Physics 170A - Midterm I, Oct 02, 2012 4 Problems

Problem 1: 25 points (you must show all of your work)

A motorcyclist attempts to jump across the river as shown. The take-off ramp is inclined at 53.13 degrees (a 345 triangle), and the river is 40.0m wide, and the far bank is 15.0m lower than the top of the ramp. The river itself is 100m below the ramp.

(a) Ignoring air resistance, what should his speed $v_0$ be at the top of the ramp to just make it to the near edge of the far bank? Hint: solve the $x$ and $y$ motion separately to give two equations for the unknowns $t$ and $v_0$.

$$y = y_0 + v_{0y}t - \frac{1}{2} g t^2$$

$$0 = 15 + v_0 \left( \frac{4}{5} \right) t - \frac{1}{2} \left( 9.8 \right) t^2$$

$$-15 = \frac{4}{5} v_0 t - 4.9 t^2$$

$$x = x_0 + v_{0x} t$$

$$0 = 0 + \frac{3}{5} v_0 t$$

$$v_0 t = 66.67$$

$$-15 = \frac{4}{5} (66.67) - 4.9 t^2$$

$$t = \sqrt{\frac{67.34}{4.9}} = 3.73 \text{ sec}.$$ 

$$v_0 = \frac{66.67}{3.73} = 17.83 \text{ m/s}$$

(b) If his speed was only half the value you found in part (a) how long will it take him to hit the water? (Hint: you have to solve a quadratic equation in $t$ and you will get two times. Explain why.

$$v_0 = \frac{17.83}{2} = 8.94 \text{ m/s}$$

$$y = y_0 + v_{0y} t - \frac{1}{2} g t^2$$

$$y - y_0 = -100 = v_0 t - \frac{1}{2} g t^2$$

$$-100 = 8.94 \left( \frac{4}{5} \right) t - 4.9 t^2$$

$$4.9 t^2 - 7.15 t - 100 = 0$$

$$t = \frac{7.15 \pm \sqrt{(11.15)^2 - 4 \left(4.9\right)\left(-100\right)}}{9.8} = 7.15 \pm \sqrt{51.12 + 196.0} \frac{9.8}{9.8} = 7.15 \pm 4.485 \frac{9.8}{9.8}$$

$$\Delta t = 3.84 \text{ sec or } -3.84 \text{ sec}$$
Problem 2: 25 points

An airplane flies in a loop (a circular path in a vertical plane) of radius 150m. The pilot’s head always points toward the center of the loop. The speed of the plane is not constant, the airplane goes slowest at the top of the loop and fastest at the bottom of the loop.

(a) At the top of the loop, the pilot feels weightless. What is the speed of the plane at this point?

\[ F = mg = ma = m \frac{v^2}{R} \quad \text{plane + pilot \ in \ free \ fall} \]

\[ a = \frac{v^2}{R} = 9.8 \quad v = \sqrt{gR} = \sqrt{9.8 \times 150m} = 38.34 \text{ m/s} \]

\[ = 38.34 \text{ m/s} \times 3600 \text{ sec/hr} \times 0.001 \text{ km/m} \]

\[ = 138 \text{ km/hr} \]

(b) At the bottom of the loop the speed of the plane is 280 km/hr. What is the apparent weight of the pilot at this point? His true weight is 700N.

\[ n = mg + m \frac{v^2}{R} \]

\[ 280 \text{ km/hr} = 280 \text{ km/hr} \times 1000 \text{ m/km} \times \frac{1}{3600} \text{ m/sec} \]

\[ = 77.78 \text{ m/sec} \]

\[ n = 700N + 700 \left( \frac{77.78^2}{9.8} \right) \]

\[ = 700N + 700 \times 115 \]

\[ = 3581 \text{ Newtons} \]

\[ = 5.115 \text{ g's} \]
Problem 3: 25 points

A proton, mass=1.67x10^{-27} kg, is propelled with an initial speed of 3.0x10^5 m/sec directly at a uranium nucleus 5.0 m away. The proton is repelled by the uranium nucleus with a force of magnitude \( F = \frac{\alpha}{x^2} \) where \( x \) is the separation between the proton and the uranium nucleus and \( \alpha = 2.12x10^{-26} \text{ Nm}^2 \).

Hint: use conservation of energy methods—determine the work done by the approaching proton and the appropriate increase in the potential.

5m \to \infty for this problem

(a) Assuming that the uranium nucleus remains at rest, what is the speed of the proton when it is 8.0x10^{-10} m from the uranium nucleus?

\[
W = \int_{x=0}^{x=x_1} F \, dx = \int_{x=0}^{x=x_1} \frac{\alpha}{x^2} \, dx = -\frac{\alpha}{x} \bigg|_{x=0}^{x=x_1} = -\frac{\alpha}{x_1} \quad W(x) = -U(x) \quad U(x) = \frac{\alpha}{x} \\
U(x) = 0 \quad K(x) + U(x) = K(8x10^{-10}) + U(8x10^{-10}) \\
K(x) = K(x_1) - K(8x10^{-10}) = \frac{1}{2} m (3x10^5)^2 - \frac{1}{2} m (8x10^{-10})^2 = \frac{1}{2} m V^2 = 2.12x10^{-26} \text{ Nm} \\
V = 2.41x10^5 \text{ m/s}
\]

(b) As the proton approaches the uranium nucleus, the repulsive force slows down the proton until it comes momentarily to rest, after which the proton moves away from the uranium nucleus. How close to the nucleus did the proton get?

\[
E_0 = K(x_0) + U(x_0) = K(0) + U(x_0) \\
U(x) = K(x) \quad \frac{1}{2} m v_1^2 \quad x_0 = \frac{2\alpha}{m v_1^2} = \frac{2 \times (2.12x10^{-26})}{(1.67x10^{-13}) (3x10^5)^2} = 2.8x10^{-10} \text{ m}
\]

(c) What is the speed of the proton when it is again 5.0 m away from the uranium nucleus? (Hint: is the collision elastic?)

Collision is elastic—Force is conservative. So

\[
E(5m) = K(5m) + U(5m) = \frac{1}{2} m (3x10^5)^2 = \frac{1}{2} (1.67x10^{-13}) (3x10^5)^2 = 7.515x10^{-17} \text{ joules} \\
V = 3x10^5 \text{ m/s}
\]

\[
F = \frac{\alpha}{x^2} = \frac{K}{x(x_0)^2} \\
F = \frac{1}{x_0^2} \quad x_0 = \sqrt{\frac{x}{x_0}} = \frac{V}{2.12x10^{-10}} \text{ m} \\
x_0 = 1.456x10^{-13} \text{ m} > 5m = \infty on this scale.
\]
Problem 4: 25 points

A particle of mass \( m \) is subject to the force \( \vec{F} = -6xy \hat{i} - 3x^2 \hat{j} \)

(a) How much work is done in moving the particle from \((0, 0)\) to \((2, 0)\) along the path \((0, 0)\) to \((2, 0)\), and then from \((2, 0)\) to \((2, 4)\)?

\[
W = \int_{x=0}^{x=2} \int_{y=0}^{y=2} -6xy \, dx \, dy + \int_{x=2}^{x=4} \int_{y=0}^{y=4} -3x^2 \, dy \, dx
\]

\[
= -6 \int_{y=0}^{y=2} y \, dy \left|_{x=0}^{x=2} \right. - 3 \int_{x=2}^{x=4} x^2 \, dx \left|_{y=0}^{y=4} \right.
\]

\[
= -6 \left[ \frac{y^2}{2} \right]_{0}^{2} - 3 \left[ \frac{x^3}{3} \right]_{2}^{4}
\]

\[
= -6 \left[ \frac{4}{2} \right] - 3 \left[ \frac{64}{3} \right] = -48
\]

(b) How much work is done in moving the particle from \((0, 0)\) to \((2, 4)\) along the path \( y = x^2 \)

\[
W = \int_{x=0}^{x=2} \int_{y=x^2}^{y=4} -6xy \, dx \, dy + \int_{x=2}^{x=4} \int_{y=x^2}^{y=4} -3x^2 \, dy \, dx
\]

\[
= -6 \int_{x=0}^{x=2} \int_{y=x^2}^{y=4} y \, dy \, dx
\]

\[
= -6 \int_{x=0}^{x=2} \left[ \frac{y^2}{2} \right]_{x^2}^{4} \, dx
\]

\[
= -6 \int_{x=0}^{x=2} \frac{16 - x^4}{2} \, dx
\]

\[
= -3 \int_{x=0}^{x=2} x^4 \, dx
\]

\[
= -3 \left[ \frac{x^5}{5} \right]_{0}^{2}
\]

\[
= -3 \frac{32}{5} = -48
\]

(c) Does your results from (a) and (b) suggest that the force may be conservative? If so, how would you prove it in general. Hint: what does the values of \( dF_x / dy \) and \( dF_y / dx \) suggest?

\[
F_x = -6xy \quad F_y = -3x^2
\]

\[
\frac{dF_x}{dy} = -6x \quad \frac{dF_y}{dx} = -6x
\]

\[
\frac{dF_x}{dy} = \frac{dF_y}{dx} \quad \text{so} \quad \frac{dU}{dx} = 0, \text{force is conservative.}
\]

(d) From your results above, can you determine the potential function \( U(x, y) \) such that \( \vec{F} = -\nabla U(x, y) \). Does your potential \( U(x, y) \) when evaluated at \((0, 0)\), \((2, 4)\) suggest that your results in found in (a) and (b) are correct?

\[
\frac{dU}{dx} = -6xy \quad \Rightarrow \quad U = 3x^2y
\]

\[
\frac{dU}{dy} = -3x^2 \quad \Rightarrow \quad U = 3xy^2
\]

\[
\begin{align*}
\frac{dU}{dx} &= -6xy \quad \Rightarrow \quad U(x, y) = 3x^2y \\
\frac{dU}{dy} &= -3x^2 \quad \Rightarrow \quad U(x, y) = 3xy^2
\end{align*}
\]

Recall \( W = -U \)

\[
W = -48 \quad U(x, y) = 3x^2y \quad U(2, 4) = 3(2)^2(4) = 48 \quad U(2, 4) = 3(2)^2(4) = 48
\]

Recall \( W = -U \)

\[
W = -48 \quad U(x, y) = 3x^2y \quad U(2, 4) = 48 \quad U(2, 4) = 48
\]

\[
\text{OK}
\]