
P = VI \cos \phi = 250 \times 0.5 \times 0.6(53.13^\circ)
\]

\[
= 250 \times 0.5 \times 0.6 = 75 \ \text{watts}
\]

heating of the resistor.