Composition of bulk silicate Earth and global geodynamics

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Overview

- Motivation: Thermal evolution of Earth
- Global mass balance and the composition of bulk silicate Earth (BSE)
- Thermal evolution revisited
Q. How was it like in the past?

- Average plate velocity \(\sim 4\text{cm/yr}\)
- Global heat flux \(\sim 44\text{TW}\)
Global heat balance equation

\[ C \frac{dT}{dt} = H(t) - Q(t) \]

- \( T \) - average internal temperature
- \( C \) - heat capacity of Earth
- \( H \) - radiogenic heating (easy to specify)
- \( Q \) - convective heat flux (not straightforward)

\[ Q(t) = H(t) - C \frac{dT}{dt} \]

Surface heat flux is a combination of internal heat production and secular cooling.
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- \( U\text{rey ratio} = \frac{H}{Q} \approx 0.3 \)

Surface heat flux is a combination of internal heat production and secular cooling.
How to parameterize $Q(t)$?

From the theory of thermal convection:

$$\text{Nu} \propto \text{Ra}^\beta$$

(surface heat flux) $\propto$ (convective vigor)$^\beta$

Mantle viscosity is strongly temperature-dependent:

$$\text{Ra} \propto \eta(T)^{-1}$$

By combining these two relations we may obtain:

$$Q = Q(T(t))$$

Hotter mantle is less viscous, convects faster, and releases more heat.
Thermal catastrophe

Whole-mantle convection does not produce a reasonable thermal history when combined with what we know about global heat budget!

Range of upper mantle temperature [Abbott et al., 1994]

Why does this happen?

Note: we are back-tracking a thermal history from the present time.
Why thermal catastrophe?

Positive feedback loop:

- $U_r=0.3$
- $\Rightarrow$ 70% secular cooling
- $\Rightarrow$ higher mantle temperature
- $\Rightarrow$ hotter mantle convects faster
- $\Rightarrow$ higher heat flux (more rapid than increase in internal heating)
- $\Rightarrow$ even lower $U_r$
How to avoid catastrophe?

A. Whole-mantle convection with Ur=0.7 (‘standard’ geophysical model)

B. Layered-mantle convection?

C. Different Q(T)?
Global mass (im)balance arguments for layered convection

• Missing argon paradox
• Missing heat-source paradox (including U-Th systematics)
• Refractory lithophile mass balance (REE, Ca, Al, Ti, U, Th, ...)
• Nd isotope mass balance

All of these arguments are based on the following negative result:

\[ \text{CC+DM} \neq \text{BSE} \]
(i.e., failure to reconstruct BSE from CC and DM)

But how do we know about BSE??
Chondrite-Sun coincidence

Carbonaceous chondrites = the most ‘primitive’ kind of meteorite

Chondrites are the ‘hand sample’ of the bulk solar system, which we can analyze in great details (to figure out the age of the solar system, etc).
Cosmochemical and geochemical jargons for element behavior

- **Regarding condensation temperatures**
  - **Volatile** - low condensation $T$
  - **Refractory** - high condensation $T$

- **Regarding chemical partitioning**
  - **Lithophile** - like to be with silicates (crust and mantle)
  - **Chalcophile** - like to be with sulfur
  - **Siderophile** - like to be with iron (core)
  - **Atmophile** - like to be in the atmosphere
Composition of Bulk Silicate Earth (=crust + mantle)

- Should be *similar* to that of carbonaceous chondrites in terms of *refractory lithophile elements* (Al, Ca, Ti, Sc, V, REE, U, Th, ...)

- But *how* similar?

- Sm-Nd & Lu-Hf isotope systems tell us BSE’s Sm/Nd and Lu/Hf should not deviation from chondritic values by more than 5%.
Nailing down the **absolute** abundance of RLEs

McDonough & Sun [1995] - the de facto standard model

Two (and only two) assumptions:

- Compotional trend in mantle peridotites is ‘melting’ trend. Along such a trend should exist the primitive mantle (or BSE).
- By imposing chondritic constraints on the ratio of RLE, we should be able to find the location of BSE on the trend.

Note: Other studies employ more assumptions, the validity of which are often questionable.
• Compotional trends in mantle peridotites are ‘melting’ trends. Along these trends should exist primitive mantle (or BSE).

• BSE contents of Ti, Al, & Ca are estimated using chondritic RLE ratios (see figure).

• Their concentrations are ~2.75 times more than those in CI-chondrite.

• This enrichment factor (~2.75) fixes the absolute concentrations of all RLEs in BSE.

Nailing down the **absolute** abundance of RLEs
McDonough & Sun [1995]
Problems with previous BSE models

- Defining ‘melting trends’ in the multi-dimensional compositional space is difficult.

- Different chondritic RLE ratios often yield different BSE estimates.

- Peridotite data have large scatters and outliers, which must influence geochemical inference.

These issues make it difficult to estimate model uncertainty (“How much should we trust one particular BSE model?”).
A new approach
[Lyubetskaya and Korenaga, JGR, 2007]

• Defining linear and nonlinear ‘melting trends’ in the multi-dimensional compositional space: the principal component analysis in the lognormalized data space

• Different chondritic RLE ratios: stochastic least-squares inversion to impose all contraints simultaneously

• Large scatters and outliers: the bootstrap resampling method to propagate data uncertainty to model uncertainty
Compositional covariation in multidimensions

- MgO $\leq 43$ wt%  
- $[\text{La/Yb}]_N \leq 2$  
- no garnet peridotites  
- no cratonic peridotites ($\text{Mg}#<91$)

[Lyubetskaya and Korenaga, 2007]
Principal component analysis
= rotation of coordinate axes

Most of variations in peridotite composition can be modeled by the 1st principal component only (i.e., “melting” trend)
Principal component analysis

>80% of total variance is explained by the first principle component

\[
\log C_i(q) \approx qa_i + \mathcal{E}(\log C_i) + \omega_i
\]
Stochastic inversion

[Lyubetskaya and Korenaga, 2007]

logarithmic enrichment factor w.r.t Cl chondrites:

\[ \varepsilon_i = \log C_i(q) - (\log C_i)_{CI} \]

If you have \( N \) refractory lithophile elements, there are \( N \) enrichment factors, and we seek to obtain the most consistent set of enrichment factors by minimizing the following cost function:

\[ \chi^2(q) = \sum_i (\varepsilon_i - \bar{\varepsilon})^2 \]

where \( \bar{\varepsilon} = \frac{1}{N} \sum_i \varepsilon_i \)

Formula for best-fit \( q \):

\[ q_{PM} = -\frac{\sum_i b_i d_i}{\sum b_i^2} \]

This determines the absolute concentration of RLEs.
$q_{PM}$ for primitive mantle (BSE)
Bootstrap resampling

Create pseudodata by sampling randomly from N data, by N times with repetitive sampling allowed.
How we actually implement the whole thing

[Lyubetskaya and Korenaga, 2007]
New BSE Model
Lyubetskaya and Korenaga [2007]

The new model of the primitive mantle is similar to the previous models in terms of major elements concentrations, Mg, Si, Fe, and is different from the previous models in terms of the RLE enrichment factor (Earth's abundance of the RLEs, normalized by their chondritic abundance).

The new estimate of the enrichment factor predicts \(~20\%\) lower bulk Earth abundances of
- Ca, Al, Ti
- REE elements
- heat producing elements K, Th, U
- volatile elements Rb, Na, Ba, As, B, etc.
This (subtle) difference in model revision turns out to be sufficient to resolve the missing heat-source and missing argon paradoxes.
Whole-mantle convection

‘Low-CaRb’ marble-cake mantle

- Given its uncertainty, the new BSE model still allows the presence of a hidden reservoir, but doesn’t require it; its size is probably small and exists as small-scale heterogeneities.
- The convecting mantle as a whole (‘interactive mantle’) may be more enriched than the pure MORB source mantle, and a recent estimate by Langmuir indicates petrological Ur~0.3 (±?).

[Lyubetskaya and Korenaga, 2007]
How to avoid catastrophe?

A. Whole-mantle convection with Ur=0.7

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C. Different $Q(T)$ for plate-tectonic convection

\[ C \frac{dT}{dt} = H(t) - Q(t) \]
Plate-tectonic convection

- Energy input from descending slab
- Energy output by dissipation at (1) bulk mantle and (2) subduction zone

[Solomatov, 1995; Conrad and Hager, 1999]

Key parameter: plate thickness

Thicker plate - more negative buoyancy, but more difficult to bend

How plate thickness changes with mantle temperature
Mantle not only convects but also melts

Plate must be bent at subduction to generate plate tectonics.

Mid-ocean ridge magmatism by decompressional melting

... and this melting removes impurities (e.g., H₂O), leaving stiffer residual mantle (dehydration stiffening)

[Karato, 1986; Hirth and Kohlstedt, 1996]

Viscosity profiles for mantle beneath a mid-ocean ridge
[Hirth and Kohlstedt, 1996]
Hotter mantle starts to melt deeper

Hotter mantle creates thicker oceanic crust and thicker depleted lithospheric mantle.

[Langmuir et al., 1992]
Plate-tectonic Q(T)

Effects of mantle melting on plate dynamics may have resulted in reduced heat flux in the past.

[Korenaga, 2006]
New evolution model

Reduced heat flux for hotter mantle breaks the positive feedback loop. Even $Ur=0.15$ (with whole-mantle convection) results in a reasonable thermal history.

[Korenaga, 2006]
Past plate motion and mantle mixing

![Graph showing past plate motion and mantle mixing](image-url)

[Note: Graphs illustrating past plate motion and mantle mixing with different rates and time scales.]

[Korenaga, 2006]
Q. Does observed isotope heterogeneity require a special mechanism (other than whole-mantle convection)?

A. Probably no. Sluggish plate tectonics in the past implies inefficient mantle mixing even with whole-mantle convection.

[Hofmann, 1997]
Conclusion

• Whole-mantle convection can satisfy a wide range of geophysical and geochemical constraints on the structure and evolution of Earth’s mantle, if we take into account the revised BSE composition and the new heat-flow scaling law for plate-tectonic convection.

• Archean dynamics is probably characterized by sluggish plate tectonics. Mantle has been mixed slowly, which has probably resulted in the presence of compositional heterogeneities with various spatial scales.