Physics 272. Practice Final Exam
On the final exam there will be 8 problems.
The final exam is Thursday May 12th, 9:45-11:45 a.m. in WAT 112

Problem 1: 25 points

A sphere has a volume charge density \( \rho(r) = \frac{B}{r} \) for \( r < R \) and \( \rho(r) = 0 \) for \( r > R \).

(a) What is the E field inside the sphere?

(b) What is the E field outside the sphere?

(c) Sketch the direction of the E field outside the sphere.

\[ \int_{-r}^{r} \vec{E} \cdot d\vec{A} = \frac{\text{Gauss Law}}{\varepsilon_0} \]

\[ \Rightarrow \vec{E} \cdot 4\pi r^2 = \frac{\text{Gauss (r)}}{\varepsilon_0} \]

Now evaluate \( \text{Gauss (r)} = \int dV \)

\[ \Rightarrow \text{Gauss (r)} = \int_{V} \frac{B}{r} \cdot 4\pi r^2 dr \]

\[ = \int_{0}^{r} \frac{B}{r} \cdot 2\pi r^2 dr \]

\[ = \frac{4\pi B r^3}{3} \]

Outside

\[ \vec{E} \cdot 4\pi r^2 = \frac{4\pi B r^2}{2\varepsilon_0} \Rightarrow \vec{E} = \frac{BR^2}{2\varepsilon_0} \]

Note \( \vec{E} \approx \frac{1}{r^2} \)
Problem 2: 25 points

The space between the conductors of a long coaxial cable used to transmit television signals has an inner radius \( r_1 = 0.15 \) mm and outer radius \( r_2 = 2.1 \) mm. Assume that each carries a charge of \( q = 0.2 \mu \text{C}. \) (Hint: Opposite in magnitude)

(a) What is the electric field between the inner cylinder and outer cylinder in terms of their radii \( r_1, r_2 \) and charge \( q \)?

(b) What is the potential difference between the inner cylinder and outer cylinder in terms of their radii \( r_1, r_2 \) and charge \( q \)?

(c) Calculate the capacitance/length of the two conductors (give a numerical answer with units)

\[
E = \frac{q}{\varepsilon_0} = \frac{q}{2\pi \varepsilon_0}
\]

\[
\Delta V = \frac{q}{2\pi \varepsilon_0} \ln \left( \frac{r_2}{r_1} \right)
\]

\[
C = \frac{\Delta V}{\Delta V} = \frac{L}{2\pi \varepsilon_0} \Rightarrow \frac{C}{L} = \frac{2\pi \varepsilon_0}{\ln \left( \frac{r_2}{r_1} \right)}
\]

\[
\frac{C}{L} = \frac{2\pi \times (6.85 \times 10^{-11} \text{ F/m})}{2.6 \ln \left( \frac{2.1}{0.15} \right)} = 2.14 \times 10^{-12} \frac{\text{F}}{\text{m}}
\]
Problem 3: 25 points

A long straight solid cylinder of radius $a$, with its axis along the $z$ direction, carries a non-uniform current density $\vec{J}(r) = J(r)\hat{k}$. The non-uniform current density is

$$J(r) = \frac{2I_0}{\pi a^2} (1 - (r/a)^2), \quad r \leq a$$

and

$$J(r) = 0, \quad r > a.$$

(a) What is the total current enclosed by the cylinder?

(b) What is the B field outside the cylinder?

$$\vec{J}(r) = \frac{2I_0}{\pi a^2}$$

$$A = \pi r^2$$

$$dA = 2\pi r \, dr$$

$$J(r) = \int_0^r \vec{J}(r) \, dA = \int_0^r \frac{2I_0}{\pi a^2} (1 - \frac{r^2}{a^2}) 2\pi r \, dr$$

$$= \frac{2I_0}{\pi a^2} \int_0^a 2\pi (r - \frac{r^3}{3a^2}) \, dr$$

$$= \frac{2I_0 (2\pi)}{\pi a^2} \left[ \frac{r^2}{2} - \frac{r^4}{4a^2} \right]_0^a = \frac{4I_0}{a^2} \left[ \frac{a^2}{2} - \frac{a^4}{4a^2} \right]$$

$$= \frac{4I_0}{a^2} \left[ \frac{a^2}{2} - \frac{a^2}{4} \right] = \frac{4I_0}{a^2} \frac{a^2}{4} = I_0$$

$$\int \vec{B} \cdot d\vec{A} = B \cdot 2\pi r = \mu_0 I_0$$

$$B = \frac{\mu_0 I_0}{2\pi r} \text{ outside.}$$
Problem 4: 25 points

Find the magnitude and direction of the magnetic field at point $P$. 

\[ dB = \frac{\mu_0}{4\pi} \frac{I \, dl \times r}{r^2} \]

On segments I & III, there is no contribution since $dl \parallel r$. On segment II, $dl \times r$ is into the page. On segment IV, $dl \times r$ is also into the page.

\[ dB = \frac{\mu_0}{4\pi} \left( \frac{I \, dl}{r_1^2} + \frac{I \, dl}{r_2^2} \right) \]

\[ = \frac{\mu_0}{4} \int \left[ \frac{1}{r_1} + \frac{1}{r_2} \right] = \frac{4\pi \times 10^{-7}}{4} (3.0\, A) \left[ \frac{1}{0.2} + \frac{1}{0.4} \right] \]

\[ = 7.07 \times 10^{-6} T \quad \text{(direction into the page)} \]
Problem 5: 25 points

Around a cylindrical core of cross-sectional area 12.2 cm² are wrapped 125 turns of insulated copper wire. The two terminals are connected to a resistor. The total resistance in the circuit is 13.3 Ω. An externally applied, uniform, longitudinal magnetic field in the core changes from 1.57 T in one direction to 1.57 T in the opposite direction in 2.88 ms.

How much charge flows through the circuit?

\[ \phi_i = N B_i A \]

\[ \phi_f = N B_f A \]

\[ \Delta \phi = \phi_f - \phi_i = N B_f A - N B_i A \]

\[ = N A (B_f - B_i) \]

\[ = 2 N A B \]

\[ \varepsilon = -\frac{\Delta \phi}{\Delta t} \Rightarrow \dot{\phi} = \frac{\varepsilon}{R} = -\frac{\Delta \phi}{\Delta t} \frac{1}{R} \Rightarrow \frac{\Delta Q}{\Delta t} = -\frac{\Delta \phi}{\Delta t} \frac{1}{R} \]

\[ \Rightarrow \Delta Q = -\frac{1}{R} \Delta \phi = -\frac{2 N A B}{R} = -\frac{2 (125) (12.2 \times 10^{-4} \text{ m}^2 \times 1.57 T)}{13.3 \Omega} \]

\[ \Rightarrow \Delta Q = 3.6 \times 10^{-2} \text{ C} \]

Note: does not depend on \( \Delta t \).
Problem 6: 25 points

An electromagnetic wave has a frequency of 100 MHz and is traveling in a vacuum. The magnetic field is given by \( \vec{B}(z, t) = \left(10^{-8} T\right) \cos(kz - \omega t)i \)

(a) Find the wavelength and direction of propagation of this wave.

(b) Find the direction and magnitude of the \( \vec{E} \) field.

(c) Find the intensity of the wave.

(d) Find the associated radiation pressure.

\( \lambda = \frac{c}{f} = \frac{3 \times 10^8 \text{ m/s}}{100 \times 10^6 \text{ Hz}} = \frac{3 \times 10^8 \text{ m}}{10^8 \text{ Hz}} = 3 \text{ m} \)

\( \vec{E} \perp \vec{B} \Rightarrow \vec{E} = 3 V/m \cos(kz - \omega t)j \)

\( k = \frac{2\pi}{\lambda} = \frac{2\pi}{3} \text{ m}^{-1}, \ c_0 = 2\pi \times 10^8 \text{ MHz} \)

\( \vec{j} = \langle \sigma v \rangle = \langle \vec{E} \times \vec{B} \rangle = \frac{1}{\mu_0 c} \cdot \frac{1}{2} \frac{E^2}{m} \)

\( I = \frac{1}{2} \frac{E^2}{(377 \Omega)^2} = 1.2 \times 10^{-12} \text{ W/m}^2 \)

\( P_r = \frac{I}{c} = 3.7 \times 10^{-11} \text{ Pa} \text{ } \text{Pascals (N/m}^2) \)
Problem 7: 25 points

A radio station on the surface of the earth (EARTH-ROCK-FM) broadcasts with an average power of 50 kW. Assume that the transmitter radiates in all directions above the ground.

(a) Find the amplitude of the electric field detected by a satellite at a distance of 100 km from the antenna.

(b) Find the amplitude of the magnetic field detected by a satellite at a distance of 100 km from the antenna.

(c) Is the magnitude of this magnetic field much smaller, much larger, or about the same size as the earth's magnetic field?

\[ P = 50 \text{ kW} \]

\[ I = \frac{50 \times 10^3 \text{ W}}{2\pi \left(100 \times 10^3 \text{ m}\right)^2} \]

\[ = \frac{50 \times 10^3 \text{ W}}{6.28 \times 10^{10} \text{ m}^2} \]

\[ I = 7.96 \times 10^{-7} \text{ W/m} \]

\[ \Rightarrow a) \quad \frac{1}{2} I = \frac{1}{2} \frac{E^2}{377 \Omega} \quad \Rightarrow \quad E^2 = I(2)(377 \Omega) \]

\[ = 1.45 \times 10^{-2} \text{ V/m} \]

\[ B = \frac{E}{c} = \frac{2.45 \times 10^{-2} \text{ V/m}}{3 \times 10^8 \text{ m/s}} = 0.82 \times 10^{-10} \text{ T} \]

\[ \Rightarrow 10^{-7} \text{ T}, \text{ so the field is much smaller} \]

\[ \text{However note that the Earth’s B field is constant.} \]
Problem 8: 25 points

A coin lies at the bottom of a pool with depth $d$ and index of refraction $n$. Show that light rays that are close to the normal appear to come from a point $d_{app} = d/n$ below the surface. This distance is the apparent depth of the pool.

\[
\frac{d}{d_{app}} = \frac{\tan \theta}{\tan \theta_n} = \frac{\theta}{\theta_n} = \frac{n \theta_n}{\theta_n} = n
\]

\[
d_{app} = \frac{d}{n}
\]
Problem 9: 25 points

A converging lens has a focal length of 20 cm. A candle is located 50 cm from the lens.

(a) Draw a simple diagram of this setup showing the position of the object, lens, and image.

(b) Where is the image of the candle?

(c) By what factor is the image magnified?

(d) Is the image inverted or erect?

\[
\frac{1}{f} = \frac{1}{o} + \frac{1}{i} \Rightarrow \frac{1}{i} = \frac{1}{f} - \frac{1}{o}
\]

\[
f = 20 \text{ cm}
\]

\[
\frac{1}{o} = \frac{50 - 20}{20 \times 50} \Rightarrow i = \frac{20 \times 150}{30} = 33 \text{ cm}
\]

\[
M = \frac{-i}{o} = \frac{-33 \text{ cm}}{50 \text{ cm}} = -0.66
\]

\[
M = \frac{h'}{h} \Rightarrow h' \text{ is inverted and slightly demagnified}
\]
Conceptual Question I: 25 points

(a) Two point charges of equal magnitude but opposite sign are on the x-axis. +Q is at \( x = -a \) and -Q is at \( x = +a \). What is the magnitude and direction of the E field at the origin?

\[
E = \frac{2kQ}{a^2} \hat{r} = \frac{1}{2\pi\varepsilon_0} \frac{Q}{a^2} \hat{r}
\]

(b) Why does a comet’s tail point away from the sun?

Radiation pressure from the sun pushes the particles in the tail away.

(c) What is the B field of a solenoid with \( n \) turns/length, carrying a current \( I \)? Draw a sketch indicating the direction.

\[
B = \frac{\mu_0 n I}{\text{solenoid}}
\]

(d) A magnetic material has a magnetic susceptibility \( \chi_M = 10^{-5} \). Is it a ferromagnet, paramagnet, superconductor, or diamagnet?

Small positive susceptibility.

(e) The output of an AC generator is \( E = E_m \sin(\omega t) \) with \( E_m = 25 \text{ V} \) and \( \omega = 10 \text{ rad/s} \). It is connected to a 3-H inductor. What is the current as a function of time?

\[
I_{\text{max}} = \frac{E_{\text{max}}}{\mathcal{L}} = \frac{E_{\text{max}}}{\omega L} = \frac{25 \text{ V}}{10 \text{ rad/s} \cdot 3 \text{ H}} = \frac{25}{30} \text{ A}
\]

\[
I = \frac{5}{6} \text{ A} \sin(\omega t - 90^\circ) = \frac{5}{6} \sin(10t - 90^\circ) \text{ (in radians)}
\]

"E LI THE ICE man."
Conceptual Question II: 25 points

(a) Three capacitors with capacitance \( C = 10 \mu F \) are connected in parallel. What is the equivalent capacitance?

\[
\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}
\]

\[
= \frac{1}{30 \mu F}
\]

(b) An oscillating electric dipole is located along the y-axis in the plane of the paper. Where will the intensity of emitted radio waves be maximal and minimal?

(c) Why is it easy to make an RLC circuit with a high resonant frequency but hard to make one with a low resonant frequency?

The resonant frequency is \( \omega^2 \approx \frac{1}{LC} \)

To obtain a low resonant frequency, need very large capacitors and inductors.

(d) The image in a convex mirror (the type in a 7-11 convenience store, \( R < 0 \)) is magnified, demagnified, unchanged real, virtual and erect, inverted. (Pick the correct choices).

\[
\frac{1}{l} = \frac{1}{f} - \frac{1}{\infty}
\]

(usually \( o > f \))

\[
\text{Since } f = + \frac{R}{2R} \Rightarrow \text{ usually } m = \frac{r}{R}
\]

(e) Three resistors \( R_1, R_2 \) and \( R_3 \) are connected in parallel, assume \( R_1 = R_2 = R \) and \( R_3 = 2R \). What is the equivalent resistance in terms of \( R \)?

\[
\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{2R}
\]

\[
= \frac{2}{R} + \frac{1}{2R}
\]

\[
= \frac{4 + 1}{2R} = \frac{5}{2R}
\]

\[
R_{eq} = \frac{2}{5} R
\]
Conceptual Question III: 25 points

(a) Consider a parallel plate capacitor charging up at a rate $dQ/dt = 2.0 \text{ A}$. What is the conduction current and displacement current between its plates?

\[
\begin{align*}
  \text{I}_{\text{conduction}} &= 0 \\
  \text{I}_{\text{displacement}} &= 2.0 \text{ A} \\
\end{align*}
\]

(b) What is Gauss' Law for magnetism? What are the consequences for magnetic monopoles?

\[
\oint \mathbf{B} \cdot d\mathbf{A} = 0
\]

\[\text{no magnetic monopoles} \quad \text{(consistent with experiment so far)}\]

(c) An electromagnetic radiation source radiates uniformly in all directions. How does the magnitude of the E field vary with distance: (i) E is constant, (ii) $E \propto 1/r^2$, (iii) $E \propto 1/r$? $E \propto 1/r^2$.

\[
\int \mathbf{E} \cdot d\mathbf{s} \Rightarrow \int 0 \mathbf{c} \frac{1}{r^2} \Rightarrow \int E \mathbf{0} \frac{1}{r}
\]

(d) A current loop is in the plane of the paper. The B field is directed to the right. What is the direction of the torque on the current loop?

\[
\mathbf{m} = IA \mathbf{n} \Rightarrow \text{points into the paper}
\]
\[
\mathbf{\tau} = \mathbf{m} \times \mathbf{B} \Rightarrow \mathbf{\tau} \text{ points downward}
\]

(e) You are stranded in outer space outside your spaceship. Fortunately, you have a 100 MW laser attached to your spacesuit. How can you get back to your spaceship?

\[\text{Fire in the direction opposite to the spaceship. The recoil momentum will send you back to the spaceship.}\]