Physics 151
Due: Friday, April 20, 2007

Roster No.: SOLUTIONS

Score: Part A: 32 pts. possible
Total: 50 pts. possible

Take-Home Midterm Exam #3, Part A

NO exam time limit. Calculator required. All books and notes are allowed, and you may obtain help from others. Complete all of Part A *AND* Part B.

For multiple-choice questions, circle the letter of the one best answer (unless more than one answer is asked for). For fill-in-the-blank and multiple-choice questions, you do NOT need to show your work.

Show your work on all free-response questions. Be sure to use **proper units** and **significant figures** in your final answers.

Ignore friction and air resistance in all problems, unless told otherwise.

Physical constants: It's an open-book test, so you can look them up in your textbook!
Useful conversions: It's an open-book test, so you can look them up in your textbook!

1. (4 pts.) **Convert** the following quantities into the given units. Fill in the blanks. (You do NOT need to show your work.) Use *scientific notation* where appropriate (very large or very small values), and express all final values to *THREE significant figures.*

a. 9.85×10^{-7} MW = 985 mW

$$\left(9.85 \times 10^{-7}\ \cancel{\text{MW}}\right)\left(\frac{10^6\ \cancel{\text{W}}}{1\ \cancel{\text{MW}}}\right)\cdot\left(\frac{1\text{ mW}}{10^{-3}\ \cancel{\text{W}}}\right) = \underline{\underline{985\text{ mW}}}.$$

b. 3.00×10^5 μm/ns = 1.08×10^9 km/h

$$\left(3.00 \times 10^5\ \frac{\cancel{\mu\text{m}}}{\cancel{\text{ns}}}\right)\left(\frac{10^{-6}\ \cancel{\text{m}}}{1\ \cancel{\mu\text{m}}}\right)\left(\frac{1\text{ km}}{10^3\ \cancel{\text{m}}}\right)\left(\frac{1\ \cancel{\text{ns}}}{10^{-9}\ \cancel{\text{s}}}\right)\left(\frac{3600\ \cancel{\text{s}}}{1\text{ h}}\right) = \underline{\underline{1.08 \times 10^9\ \frac{\text{km}}{\text{h}}}}.$$

c. 12,500 rpm = 0.208 kHz (“rpm” = rev/min)

$$\left(12{,}500\ \frac{\text{rev}}{\cancel{\text{min}}}\right)\cdot\left(\frac{1\ \cancel{\text{min}}}{60\text{ s}}\right) = 208\ \frac{\text{rev}}{\text{s}} = 208\ \cancel{\text{Hz}}\left(\frac{1\text{ kHz}}{10^3\ \cancel{\text{Hz}}}\right) = \underline{\underline{0.208\text{ kHz}}}$$

d. 19,300 kg/m³ = 19.3 mg/mm³ (this is the density of gold)

$$\left(19{,}300\ \frac{\cancel{\text{kg}}}{\cancel{\text{m}^3}}\right)\left(\frac{10^3\ \cancel{\text{g}}}{1\ \cancel{\text{kg}}}\right)\left(\frac{1\text{ mg}}{10^{-3}\ \cancel{\text{g}}}\right)\left(\frac{10^{-3}\ \cancel{\text{m}}}{1\text{ mm}}\right)^3 = \underline{\underline{19.3\ \frac{\text{mg}}{\text{mm}^3}}}.$$

2. A 430-g soccer ball flying directly to the left at speed v_b strikes the head of a soccer player. The entire impact lasts 15 ms. Afterward, the ball rebounds directly to the right at the same speed v_b. Express all answers in **MKS units**:

a. (2 pts.) If $v_b = 21$ m/s, what is the total **impulse** that the ball receives during the impact? 18 $\frac{\text{kg}\cdot\text{m}}{\text{s}}$ or N·s

b. (2 pts.) What is the **average force** that the ball exerts on the player's head during part (a)? 1.2×10^3 N or 1200 N

c. (2 pts.) The player's head has a mass of 4.8 kg. The player will experience a concussion (brain injury) if his head undergoes an average acceleration greater than 75 gees. (1 gee = 9.80 m/s².) Ignore the head's attachment to the rest of the player's body; just consider the head alone.) Assuming that an impact always lasts 15 ms regardless of the value of v_b, what is the **minimum ball speed** v_b that will cause a concussion? 62 m/s
(Fortunately, speeds this large are *not* normally attained by soccer balls during play.)

(a) Choose +x-direction to the right: $v_{\text{ball, init}} = -v_b$, $v_{\text{ball, final}} = +v_b$.

Impulse: $\Delta p = m \cdot \Delta v = m \cdot (v_f - v_i) = m \cdot (+v_b - (-v_b))$

$\Rightarrow \Delta p = m \cdot 2v_b = (0.43\text{ kg})(2)(21\text{ m/s}) = \underline{18.06\ \frac{\text{kg}\cdot\text{m}}{\text{s}}}$.

(b) $F_{av} = \frac{\Delta p}{\Delta t}$ ← impulse received by head

↑ ave. force acting on head

By Newton's 3rd Law, F_{av} on head = F_{av} on ball, and Δt is same for both head and ball,
∴ Δp of head = Δp of ball.

2.(b) continued:

$$\therefore \quad \underset{\text{on head}}{F_{av}} = \frac{\Delta p_{head}}{\Delta t} = \frac{\Delta p_{ball}}{\Delta t} = \frac{18.06\ \frac{kg\cdot m}{s}}{15\ \cancel{ms}\left(\frac{10^{-3} s}{1\ \cancel{ms}}\right)} = 1204\ N \approx \underline{\underline{1.2\times 10^{3} N}}$$

(c) Take limiting case of minimum acceleration and force to cause concussion:

<u>Force</u>: $\underset{\text{on head}}{F_{av}} = m \cdot \underset{\text{on head}}{a_{av}} = (4.8\ kg)(75\ \text{gees})\left(\frac{9.80\ m/s^2}{1\ \text{gee}}\right) = \underline{\underline{3528\ N}}$ (minimum force)

<u>Impulse</u>: $\Delta p_{head} = F_{av} \cdot \Delta t = (3528\ N)(0.015\ s) = 52.92\ \frac{kg\cdot m}{s}$

Impulse of <u>head</u> and impulse of <u>ball</u> must be <u>equal</u> and opposite, due to Newton's 3rd Law. Thus: $\Delta p_{ball} = m_{ball} 2 v_b$ (see part (a))

$$52.92\ \frac{kg\cdot m}{s} = (0.43\ kg)\cdot 2 \cdot v_b$$

$$\Rightarrow \quad \underline{\underline{v_b = 61.53\ m/s}}\ .$$

(This is approx. 138 mph! Fortunately, soccer balls are usually much slower.)

<u>Alternative solution:</u> $a_{av} = \frac{\Delta v}{\Delta t}$ for <u>head</u>

$$\Rightarrow \underline{\Delta v_{head}} = a_{av} \cdot \Delta t = (75\ \text{gees})\left(\frac{9.80\ m/s^2}{1\ \text{gee}}\right)(0.015\ s) = \underline{\underline{11.0\ m/s}}\ \text{(to the left)}$$

<u>Conserv. of momentum</u> in ball/head collision: $m_h \cdot v_{hi} + m_b \underset{\overset{\uparrow}{-v_b}}{v_{bi}} = m_h v_{hf} + m_b \underset{\overset{\uparrow}{v_b}}{v_{bf}}$

(Choose +x to the right) ⇒

$$\Rightarrow m_h v_{hi} - m_h v_{hf} = m_b v_b + m_b v_b$$

$$\Rightarrow -m_h \cdot \Delta v_{head} = 2 m_b v_b$$

$$\Rightarrow \quad v_b = \frac{-m_h \cdot \Delta v_{head}}{2\ m_b}$$

$$= \frac{-(4.8\ kg)(-11.0\ m/s)}{2\ (0.43\ kg)} = 61.5\ m/s$$

$\underline{\underline{v_b = 61.5\ m/s}}$ — same answer as above.

3. An airplane propeller with a moment of inertia (about its center) equal to 55 kg·m^2 starts out spinning at 330 rpm. The airplane's engine then takes 25 s to accelerate the propeller (at a constant acceleration) up to a new rate of 1600 rpm. Express all answers in **MKS units**:

a. (2 pts.) What is the propeller's **angular acceleration** during this time? $5.3\ \mathrm{rad/s^2}$

b. (2 pts.) If the propeller measures 1.5 m from center to tip, what is the **total distance traveled** by the tip of the propeller during the acceleration in part (a)? 3.8×10^3 m or 3.8 km
(You may express your answer in either meters or kilometers.)

c. (2 pts.) Suppose that air resistance opposes the spinning propeller with a constant torque of 85 N·m, regardless of the propeller's speed. (Not actually true! But we'll assume it for simplicity.) In that case, what is the **engine's torque** acting on the propeller during the acceleration in part (a)? 3.8×10^2 N·m or 380 N·m

d. (2 pts.) What is the total **average power output** of the engine during this acceleration? Be sure to include the energy spent overcoming air resistance. Convert your answer to units of horsepower: 51 **hp**

(a) $\alpha_{av} = \dfrac{\Delta\omega}{\Delta t}$

$$\omega_i = 330\ \frac{\cancel{\text{rev}}}{\cancel{\text{min}}}\cdot\left(\frac{1\ \cancel{\text{min}}}{60\ \text{s}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \cancel{\text{rev}}}\right) = 34.56\ \frac{\text{rad}}{\text{s}}$$

$$\omega_f = 1600\ \frac{\cancel{\text{rev}}}{\cancel{\text{min}}}\left(\frac{1\ \cancel{\text{min}}}{60\ \text{s}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \cancel{\text{rev}}}\right) = 167.55\ \frac{\text{rad}}{\text{s}}$$

$$\alpha_{av} = \frac{\omega_f-\omega_i}{\Delta t} = \frac{167.55\ \frac{\text{rad}}{\text{s}} - 34.56\ \frac{\text{rad}}{\text{s}}}{25\ \text{s}} = \frac{132.99\ \frac{\text{rad}}{\text{s}}}{25\ \text{s}} = \underline{\underline{5.32\ \text{rad}/\text{s}^2}}$$

(b) $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$$\theta_f = 0 + \left(34.56\ \tfrac{\text{rad}}{\text{s}}\right)(25\ \text{s}) + \tfrac{1}{2}\left(5.32\ \tfrac{\text{rad}}{\text{s}^2}\right)(25\ \text{s})^2 = 864\ \text{rad} + 1662\ \text{rad}$$

$$= \underline{2526\ \text{rad}}.$$

$$s_f = \theta_f\cdot r = (2526\ \text{rad})(1.5\ \text{m}) = 3790\ \text{m} = \underline{\underline{3.8\times10^3\ \text{m}}} = \underline{\underline{3.8\ \text{km}}}$$

(c) Suppose τ_{engine} is counter-clockwise, then τ_{air} will be clockwise:

$$\Sigma\tau = I\cdot\alpha$$

$$\tau_{engine} - \tau_{air} = I\cdot\alpha \Rightarrow \tau_{engine} = I\cdot\alpha + \tau_{air}$$

$$= (55\ \text{kg}\cdot\text{m}^2)\left(5.32\ \tfrac{\text{rad}}{\text{s}^2}\right) + 85\ \text{N}\cdot\text{m}$$

$$= 292.6\ \text{N}\cdot\text{m} + 85\ \text{N}\cdot\text{m}$$

$$\tau_{engine} = 377.6\ \text{N}\cdot\text{m} \approx \underline{\underline{3.8\times10^2\ \text{N}\cdot\text{m}}}$$

(d) $W_{engine} = \tau_{eng}\cdot\Delta\theta = (377.6\ \text{N}\cdot\text{m})(2526\ \text{rad}) = \underline{953{,}900\ \text{J}}$

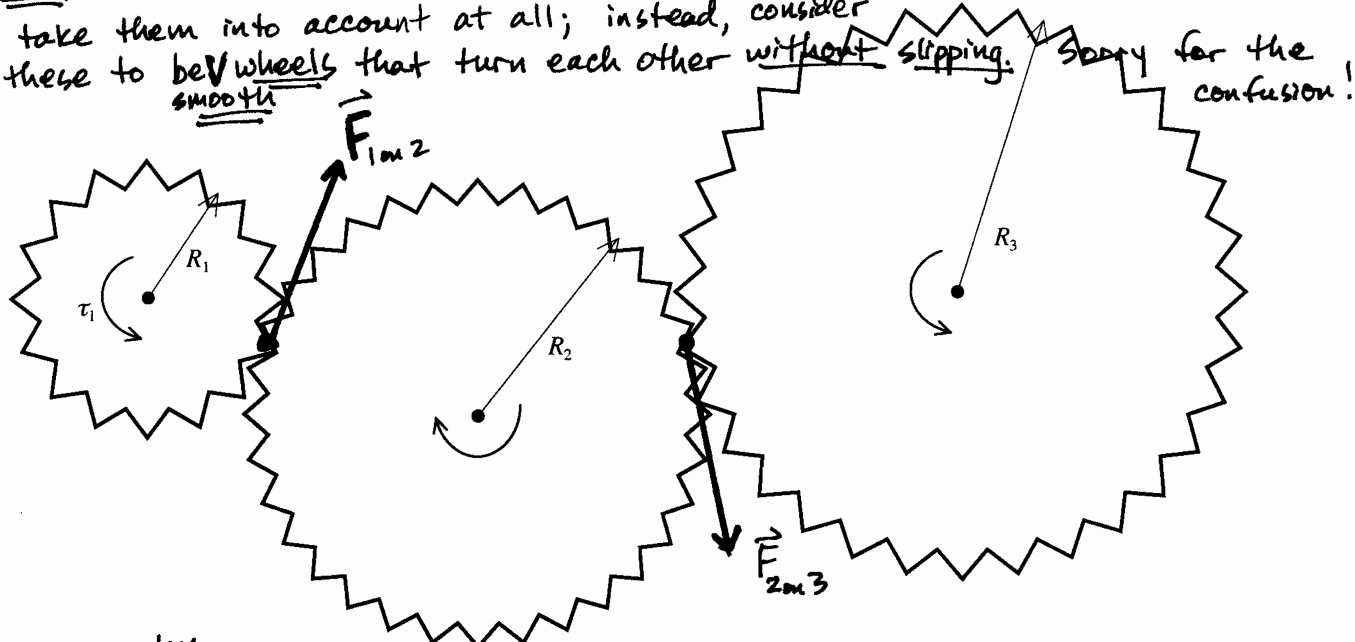
MISSING! CANNOT USE.

[NOTE: $W_{engine} \, \textcircled{- W_{air}} = \textcircled{\Delta K} = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 = \frac{1}{2}(55\ \text{kg}\cdot\text{m}^2)\left[\left(167.55\ \tfrac{\text{rad}}{\text{s}}\right)^2 - \left(34.56\ \tfrac{\text{rad}}{\text{s}}\right)^2\right] = \textcircled{739{,}200\ \text{J}}$]

CANNOT USE.

Then: $P_{av} = \dfrac{W_{eng}}{\Delta t} = \dfrac{953{,}900\ \text{J}}{25\ \text{s}} = \underline{38{,}160\ \text{W}}\cdot\left(\dfrac{1\ \text{hp}}{746\ \text{W}}\right) = \underline{51.1\ \text{hp}}$

Note: It was NOT my intention that you count the number of gear teeth or take them into account at all; instead, consider these to be smooth wheels that turn each other without slipping. Sorry for the confusion!



4. Three massless interlocking gears have average radii of R_1, R_2 and R_3, with relative sizes as shown above. (Their centers are all fixed, and there is no friction at their pivots. Assume that the size of the gears' teeth is much smaller than the gears' radii.)

a. (2 pts.) An electric motor exerts torque τ_1 on Gear #1, ultimately resulting in torque τ_3 on Gear #3. If the system's mechanical advantage is equal to the ratio τ_{out}/τ_{in}, what is the **mechanical advantage** (τ_3/τ_1) of this gear system?

Express your answer *ONLY* in terms of R_1, R_2, R_3, and numerical constants: R_3/R_1

b. (1 pt.) Suppose that the electric motor rotates Gear #1 at a constant angular speed. As you watch all three gears turn, which gear will exhibit the **greatest angular speed**?

[✓ A. Gear #1] B. Gear #2 C. Gear #3 D. all three will have equal angular speeds

c. (1 pt.) Again, Gear #1 is rotated at a constant angular speed. Each gear has a small dot painted near its outer edge. During a particular time interval, which gear's dot travels the **greatest distance**?

A. Gear #1 B. Gear #2 C. Gear #3 [✓ D. all three will travel equal distances]

(a) Torque: $\tau = r \cdot F \cdot \sin\phi$, but $\phi = 90°$ in all cases here, since $\vec{F}$ is always tangent to edge of gear.

$$\tau_3 = R_3 \cdot F_{2 on 3} = R_3 \cdot \left(\frac{\tau_2}{R_2}\right) = \frac{R_3}{R_2} \cdot \tau_2 = \frac{R_3}{\cancel{R_2}}\left(\cancel{R_2} \cdot F_{1 on 2}\right) = R_3\left(\frac{\tau_1}{R_1}\right)$$

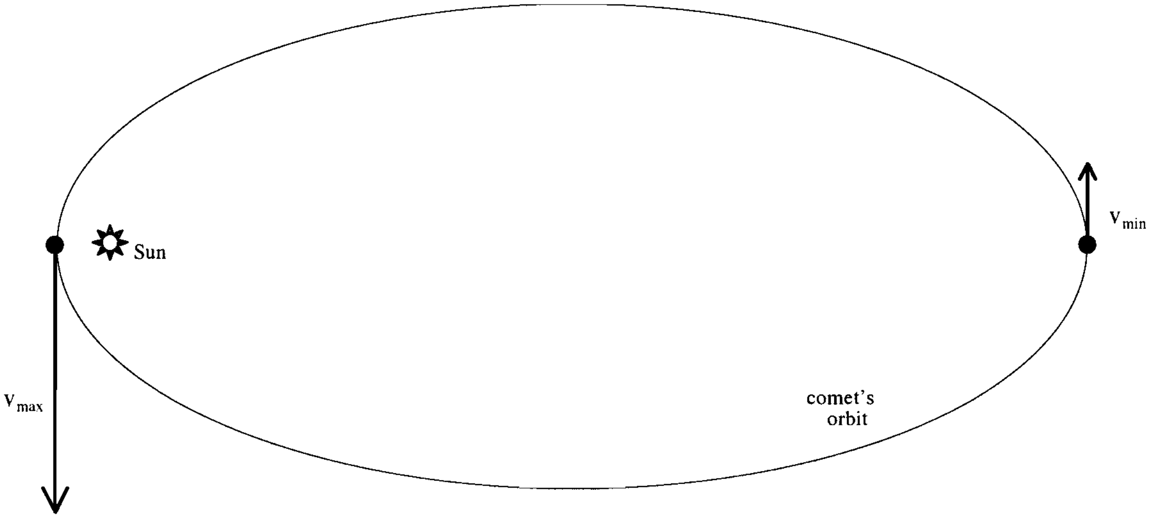
$$\Rightarrow \frac{\tau_3}{\tau_1} = \frac{R_3}{R_1}.$$

(b) The linear speed of edge of Gear #1 must be same as linear speed of edge of Gear #2, (since both gears' teeth must travel same distance in same time).

$\Rightarrow v_1 = v_2 \Rightarrow \omega_1 R_1 = \omega_2 R_2 \Rightarrow \omega_1 > \omega_2$ since $R_2 > R_1$

same argument $\Rightarrow \omega_2 > \omega_3$ since $R_3 > R_2$

(c) $\Delta s_1 = v_1 \cdot \Delta t$, and since $v_1 = v_2 = v_3$, $\Delta s_1 = \Delta s_2 = \Delta s_3$ (for points on edge of wheel).



5. Halley's Comet (mass = 1.7×10^{15} kg) has a very elliptical orbit: it varies from a distance of 0.586 AU (at closest approach) to 35.1 AU (at greatest distance) away from the Sun. (*Recall*: 1 AU = average Earth–Sun distance = 1.50×10^{11} m.) Assume that the comet is a "point mass" in space.

Kepler's 2nd Law tells us that any body orbiting the Sun should have greatest velocity when closest to the Sun and slowest velocity when farthest. Kepler discovered this by trial-and-error, but Newton later showed that this law can be derived as the result of conservation of angular momentum:

a. (3 pts.) If the comet's speed is 54 km/s at its closest approach to the Sun, find its **angular momentum** at that point. Express your answer in **MKS units**: $8.1 \times 10^{30}\ \frac{kg\cdot m^2}{s}$

b. (1 pt.) In which **direction** does the comet's **angular momentum vector** point?
A. into the page [✓ B.] out of the page C. direction varies during orbit

"right-hand rule": curl right-hand fingers in direction of rotation (CCW) ⇒ thumb points <u>out of page</u>.

c. (2 pts.) Knowing that the comet's angular momentum remains constant as it orbits the Sun, find the comet's **speed** when it reaches its **farthest point** from the Sun, in [km/s]: 0.90 **km/s**

(a) $L = m \cdot v \cdot r \cdot \sin\phi$, where ϕ = (angle between $\vec{r}$ and $\vec{v}$) = 90° at both points shown above

At closest approach: $L = m \cdot v \cdot r = (1.7\times10^{15}\,kg)(54{,}000\,\frac{m}{s})(0.586\,\cancel{AU})\left(\frac{1.50\times10^{11}\,m}{1\,\cancel{AU}}\right)$

$L = 8.07\times10^{30}\ \frac{kg\cdot m^2}{s}$

(c) L_{comet} remains constant throughout orbit!

$$L_{closest} = L_{farthest}$$

$$L_{closest} = (m\, v \cdot r)_{farthest}$$

$$8.07\times10^{30}\ \frac{kg\cdot m^2}{s} = (1.7\times10^{15}\,kg)\ v_{far}\left[(35.1\,\cancel{AU})\cdot\left(\frac{1.50\times10^{11}\,m}{1\,\cancel{AU}}\right)\right]$$

$$\Rightarrow\quad v_{far} = \underline{\underline{902\ \frac{m}{s}}} \approx \underline{\underline{0.90\ \frac{km}{s}}}.$$

Alternative solution:

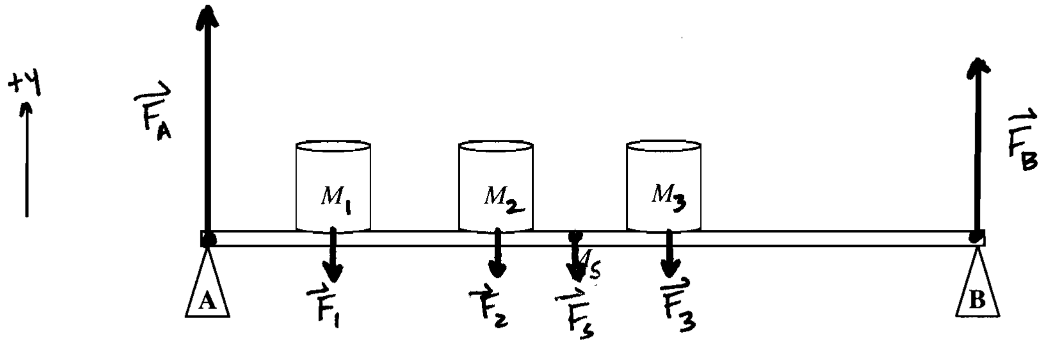
$$L_{closest} = L_{farthest}$$

$$\cancel{m}\, v_{close}\, r_{close} = \cancel{m}\, v_{far}\, r_{far}$$

$$\Rightarrow v_{far} = \frac{r_{close}}{r_{far}} \cdot v_{close}$$

(no need to convert units!)

$$v_{far} = \frac{0.586\,AU}{35.1\,AU}\cdot 54\,\frac{km}{s} = \underline{\underline{0.90\ \frac{km}{s}}}.$$



6. A shelf is supported by two posts at a distance *L* apart from each other. Three canisters, each of mass *M*, are located at distances of (1/5)*L*, (2/5)*L*, and (3/5)*L* from post *A*. The shelf itself also has mass *M*. The entire configuration is at rest in equilibrium. Express your answers below *ONLY* in terms of *M*, *L*, *g*, and numerical constants:

a. (2 pts.) Find the upward **force** exerted by **post *A*** on the shelf: $\frac{23}{10}M\cdot g$ or $2.3\,M\cdot g$

b. (2 pts.) Find the upward **force** exerted by **post *B*** on the shelf: $\frac{17}{10}M\cdot g$ or $1.7\,M\cdot g$

The shelf is in <u>Static equilibrium</u>: ① $\underline{\Sigma F_y = 0}$ and ② $\underline{\Sigma\tau = 0}$ about any pivot (axis of rotation).

①: $\Sigma F_y = F_A + F_B - F_1 - F_2 - F_3 - F_s$

$0 = F_A + F_B - M_1 g - M_2 g - M_3 g - M_s g \Rightarrow \underline{F_A + F_B = 4M\cdot g}$ (★)

②: We can select pivot anywhere along shelf... choose point "A" (left end). Then:

$$\Sigma\tau = \tau_A - \tau_1 - \tau_2 - \tau_s - \tau_3 + \tau_B$$

$$0 = \underset{\downarrow}{r_A} F_A - r_1 F_1 - r_2 F_2 - r_s F_s - r_3 F_3 + r_B F_B$$

$$0 = 0 - \left(\frac{L}{5}\right)(M_1 g) - \left(\frac{2L}{5}\right)(M_2 g) - \left(\frac{L}{2}\right)(M_s g) - \left(\frac{3L}{5}\right)(M_3 g) + L\cdot F_B$$

$$\Rightarrow \cancel{L}\cdot F_B = \cancel{L}\cdot M\cdot g\left(\frac{1}{5} + \frac{2}{5} + \frac{1}{2} + \frac{3}{5}\right)$$

$$\Rightarrow \underline{\underline{F_B = \frac{17}{10}M\cdot g}} = \underline{\underline{1.7\,M\cdot g}}$$

Substitute this back into (★): $F_A + \left(\frac{17}{10}Mg\right) = 4Mg$

$$\Rightarrow \underline{\underline{F_A = \frac{23}{10}M\cdot g}} = \underline{\underline{2.3\,Mg}}$$

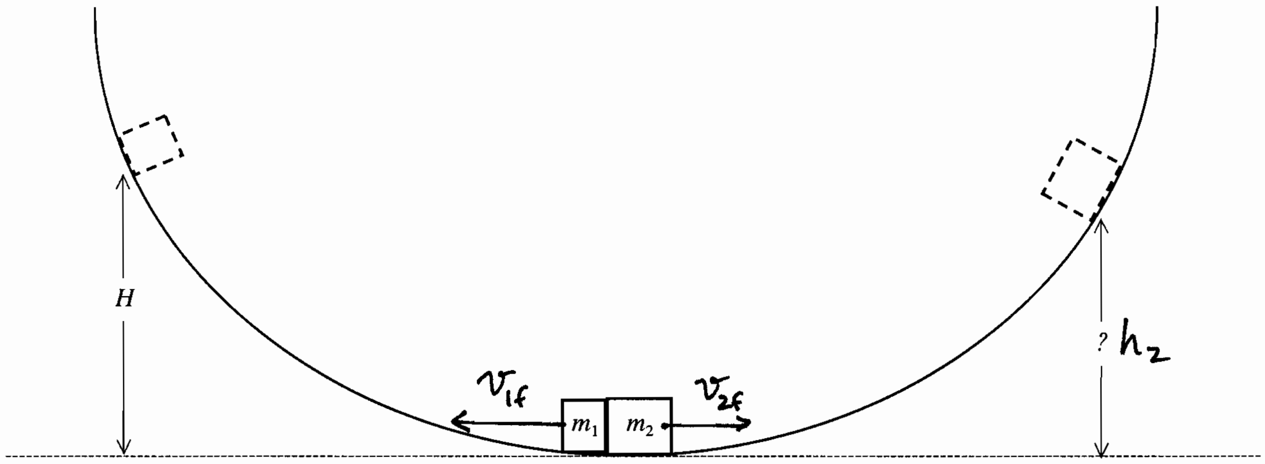
<u>Alternate solution</u>: Can choose pivot at point "B" (right end), or center of shelf, or any other point, and still get same final answers for F_A and F_B. (Note that, by choosing pivot right <u>at</u> a particular forces' position, <u>that</u> force exerts zero torque, and it drops out of the equation.)

Physics 151
Spring 2007

Roster No.: SOLUTIONS

Score: Part B: 18 pts. possible

Take-Home Midterm Exam #3, Part B



1. Two *unequal* masses m_1 and m_2 are located in a large, frictionless bowl, and both start at rest at the bottom of the bowl, touching each other. An explosion between the two masses pushes them apart horizontally in opposite directions. Mass m_1 slides to a maximum height H up the left side of the bowl.

(8 pts.) For both parts of this question, your answers will be algebraic expressions. You may solve parts (a) and (b) in whichever order you wish. For both parts:

- Express each final answer ***ONLY*** in terms of m_1, m_2, H, g, and any necessary mathematical constants.
- Show all steps of your derivation.
- ***Simplify*** each final answer to its most compact algebraic form.

a. To what **maximum height** does **mass m_2** slide up the right side of the bowl?
b. Find the **total** amount of **kinetic energy** gained by both masses in the explosion.

(a) Conservation of momentum in explosion:

$$\Sigma p_i = \Sigma p_f$$

$$(m_1+m_2)\cancelto{0}{v_i} = m_1 v_{1f} + m_2 v_{2f}$$

$$0 = m_1 v_{1f} + m_2 v_{2f}$$

$$\Rightarrow \quad \underline{\underline{v_{2f} = \frac{m_1}{m_2}(-v_{1f})}}. \quad (*)$$

Then, Conservation of energy as masses slide up ramps:

$$E_{bottom} = E_{top}$$

$$(U_{gr}+K)_{bottom} = (U_{gr}+K)_{top}$$

$$mg\cancelto{0}{y_{bottom}} + \frac{1}{2}mv^2_{bottom} = mgy_{top} + \frac{1}{2}m\cancelto{0}{v^2_{top}}$$

$$\frac{1}{2}\cancel{m}v^2_{bottom} = \cancel{m}gh_{top}$$

Mass 1: $\Rightarrow \underline{\underline{v_{1f} = \sqrt{2gH}}}$ (**) and Mass 2: $\underline{\underline{v_{2f} = \sqrt{2gh_2}}}$ (***)

continued on next page…

1. *continued:* Substitute (★★) and (★★★) into equation (★):

$$v_{2f} = \frac{m_1}{m_2}\left(-v_{1f}\right)$$

(Ignore negative sign – it simply denotes <u>leftward</u> motion.)

$$\Rightarrow \sqrt{\cancel{2g}h_2} = \frac{m_1}{m_2}\left(\sqrt{\cancel{2g}H}\right)$$

$$\Rightarrow \boxed{h_2 = \left(\frac{m_1}{m_2}\right)^2 \cdot H}$$

$$\text{(b)}\ \Delta K = \overset{\text{after explosion}}{K_f} - \underset{\text{before explosion}}{K_i}$$

$$= \left[\frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2\right] - \frac{1}{2}(m_1+m_2)\overset{0}{\cancel{v_i}}^2$$

$$= \left[\frac{1}{2}m_1\left(\sqrt{2gH}\right)^2 + \frac{1}{2}m_2\left(\sqrt{2gh_2}\right)^2\right] - 0$$

$$= \frac{1}{2}m_1(2gH) + \frac{1}{2}m_2\left(2g\left(\frac{m_1}{m_2}\right)^2 H\right)$$

$$= m_1 gH + m_2 g\left(\frac{m_1}{m_2}\right)^2 H$$

$$\boxed{\Delta K = \left(m_1 + \frac{m_1^2}{m_2}\right)gH} \quad \text{or} \quad \boxed{(m_1+m_2)\frac{m_1}{m_2}gH}$$

Suppose that $m_2 > m_1$. Use this fact along with your work above to answer the following questions:

c. (1 pt) *Immediately* after the explosion…

[✓ A.] m_1 has a *faster* **speed** than m_2
B. m_1 has the *same* **speed** as m_2
C. m_1 has a *slower* **speed** than m_2

Part (a): $|v_{2f}| = \frac{m_1}{m_2} \cdot v_{1f}$ (magnitude only, ignoring direction)

If $m_2 > m_1$, then $\left(\frac{m_1}{m_2}\right) < 1$, and $|v_{2f}| < v_{1f}$.

d. (1 pt) *Immediately* after the explosion…

[✓ A.] m_1 has *more* **kinetic energy** than m_2
B. m_1 has the *same* **kinetic energy** as m_2
C. m_1 has *less* **kinetic energy** than m_2

Part (b): $K_{1f} = m_1 gH$.

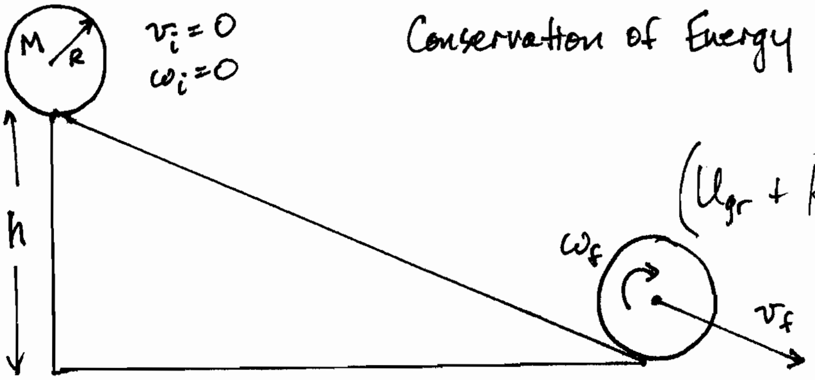
$K_{2f} = m_2 g h_2 = \left(\frac{m_1}{m_2}\right) m_1 gH$

$\Rightarrow K_{2f} = \left(\frac{m_1}{m_2}\right) \cdot K_{1f}$. Since $\left(\frac{m_1}{m_2}\right) < 1$, $K_{2f} < K_{1f}$.

2. You want to determine the total moment of inertia of an entire car wheel (rim, tire, and all). Although it is a complicated arrangement of many parts, it is still radially symmetric about its center. You measure the wheel's overall outer radius to be 25.0 cm, and you weigh it to find that its total mass is 23.0 kg.

You release the wheel from rest at the top of a 1.50-meter-tall ramp, and it rolls down without slipping. Averaging together several trials, you find that the wheel's final linear speed is 4.90 m/s at the bottom of the ramp.

a. (5 pts.) Find the **moment of inertia** of the car wheel. Express your final answer in MKS units. Show your work completely.



Conservation of Energy as wheel descends:

$$E_i = E_f$$

$$(U_{gr} + K_{trans} + K_{rot})_i = (U_{gr} + K_{trans} + K_{rot})_f$$

$$Mgy_i + \frac{1}{2}M\overset{0}{\cancel{v_i}}^2 + \frac{1}{2}I\overset{0}{\cancel{\omega_i}}^2 = Mg\overset{0}{\cancel{y_f}} + \frac{1}{2}Mv_f^2 + \frac{1}{2}I\omega_f^2$$

$$Mgh = \frac{1}{2}Mv_f^2 + \frac{1}{2}I\omega_f^2$$

For objects that roll without slipping, $\omega = \frac{v}{R}$ at all times:

$$Mgh = \frac{1}{2}Mv_f^2 + \frac{1}{2}I\left(\frac{v_f}{R}\right)^2.$$

Inserting values:

$$(23.0\,\text{kg})(9.80\,\tfrac{\text{m}}{\text{s}^2})(1.50\,\text{m}) = \frac{1}{2}(23.0\,\text{kg})(4.90\,\tfrac{\text{m}}{\text{s}})^2 + \frac{1}{2}I\left(\frac{4.90\,\text{m/s}}{0.250\,\text{m}}\right)^2$$

$$338.1\,\text{J} = 276.1\,\text{J} + (192.1/\text{s}^2)\,I$$

$$\Rightarrow (192.1\,\text{s}^{-2})\,I = \underset{\text{final rotational kinetic energy}}{62.0\,\text{J}}$$

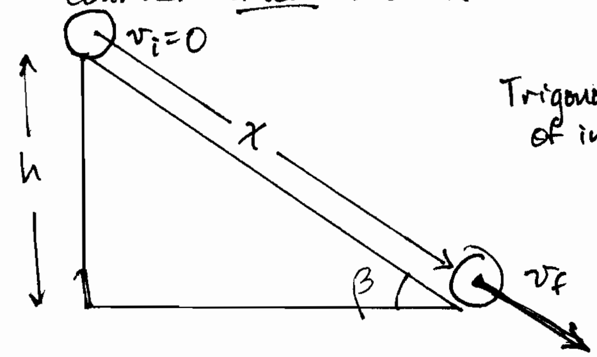
$$\Rightarrow \quad I = 0.3227\,\text{kg}\cdot\text{m}^2 = \boxed{0.323\,\text{kg}\cdot\text{m}^2}$$

continued on next page…

2. *continued:*

b. (3 pts.) If the ramp is a 30.0° incline of constant slope, find the total **time of descent** for the car wheel. Show your work.

Consider linear motion of wheel, and don't worry about rotation:



Trigonometry of incline: $x = \dfrac{h}{\sin\beta} = \dfrac{1.50\,\text{m}}{\sin 30.0^\circ} = \underline{\underline{3.00\,\text{m}}}$.

Assuming a constant downhill acceleration:

$$v_f^2 = v_i^2 + 2 \cdot a_x \Delta x$$

$$\left(4.90\,\tfrac{\text{m}}{\text{s}}\right)^2 = 0^2 + 2 \cdot a_x \cdot (3.00\,\text{m})$$

$$\Rightarrow \quad \underline{\underline{a_x = 4.00\,\text{m}/\text{s}^2}}.$$

Note: This downhill acceleration is slower than that of an equally heavy, but non-rotating object, since the wheel is rotating.

Finally, $v_f = v_i + a_x \cdot t$

$$4.90\,\tfrac{\text{m}}{\text{s}} = 0 + 4.00\,\text{m}/\text{s}^2 \cdot t$$

$$\Rightarrow \quad \boxed{t = 1.22\,\text{s}}$$

Alternative solution: As wheel rolls down 3.00m-long incline,

$$\Delta\theta = \frac{\Delta x}{R} = \frac{3.00\,\text{m}}{0.250\,\text{m}} = 12.0\,\text{rad} \quad \text{(which is 1.91 revolutions)}$$

Then: $\omega_f^2 = \omega_i^2 + 2 \cdot \alpha \cdot \Delta\theta$, where $\omega_f = \dfrac{v_f}{R} = \dfrac{4.90\,\text{m/s}}{0.250\,\text{m}} = 19.6\,\tfrac{\text{rad}}{\text{s}}$.

$$\left(19.6\,\tfrac{\text{rad}}{\text{s}}\right)^2 = 0 + 2 \cdot \alpha \cdot (12.0\,\text{rad})$$

$$\Rightarrow \quad \underline{\underline{\alpha = 16.0\,\text{rad}/\text{s}^2}}.$$

Then: $\omega_f = \omega_i + \alpha t$

$$19.6\,\tfrac{\text{rad}}{\text{s}} = 0 + \left(16.0\,\tfrac{\text{rad}}{\text{s}^2}\right) \cdot t$$

$$\Rightarrow \quad \boxed{t = 1.22\,\text{s}}.$$