Physics 151
Due: Friday, April 24, 2009

Roster No.: SOLUTIONS

Score: Part A: 55 pts. (+4 pts. Bonus)

Total: 75 pts.

Take-Home Midterm Exam #3, Part A

NO exam time limit. Calculator required. All books and notes are allowed, and you may obtain help from others. Complete all of Part A *AND* Part B.

For multiple-choice questions, circle the letter of the one best answer (unless more than one answer is asked for). *For fill-in-the-blank and multiple-choice questions, you do NOT need to show your work.*

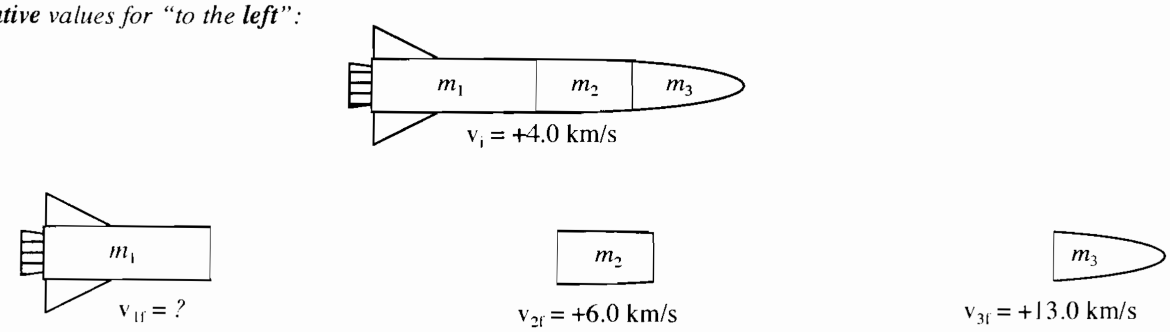
Show your work on all free-response questions. Be sure to use **proper units** and **significant figures** in your final answers.

Ignore friction and air resistance in all problems, unless told otherwise.

Physical constants: It's an open-book test, so you can look them up in your textbook!
Useful conversions: It's an open-book test, so you can look them up in your textbook!

1. A three-part rocket begins intact as a single object in distant outer space, traveling to the right at 4.0 km/s. The first "stage" (m_1 = 950,000 kg) explodes away from the rear of the rocket, with an unknown final velocity (v_{1f}). Later, the second stage (m_2 = 550,000 kg) explodes away from the rear, with a final velocity to the right at 6.0 km/s. The third stage (m_3 = 350,000 kg) ends up with a final velocity to the right of 13.0 km/s.

(All three masses move only along the x-axis. Ignore gravity throughout this problem. Assume that all parts of the rocket have constant masses.) For ALL answers to this problem, use ***positive*** *values for "to the* ***right***,*" and* ***negative*** *values for "to the* ***left***":



a. (3 pts.) What is the **final velocity** v_{1f} of the first stage? $-0.5\ \frac{km}{s}$ or $-0.47\ \frac{km}{s}$

Conservation of Momentum: $\Sigma p_{init} = \Sigma p_{final}$

$$(m_1+m_2+m_3)v_i = m_1 v_{1f} + m_2 v_{2f} + m_3 v_{3f}$$

$$(950{,}000\ kg + 550{,}000\ kg + 350{,}000\ kg)(4.0\ \tfrac{km}{s}) = (950{,}000\ kg)v_{1f} + (550{,}000\ kg)(6.0\ \tfrac{km}{s}) + (350{,}000\ kg)(13.0\ \tfrac{km}{s})$$

$$7.4\times10^6\ \tfrac{kg\cdot km}{s} = (950000\ kg)v_{1f} + 3.3\times10^6\ \tfrac{kg\cdot km}{s} + 4.55\times10^6\ \tfrac{kg\cdot km}{s}$$

$$-4.5\times10^5\ \tfrac{kg\cdot km}{s} = (950{,}000\ kg)v_{1f}$$

(Technically, loss of one sig fig. here due to subtraction)

$$\Rightarrow v_{1f} = -0.47\ \tfrac{km}{s}$$

b. (2 pts.) Find the **impulse** received by the first rocket ~~stage (in its~~ explosion): $-4.3\times10^6\ \frac{kg\cdot km}{s}$

Impulse: $\Delta p_1 = (p_f - p_i)_1$

$$= m_1 v_{1f} - m_1 v_{1i}$$

$$= (950{,}000\ kg)(-0.47\ \tfrac{km}{s}) - (950{,}000\ kg)(4.0\ \tfrac{km}{s})$$

$$= -450{,}000\ \tfrac{kg\cdot km}{s} - 3{,}800{,}000\ \tfrac{kg\cdot km}{s}$$

$$= -4{,}250{,}000\ \tfrac{kg\cdot km}{s}$$

c. (2 pts.) Find the **total impulse** received by the second rocket stage (after both explosions): $1.1\times10^6\ \frac{kg\cdot km}{s}$

$$\Delta p_2 = (p_f - p_i)_2$$

$$= m_2 v_{2f} - m_2 v_{2i}$$

$$= (550{,}000\ kg)(6.0\ \tfrac{km}{s}) - (550{,}000\ kg)(4.0\ \tfrac{km}{s})$$

$$= 3{,}300{,}000\ \tfrac{kg\cdot km}{s} - 2{,}200{,}000\ \tfrac{kg\cdot km}{s} = 1{,}100{,}000\ \tfrac{kg\cdot km}{s}$$

d. (2 pts.) Find the **total impulse** received by the third rocket stage (after both explosions): $3.2 \times 10^6 \frac{kg \cdot km}{s}$

$$\Delta p_3 = (p_f - p_i)_3 = m_3 v_{3f} - m_3 v_{3i}$$
$$= (350{,}000\ kg)(13.0 \tfrac{km}{s}) - (350{,}000\ kg)(4.0 \tfrac{km}{s})$$
$$= 4{,}550{,}000 \tfrac{kg \cdot km}{s} - 1{,}400{,}000 \tfrac{kg \cdot km}{s} = \underline{\underline{3{,}150{,}000 \tfrac{kg \cdot km}{s}}}$$

e. (1 pt.) Find the **sum** of these three impulses: **0**

Hint: According to conservation of momentum, what should the sum of all impulses be equal to?

Conservation of momentum: $\Sigma p_i = \Sigma p_f \Rightarrow \underline{\underline{\Sigma(\Delta p) = 0}}$. This agrees with the sum of the above three values: $\Delta p_1 + \Delta p_2 + \Delta p_3 = \underline{\underline{0}}$.

f. (1 pt.) During the second explosion, the **force** acting on m_2 was ________ the **force** acting on m_3.

A. less than B. the same as C. greater than

Newton's 3rd Law: The impulsive force of m_2 on m_3 is equal and opposite to the (simultaneous) impulsive force of m_3 on m_2.

g. (1 pt.) After both explosions, the **total kinetic energy** of the system…

A. increased B. decreased C. remained unchanged

$$\Delta K = \Sigma K_f - \Sigma K_i$$
$$= \left[\tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2 + \tfrac{1}{2} m_3 v_{3f}^2\right] - \tfrac{1}{2}(m_1 + m_2 + m_3) v_i^2$$
$$= \left[\tfrac{1}{2}(950{,}000\,kg)(-474 \tfrac{m}{s})^2 + \tfrac{1}{2}(550{,}000\,kg)(6000 \tfrac{m}{s})^2 + \tfrac{1}{2}(350{,}000\,kg)(13{,}000 \tfrac{m}{s})^2\right] - \tfrac{1}{2}(1{,}850{,}000\,kg)(4{,}000 \tfrac{m}{s})^2$$
$$= \left[1.07 \times 10^{11}\,J + 9.9 \times 10^{12}\,J + 2.96 \times 10^{13}\,J\right] - 1.48 \times 10^{13}\,J$$

$\Delta K = 2.48 \times 10^{13}\,J$ ← ΔK is positive, so K_{tot} increases.

h. (1 pt.) After both explosions, what is the **final velocity of the center-of-mass** of the three parts? $4.0 \frac{km}{s}$

Before any explosions, the rocket's $V_{CM} = +4.0 \frac{km}{s}$.

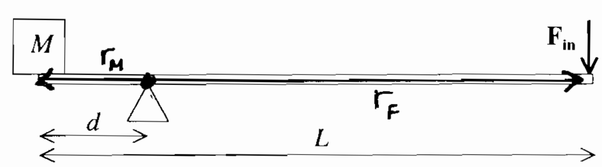
Since momentum is conserved, $\Sigma p_{init} = \Sigma p_{final} \Rightarrow (M \cdot V_{cm})_i = (M V_{cm})_f \Rightarrow (V_{cm})_f = (V_{cm})_i = \underline{\underline{4.0 \tfrac{km}{s}}}$.

OR: $$(v_{cm})_f = \frac{m_1 v_{1f} + m_2 v_{2f} + m_3 v_{3f}}{m_1 + m_2 + m_3} = \frac{(950{,}000\,kg)(-0.474 \tfrac{km}{s}) + (550{,}000\,kg)(6.0 \tfrac{km}{s}) + (350{,}000\,kg)(13.0 \tfrac{km}{s})}{950{,}000\,kg + 550{,}000\,kg + 350{,}000\,kg}$$

$$\Rightarrow (v_{cm})_f = 7{,}400{,}000 \tfrac{kg \cdot km}{s} / 1{,}850{,}000\,kg = 4.0 \tfrac{km}{s}$$

2. A simple lever gives the user a mechanical advantage: pushing down with a small force F_{in} on the long end of the lever creates a large upward force F_{out} on the short end.

Suppose that you want to support mass M at rest by using a lever of total length L whose fulcrum is located at a distance d from the short end of the lever. (Assume that the mass of the lever itself is negligibly small.)



a. (2 pts.) What is the **mechanical advantage** ($F_{out} \div F_{in}$) of this lever? *Express your answer **ONLY** in terms of **L, d,** and mathematical constants:* $\frac{L-d}{d}$

If lever and M are at rest, then:

$$\Sigma \tau = 0.$$
$$\tau_M - \tau_F = 0 \quad \text{(CCW positive, CW negative)}$$
$$F_M \cdot r_M \cdot \sin 90^\circ - F_{in} \cdot r_F \cdot \sin 90^\circ = 0$$
$$F_M \cdot d - F_{in}(L - d) = 0$$
$$\Rightarrow \frac{F_M}{F_{in}} = \underline{\underline{\frac{L-d}{d}}}.$$

b. (1 pt.) At what **distance *d*** should the fulcrum be positioned so that the lever has a mechanical advantage equal to exactly 1? *Express your answer **ONLY** in terms of **L** and mathematical constants:* $\frac{1}{2}L$

$$\text{Mech. Adv.} = \frac{F_M}{F_{in}} = \frac{L-d}{d}$$

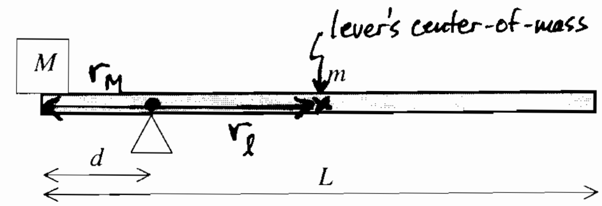
$$1 = \frac{L-d}{d} \Rightarrow d = L-d \Rightarrow \underline{\underline{d = \frac{1}{2}L}}$$

c. (1 pt.) As $d \to 0$, what **value** does the **mechanical advantage** approach?

A. 0 B. 1 C. ∞

$$\lim_{d \to 0}\left(\frac{L-d}{d}\right) = \frac{L}{0} = \underline{\underline{\infty}}.$$

We can eliminate the need for a human F_{in} by instead using a thick, massive lever with mass m, so that the weight of the lever itself balances M. (Assume that the lever has uniform thickness and density.)



d. (2 pts.) At what **distance *d*** should the fulcrum be positioned so that the lever's mass m exactly balances M? *Express your answer **ONLY** in terms of **L, m, M,** and mathematical constants:* $\frac{mL}{2(M+m)}$

When in balance, $\Sigma\tau = 0$

$$\tau_M - \tau_{lever} = 0$$

$$F_M \cdot r_M \cdot \cancel{\sin 90^\circ} - F_\ell \cdot r_\ell \cdot \cancel{\sin 90^\circ} = 0$$

$$(M \cdot g) \cdot d - (mg) \cdot \left(\frac{L}{2} - d\right) = 0$$

$$M\cancel{g}d = m\cancel{g}\left(\frac{L}{2} - d\right)$$

$$(M+m)d = \frac{mL}{2} \Rightarrow d = \frac{mL}{2(m+M)}$$

OR:

System will balance when fulcrum is positioned at center-of-mass: $\left(x_{cm} = \frac{M \cdot x_M + m \cdot x_\ell}{M+m}\right)$

$$\Rightarrow d = \frac{\cancel{M \cdot (0)} + m \cdot (L/2)}{M+m} = \frac{mL}{2(M+m)}.$$

e. (2 pts.; –1 for each error) Which of the following statements is/are **TRUE** for a balanced system? *Circle ALL that apply:*

A. If $M >> m$, $d \to 0$. $\longrightarrow \lim_{m \to 0}\left(\frac{mL}{2(M+m)}\right) = \frac{0}{2M} = \underline{\underline{0}}.$

B. If $M = m$, $d = L/2$. $\longrightarrow \left\{\lim_{M \to m}\left(\frac{mL}{2(M+m)}\right) = \frac{mL}{2(2m)} = \underline{\underline{\frac{L}{4}}}\right.$

C. If $M << m$, $d \to L$. $\longrightarrow \lim_{M \to 0}\left(\frac{mL}{2(M+m)}\right) = \frac{mL}{2m} = \underline{\underline{\frac{L}{2}}}$

f. (3 pts.) Suppose that d is positioned *incorrectly*, so that the lever system is *NOT* balanced. Let $d = 25$ cm, $L = 100.$ cm, $M = 7.0$ kg, and $m = 5.0$ kg. Immediately after the system is released from horizontal rest (as shown in the diagram above), what is the **net torque** about the fulcrum? **4.9 N·m** (ccw)

$$\Sigma\tau = \tau_M - \tau_\ell = F_M \cdot r_M \cdot \cancel{\sin 90^\circ} - F_\ell \cdot r_\ell \cdot \cancel{\sin 90^\circ}$$

$$= (M \cdot g) \cdot d - (m \cdot g) \cdot \left(\frac{L}{2} - d\right)$$

$$= \left[(7.0\text{kg})(9.80\text{m/s}^2)(0.25\text{m})\right] - \left[(5.0\text{kg})(9.80\text{m/s}^2)\left(\tfrac{1}{2}(1.00\text{m}) - 0.25\text{m}\right)\right] = 17.15\text{ N·m} - 12.25\text{ N·m} = \underline{\underline{4.9\text{ N·m}}}$$

(ccw = positive, cw = negative)

g. (3 pts.) In part (f), what is the system's **angular acceleration** immediately after it is released? (Assume that the lever is a thin rod with $I = \frac{7}{48}mL^2$, and that M is a "point mass" located at the very end of the lever.)

$$\Sigma\tau = I \cdot \alpha \Rightarrow \alpha = \frac{\Sigma\tau}{I_{tot}} = \frac{4.9\text{ N·m}}{1.167\text{ kg·m}^2} = \underline{\underline{4.2\ \frac{\text{rad}}{\text{s}^2}}}$$

4.2 rad/s² (ccw)

$$I_{tot} = I_M + I_{lever} = Md^2 + \frac{7}{48}mL^2 = (7.0\text{kg})(0.25\text{m})^2 + \frac{7}{48}(5.0\text{kg})(1.00\text{m})^2$$

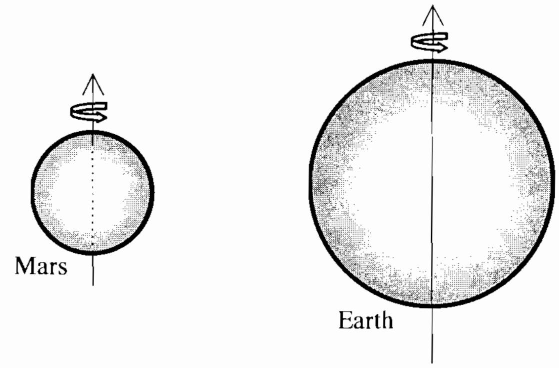
$$= 0.4375\text{ kg·m}^2 + 0.7292\text{ kg·m}^2 = \underline{1.167\text{ kg·m}^2}$$

h. (1 pt.) The angular acceleration in part (g) is…

A. **clockwise** B. **counter-clockwise**

3. Earth ("E") and Mars ("M") have rotation periods that are surprisingly similar, even though Mars has a radius that is only about half as large as Earth's.

For parts (a)–(e) of this problem, assume that both planets are solid, uniform-density spheres with the following properties (*NOT* actually true, but use *these* values):



rotation periods:	$T_E = T_M$
planetary radii:	$R_E = 2R_M$
planetary masses:	$M_E = 10M_M$

You may give the next five answers as pure integers, as simplified pure rational numbers, or as 3-sig.-fig. decimals:

$$\omega = \frac{2\pi}{T} \Rightarrow \frac{\omega_E}{\omega_M} = \frac{2\pi/T_E}{2\pi/T_M} = \frac{T_M}{T_E} = \underline{1}.$$

a. (1 pt.) The **angular speed** of Earth = **1** times the **angular speed** of Mars.

solid sphere:

$$I = \frac{2}{5}MR^2. \Rightarrow \frac{I_E}{I_M} = \frac{\cancel{\frac{2}{5}}M_E R_E^2}{\cancel{\frac{2}{5}}M_M R_M^2} = (10)(2)^2 = \underline{40}.$$

b. (1 pt.) The **moment of inertia** of Earth = **40** times the **moment of inertia** of Mars.

$$L = I \cdot \omega \Rightarrow \frac{L_E}{L_M} = \frac{I_E \cdot \omega_E}{I_M \omega_M} = (40)(1) = \underline{40}.$$

c. (1 pt.) The **angular momentum** of Earth = **40** times the **angular momentum** of Mars.

$$K = \frac{1}{2}I\omega^2 \Rightarrow \frac{K_E}{K_M} = \frac{\cancel{\frac{1}{2}}I_E\omega_E^2}{\cancel{\frac{1}{2}}I_M\omega_M^2} = (40)(1)^2 = \underline{40}.$$

d. (1 pt.) The **rotational kinetic energy** of Earth = **40** times the **rotational kinetic energy** of Mars.

$$\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3} \Rightarrow \frac{\rho_E}{\rho_M} = \frac{\frac{M_E}{\cancel{\frac{4}{3}\pi} R_E^3}}{\frac{M_M}{\cancel{\frac{4}{3}\pi} R_M^3}} = \frac{M_E}{M_M}\left(\frac{R_M}{R_E}\right)^3 = (10)\left(\frac{1}{2}\right)^3 = \underline{\frac{5}{4}}.$$

sphere

e. (1 pt.) The **density** of Earth = $\mathbf{\frac{5}{4}}$ times the **density** of Mars.

For the remaining parts of this question, use the *actual* values for Mars's physical properties (again, assume Mars is a solid sphere of uniform density):

$T_M = 24.6$ h
$R_M = 3.40 \times 10^3$ km
$M_M = 6.42 \times 10^{23}$ kg

solid sphere

$$I_{Mars} = \frac{2}{5}M_M R_M^2 = \frac{2}{5}(6.42\times10^{23}\,kg)(3.40\times10^{6}\,m)^2 = \underline{2.969\times10^{36}\,kg\cdot m^2}$$

$$\omega_{Mars} = \frac{2\pi}{T_M} = \frac{2\pi}{24.6\,h\left(\frac{3600\,s}{1\,h}\right)} = \underline{7.095\times10^{-5}\,\frac{rad}{s}}$$

$$L_M = I_M \cdot \omega_M = (2.969\times10^{36}\,kg\cdot m^2)(7.095\times10^{-5}\,\tfrac{rad}{s}) = \underline{2.106\times10^{32}\,\tfrac{kg\cdot m^2}{s}}$$

BONUS (+2 pts.) Calculate Mars's **angular momentum**, including MKS units: $\mathbf{2.11\times10^{32}\,\frac{kg\cdot m^2}{s}}$

$$K_{rot} = \frac{1}{2}I_M\omega_M^2 = \frac{1}{2}(2.969\times10^{36}\,kg\cdot m^2)(7.095\times10^{-5}\,\tfrac{rad}{s})^2 = \underline{7.471\times10^{27}\,J}$$

BONUS (+2 pts.) Calculate Mars's **rotational kinetic energy**, including MKS units: $\mathbf{7.47\times10^{27}\,J}$

f. (3 pts.) Calculate Mars's **density**, including MKS units: $3900\ \frac{kg}{m^3}$ or $3.90\times10^{3}\ \frac{kg}{m^3}$

Volume of sphere: $V_M = \frac{4}{3}\pi R_M^3 = \frac{4}{3}\pi\left(3.40\times10^{6}\ m\right)^3 = \underline{1.646\times10^{20}\ m^3}$

density: $\rho_M = \frac{M_M}{V_M} = \frac{6.42\times10^{23}\ kg}{1.646\times10^{20}\ m^3} = \underline{\underline{3.900\times10^{3}\ \frac{kg}{m^3}}}$

g. (1 pt.) A typical density for rock is ~2500 kg/m³, while iron (the most common metal in the solar system) has a density of ~9000 kg/m³. We can conclude that Mars's interior mass consists, very roughly, of a mixture of...

A. 99% rock and 1% metal $\Rightarrow \rho \approx 2600\ kg/m^3$
B. 80% rock and 20% metal $\Rightarrow \rho \approx 3800$
C. 50% rock and 50% metal $\Rightarrow \rho \approx 5800$
D. 20% rock and 80% metal $\Rightarrow \rho \approx 7700$
E. 1% rock and 99% metal $\Rightarrow \rho \approx 8900\ kg/m^3$

4. A water polo ball (which looks like a waterproof yellow volleyball) is a hollow sphere of radius 11.1 cm and mass 450. grams. Assume that the density of water is 1.000 g/cm³.

a. (2 pts.) Find the **volume** of the ball: $5.73\times10^{-3}\ m^3$ or **5.73 L**

sphere: $V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(0.111\ m)^3 = \underline{\underline{5.729\times10^{-3}\ m^3}}$

b. (1 pt.) While floating, what **mass** of water does the ball displace? **450. g** or **0.450 kg**

c. (2 pts.) While floating, what **volume** of water does the ball displace? **0.450 L** or $450.\ cm^3$ or $4.50\times10^{-4}\ m^3$

(b) Archimedes' principle: A floating object displaces a mass of fluid equal to the object's mass $= \underline{\underline{450.\ g}}$

(c) $V_w = \frac{m_w}{\rho_w} = \frac{450.\ g}{1.000\ g/cm^3} = \underline{\underline{450.\ cm^3}}$

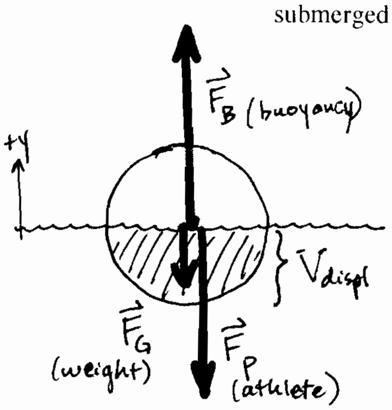
OR: Floating $\Rightarrow$ $F_B = F_G$

$\rho_w \cdot \cancel{g} \cdot V_{displ} = m_b \cancel{g}$ $\Rightarrow$ $V_{displ} = \frac{m_b}{\rho_w}$

$V_{displ} = \frac{450.\ g}{1.000\ g/cm^3} = \underline{\underline{450.\ cm^3}}$

AND: $m_w = \rho_w \cdot V_{disp} = \cancel{\rho_w}\cdot\left(\frac{m_b}{\cancel{\rho_w}}\right)$ $\Rightarrow$ $m_w = m_b = \underline{\underline{450.\ g}}$

d. (3 pts.) Suppose that a polo player starts pushing downward on the floating ball, gradually increasing his force. What is the magnitude of the **athlete's downward force** when the ball has exactly *half* of its volume submerged below the surface? **23.7 N**



$\Sigma F_y = m \cdot \cancel{a_y}^{\,0}$ ← equilibrium/static

$F_B - F_G - F_P = 0$

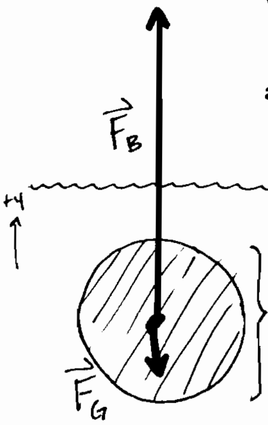
$\Rightarrow F_P = F_B - F_G$

$= \rho_w \cdot g \cdot V_{displ} - m_b \cdot g$

$= \left(1000\ \frac{kg}{m^3}\right)\left(9.80\ m/s^2\right)\left[\frac{1}{2}\left(5.73\times10^{-3}\ m^3\right)\right] - (0.450\ kg)(9.80\ m/s^2)$

$= 28.07\ N - 4.41\ N = \underline{23.66\ N}$

e. (3 pts.) Later, the polo player *completely* submerges the ball, pushing it a short depth below the surface of the water. Then he releases the ball from rest, underwater. *Immediately* afterward, what is the ball's upward



acceleration? (Neglect any drag force.) **115 m/s^2**

$$\Sigma F_y = m \cdot a_y$$

$$F_B - F_G = m \cdot a_y$$

$$\rho_w \cdot g \cdot V_{disp} - m \cdot g = m \cdot a_y$$

$$\left[(1000. \tfrac{kg}{m^3})(9.80 \, m/s^2)(5.73 \times 10^{-3} \, m^3)\right] - (0.450 \, kg)(9.80 \, m/s^2) = (0.450 \, kg) \cdot a_y$$

$$56.14 \, N - 4.41 \, N = (0.450 \, kg) \cdot a_y$$

$$51.73 \, N = (0.450 \, kg) \cdot a_y \Rightarrow a_y = 115.0 \, m/s^2$$

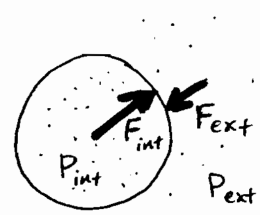
Suppose the water polo ball has an internal pressure (absolute pressure, *not* "gauge pressure") of 185 kPa.

f. (2 pts.) Find the **surface area** of the ball: **0.155 m^2**

sphere: $A = 4\pi R^2 = 4\pi(0.111 \, m)^2 = 0.1548 \, m^2$

g. (2 pts.) While the ball sits at rest in 1.00-atm air, what is the **net outward force** acting on the inside

of the ball? **1.30×10^4 N** (this is almost 3000 lbs!)



$$F_{net} = F_{int} - F_{ext} = P_{int} \cdot A - P_{ext} \cdot A$$

$$= (1.85 \times 10^5 \, Pa)(0.1548 \, m^2) - \overbrace{(1.013 \times 10^5 \, Pa)}^{1.00 \, atm}(0.1548 \, m^2)$$

$$= 2.864 \times 10^4 \, N - 1.568 \times 10^4 \, N = 1.296 \times 10^4 \, N$$

h. (3 pts.) At what **depth** in a swimming pool would the absolute pressure on the outside of the ball exceed

185 kPa, causing the ball to start to collapse? **8.5m** (This is approx. 28 feet deep.)
(Assume that the pool is located near sea level, so that there is 1.00 atm of *atmospheric* pressure at the water's surface. Your answer will be deeper than most swimming pools… but not by much!)

$$P_{tot} = P_{water} + P_{air}$$

$$P_{tot} = \rho_w \cdot g \cdot h + P_{air} \qquad \left(P_{air} = 1.00 \, atm = 1.013 \times 10^5 \, Pa\right)$$

depth

$$1.85 \times 10^5 \, Pa = (1000. \tfrac{kg}{m^3})(9.80 \tfrac{m}{s^2}) \cdot h + 1.013 \times 10^5 \, Pa$$

$$8.37 \times 10^4 \, Pa = (1000. \tfrac{kg}{m^3})(9.80 \tfrac{m}{s^2}) \cdot h$$

$$\Leftrightarrow \quad h = 8.54 \, m$$

Note: loss of 1 sig. fig. due to subtraction.

Physics 151
Spring 2009

Roster No.: **SOLUTIONS**

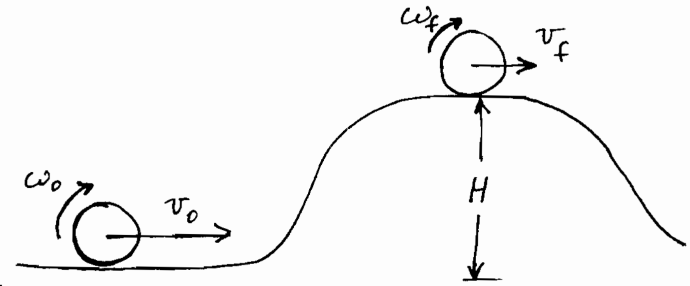
Score: Part B: 20 pts. possible

Take-Home Midterm Exam #3, Part B

1. A bowling ball (solid, uniform-density sphere of mass M and radius R) rolls *without slipping* toward a hill of maximum height H and varying slope.

a. (5 pts.) If the bowling ball starts with linear speed v_0 at the bottom of the hill, what is its **linear speed** as it rounds the crest of the hill? *Show your work completely.*

Express your final answer algebraically ***ONLY*** *in terms of "known" variables* ***M, R, H, v_0, g,*** *and any necessary numerical constants (but NOT ω or any other variables!).* ***Simplify*** *your final answer as much as possible!*



Conservation of Energy:

$$E_i = E_f$$

$$(K_{trans} + K_{rot} + U_{gr})_i = (K_{trans} + K_{rot} + U_{gr})_f$$

$$\frac{1}{2}Mv_0^2 + \frac{1}{2}I\omega_0^2 + Mg\cancel{y_i}^{\,0} = \frac{1}{2}Mv_f^2 + \frac{1}{2}I\omega_f^2 + MgH$$

$$\Rightarrow \frac{1}{2}Mv_f^2 = \frac{1}{2}Mv_0^2 + \frac{1}{2}I(\omega_0^2 - \omega_f^2) - MgH.$$

Rolling without slipping: $\omega = \frac{v}{R}$.
Solid sphere: $I = \frac{2}{5}MR^2$

$$\Rightarrow v_f^2 = v_0^2 + \frac{I}{M}(\omega_0^2 - \omega_f^2) - 2gH$$

substitute to eliminate I and ω.

$$v_f^2 = v_0^2 + \frac{\frac{2}{5}MR^2}{M}\left(\left(\frac{v_0}{R}\right)^2 - \left(\frac{v_f}{R}\right)^2\right) - 2gH$$

$$v_f^2 = v_0^2 + \frac{2}{5}(v_0^2 - v_f^2) - 2gH$$

$$\frac{7}{5}v_f^2 = \frac{7}{5}v_0^2 - 2gH$$

$$\boxed{v_f = \sqrt{v_0^2 - \frac{10}{7}gH}}$$

b. (1 pt.) Suppose someone makes a "hollow" bowling ball whose center is mostly empty, but whose outer edge is loaded with dense metal. Overall, it has the same mass M and outer radius R as a standard bowling ball.

If you roll the *hollow* bowling ball with the *same* initial linear speed v_0 toward the same hill H, the hollow ball will reach the top of the hill with ______________ **linear speed** than the standard bowling ball.

A. **greater**
B. **less**
C. **the same**

Hollow sphere: $I = \frac{2}{3}MR^2$ (instead of solid sphere: $\frac{2}{5}MR^2$)

Same method as above gives:

$$v_f = \sqrt{v_0^2 - \frac{6}{5}gH} \text{ for hollow sphere.}$$

Compare "v_f" $\Rightarrow \sqrt{v_0^2 - \frac{10}{7}gH} < \sqrt{v_0^2 - \frac{6}{5}gH}$

$$\Rightarrow \boxed{v_{solid} < v_{hollow}}$$

2. An ice skater begins by spinning with a rotation period of 1.20 s when her arms and legs are outstretched, giving her whole body an initial moment-of-inertia of 2.56 kg·m^2. By pulling her arms and legs in close to her spin axis, her moment of inertia decreases to 0.850 kg·m^2. (Ignore all friction.)

a. (5 pts.) What is the skater's **final rotation period**? *Show your work completely.*

Initial angular speed: $\omega_i = \dfrac{2\pi}{T_i} = \dfrac{2\pi}{1.20\,s} = \underline{5.236\,\frac{rad}{s}}$.

Conserv. of Angular Momentum: $L_i = L_f$

$$I_i \cdot \omega_i = I_f \cdot \omega_f$$

$$(2.56\,kg\cdot m^2)(5.236\,\tfrac{rad}{s}) = (0.850\,kg\cdot m^2)\,\omega_f$$

$$\Rightarrow \quad \omega_f = \underline{15.77\,\tfrac{rad}{s}}$$

Final period: $T_f = \dfrac{2\pi}{\omega_f} = \dfrac{2\pi}{15.77\,\frac{rad}{s}} = 0.3984\,s \approx \boxed{0.398\,s}$

b. (4 pts.) How much total **work do** the skater's muscles perform while pulling in her arms and legs? (*Hint:* What is her increase in kinetic energy?)

Work-energy Theorem: $W_{tot} = \Delta K_{rot}$

$$= K_f - K_i$$

$$= \tfrac{1}{2} I_f \omega_f^2 - \tfrac{1}{2} I_i \omega_i^2$$

$$= \tfrac{1}{2}(0.850\,kg\cdot m^2)(15.77\,\tfrac{rad}{s})^2 - \tfrac{1}{2}(2.56\,kg\cdot m^2)(5.236\,\tfrac{rad}{s})^2$$

$$= 105.7\,J - 35.1\,J$$

$$W_{tot} = \underline{70.6\,J} \approx \boxed{71\,J}$$

loss of 1 sig fig due to subtraction

c. (5 pts.) If the skater undergoes a constant angular acceleration over a span of 7.0 s, how many (fractional) **revolutions** does she execute during the acceleration? (blaarrrp… how dizzying!) *Show your work completely.*

Find angular acceleration: $\omega_f = \omega_0 + \alpha t$

$$\Rightarrow \alpha = \frac{\omega_f - \omega_0}{t} = \frac{15.77\,\frac{rad}{s} - 5.236\,\frac{rad}{s}}{7.0\,s} = \underline{1.50\,\tfrac{rad}{s^2}}$$

Angular Displacement: $\Delta\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2$

$$= (5.236\,\tfrac{rad}{s})(7.0\,s) + \tfrac{1}{2}(1.50\,\tfrac{rad}{s^2})(7.0\,s)^2 = \underline{73.5\,rad}$$

OR: $\omega_f^2 = \omega_0^2 + 2\cdot\alpha\cdot\Delta\theta$

$$(15.77\,\tfrac{rad}{s})^2 = (5.236\,\tfrac{rad}{s})^2 + 2(1.50\,\tfrac{rad}{s^2})\,\Delta\theta \Rightarrow \Delta\theta = \underline{73.5\,rad}$$

Convert to revolutions: $\Delta\theta = 73.5\,rad\left(\dfrac{1\,rev}{2\pi\,rad}\right) = 11.7\,rev \approx \boxed{12\,rev}$