Physics 151
October 22, 2010

Roster No.: SOLUTIONS

Score: Part A: 25 pts. possible
Total: 50 pts. possible

Midterm Exam #2, Part A

Exam time limit: 50 minutes. You may use a calculator and both sides of ONE sheet of notes, *handwritten only*. Closed book; no collaboration. Ignore friction and air resistance in all problems, unless told otherwise.

Part A: For each question, fill in the letter of the one best answer on your bubble answer sheet.

Physical constants:	$g = 9.80$ m/s^2	$G = 6.67 \times 10^{-11}$ N·m^2/kg^2	
Useful conversions:	1 year = 3.156×10^7 s		
Sun, Earth, & Moon data:			
masses	*physical radii*	*orbital distances*	*orbital periods*
$M_{Sun} = 2.00 \times 10^{30}$ kg	$R_{Sun} = 6.95 \times 10^8$ m		
$M_{Earth} = 5.97 \times 10^{24}$ kg	$R_{Earth} = 6.37 \times 10^6$ m	$d_{Earth\text{-}Sun} = 1.50 \times 10^{11}$ m	T_{Earth} = 1 year (exact)
$M_{Moon} = 7.35 \times 10^{22}$ kg	$R_{Moon} = 1.74 \times 10^6$ m	$d_{Earth\text{-}Moon} = 3.84 \times 10^8$ m	T_{Moon} = 27.3 days

(2 pts. each) **Convert** the following quantities into the given units:

1. 99 kW = 9.9×10^7 mW (*Note:* "W" = watt, the MKS unit for power)

A. 9.9×10^{-10} mW
B. 9.9×10^{-7} mW
C. 9.9×10^{-4} mW
D. 9.9×10^{4} mW
E. 9.9×10^{7} mW

$$99\,\cancel{kW}\left(\frac{10^3\,\cancel{W}}{1\,\cancel{kW}}\right)\left(\frac{1\,mW}{10^{-3}\,\cancel{W}}\right) = 99\times10^6\,mW$$
$$= \underline{\underline{9.9\times10^7\,mW.}}$$

2. 3.3×10^{11} m^3 = 3.3×10^2 km^3

A. 330 km^3
B. 3.3×10^3 km^3
C. 3.3×10^4 km^3
D. 3.3×10^5 km^3
E. 3.3×10^6 km^3

$$3.3\times10^{11}\,\cancel{m^3}\left(\frac{1\,km}{10^3\,\cancel{m}}\right)^3 = \underline{\underline{3.3\times10^2\,km^3}}$$

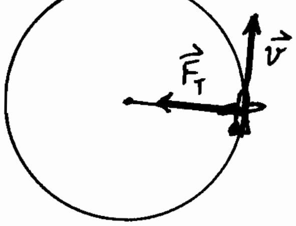
3. 25 cm/s = 0.90 km/h (*Note:* "h" = hour)

A. 9.0×10^{-3} km/h
B. 9.0×10^{-2} km/h
C. 0.90 km/h
D. 9.0 km/h
E. 90. km/h

$$25\,\frac{\cancel{cm}}{\cancel{s}}\left(\frac{1\,\cancel{m}}{100\,\cancel{cm}}\right)\left(\frac{1\,km}{1000\,\cancel{m}}\right)\left(\frac{3600\,\cancel{s}}{1\,h}\right) = \underline{\underline{0.90\,\frac{km}{h}}}$$

Questions #4–5: A child's toy airplane flies in uniform circular motion at the end of a massless tether (cord). The plane of the circle is exactly horizontal (parallel to the ground). (*Neglect gravity and air resistance.*)

top view:



4. (1 pt.) The **acceleration** of the airplane is always…

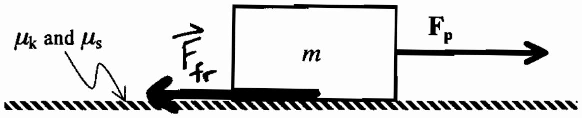
A. tangent to the circle, in the direction of the airplane's velocity
B. exactly toward the center of the circle – in direction of $\vec{F}_T$
C. exactly away from the center of the circle
D. zero

5. (2 pts.) The tether will break if its tension exceeds 95 N. If the length of the tether is 1.5 m, and the airplane has a mass of 0.20 kg, what is the toy airplane's **maximum linear speed**?

A. 11 m/s
B. 16 m/s
C. 22 m/s
D. 27 m/s
E. 34 m/s

$$\Sigma F_{rad} = \frac{mv^2}{r} \text{ (centripetal force)}$$
$$(F_T)_{max} = \frac{m v_{max}^2}{r}$$
$$95\,N = \frac{(0.20\,kg)\,v_{max}^2}{1.5\,m} \Rightarrow \underline{\underline{v_{max} = 26.7\,\frac{m}{s}}}$$

Questions #6–8: A block of mass m initially sits *at rest* on a horizontal surface. The coefficients of friction between the block and the surface are μ_k and μ_s. A person pushes on the block with a horizontal force $\mathbf{F}_p$ in an attempt to dislodge it.



6. (2 pts.) What is the **minimum magnitude of $\mathbf{F}_p$** needed for the block to start sliding?

A. $\mu_k mg$

B. $\mu_s mg$

C. $\dfrac{mg}{\mu_k}$

D. $\dfrac{mg}{\mu_s}$

E. $\dfrac{mg}{\mu_k + \mu_s}$

F_p needs to match (and slightly exceed) the maximum magnitude of static friction:

$F_p \geq (F_{fr,s})_{max} = \mu_s \cdot F_N$, and we see (by inspection) $F_N = m \cdot g$

$\Rightarrow \boxed{F_p \geq \mu_s (mg)}$

7. (2 pts.) Later, suppose the block is sliding to the right. If the block has a rightward acceleration a, what is the **magnitude of $\mathbf{F}_p$**?

A. ma

B. $\mu_k ma$

C. $m(a + \mu_k g)$

D. $m(a - \mu_k g)$

E. $\dfrac{ma}{\mu_k}$

$\Sigma F_x = m \cdot a$

$F_p - F_{fr,k} = m \cdot a$, where $F_{fr,k} = \mu_k \cdot F_N = \mu_k \cdot mg$.

$\Rightarrow \boxed{F_p = m \cdot a + \mu_k \cdot mg}$

8. (1 pt.) In the previous question, the person exerts a rightward force of F_p on the crate, and the crate accelerates to the right. *At the same time*, the **crate** exerts a **leftward force on the person** that is…

A. zero

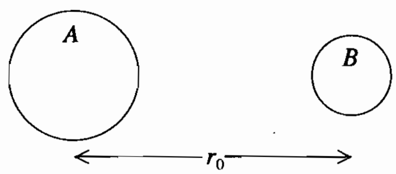
B. weaker than F_p

C. equal to F_p

D. stronger than F_p

by Newton's 3rd Law: as person exerts magnitude F_p on crate to right, crate exerts magnitude F_p on person to left! (Regardless of speed or acceleration!)

Questions #9–11: Consider two spherical masses, A and B, as shown, released *from rest* at an initial separation r_0. Mass A is larger than mass B. (*Assume that NO other masses exist in the universe.*)



9. (1 pt.) As the two masses fall toward each other, the gravitational **force** acting **on mass A** is ____________ the gravitational force acting **on mass B**, at all times.

A. stronger than

B. equal strength as

C. weaker than

D. None of the above answers is true at all times.

Newton's 3rd Law: both attractive forces are equal in magnitude (and opposite in direction), even if two masses are unequal!

Newton's 2nd Law: $a = \dfrac{F}{m}$.

Both forces are equal, but $m_A > m_B$,

so $a_A < a_B$, and A will move more slowly than B.

10. (1 pt.) The two masses will finally collide at a **location**…

A. closer to the starting position of mass A

B. closer to the starting position of mass B

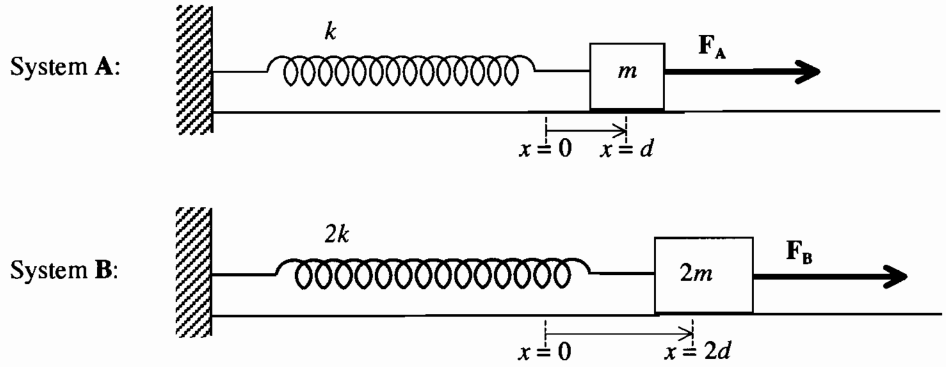
C. exactly halfway between their original positions

11. (1 pt.) Just before the two masses collide, the **speed** of mass A will be ____________ the speed of mass B.

A. faster than

B. equal to

C. slower than

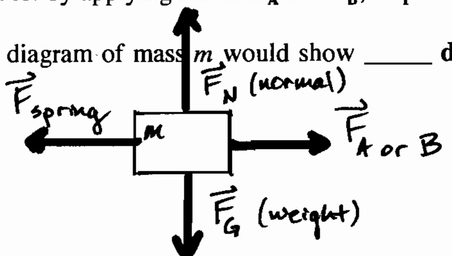


Questions #12–16: Masses m and $2m$ are attached to ideal, massless springs, k and $2k$, respectively, as shown above. The mass in system A is initially pulled aside to a displacement of $x = d$, while the mass in system B is initially displaced *twice* as far. (*The surfaces are frictionless. The force vectors* $\mathbf{F}_A$ *and* $\mathbf{F}_B$ *are NOT necessarily drawn to scale.*)

For the next 3 questions, the two systems are held *at rest* by applying forces $\mathbf{F}_A$ and $\mathbf{F}_B$, respectively.

12. (1 pt.) For either system, a *complete* free-body diagram of mass m would show _____ **distinct force vectors** acting on m.

A. 1
B. 2
C. 3
D. 4
E. 5



(no friction.)

13. (2 pts.) Suppose that $k = 55$ N/m, $m = 1.8$ kg, and $d = 7.5$ cm. What is the **magnitude** of force $\mathbf{F}_A$?

A. 2.9 N
B. 4.1 N
C. 6.5 N
D. 8.0 N
E. 9.2 N

$\Sigma F_x = m \cdot a_x = 0$ (at rest)

$\Rightarrow (F_A)_x + (F_{spr})_x = 0$, where $F_{spr,x} = -kx$.

$\Rightarrow F_A = kd = (55\frac{N}{m})(0.075m) = \underline{\underline{4.1\text{ N}}}$

14. (2 pts.) The magnitude of **force** $\mathbf{F}_A$ is __________ times the magnitude of **force** $\mathbf{F}_B$.

A. $\frac{1}{4}$
B. $\frac{1}{2}$
C. 1 (equal)
D. 2
E. 4

$F_A = k \cdot d$ (see above)

Similarly, $F_B = (2k) \cdot (2d)$. $\Rightarrow \underline{\underline{F_A = \frac{1}{4} F_B}}$.

Now, both forces $\mathbf{F}_A$ and $\mathbf{F}_B$ are *removed* simultaneously, and both masses are free to move without friction.

15. (1 pt.) Immediately after release, both masses will return to $x = 0$ ...

A. at constant speed
B. with increasing speed, but with diminishing acceleration
C. with increasing speed, and with constant acceleration
D. with increasing speed, and with strengthening acceleration

At release, $v_{init} = 0$.

Newton II: $F_{spr,x} = m \cdot a_x$, where $F_{spr,x} = -kx$.

$\Rightarrow \underline{a_x = -\frac{k}{m} \cdot x}$. Thus, m accelerates to the left, but as x gets smaller, acceleration diminishes.

16. (2 pts.) *Immediately after release*, the mass's **acceleration** in **system** A is __________ times the mass's **acceleration** in **system** B.

A. $\frac{1}{4}$
B. $\frac{1}{2}$
C. 1 (equal)
D. 2
E. 4

System A, immediately after release ($x = d$):

$$\left.\begin{aligned} a_A &= -\frac{k}{m} \cdot d. \\ \text{Similarly, System B: } a_B &= -\frac{2k}{2m}(2d). \end{aligned}\right\} \Rightarrow \underline{\underline{a_A = \frac{1}{2} \cdot a_B}}.$$

Physics 151
October 22, 2010

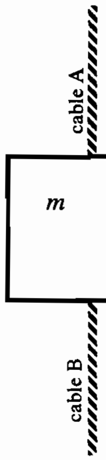
Roster No.: **SOLUTIONS**

Score: Part B: 25 pts. possible

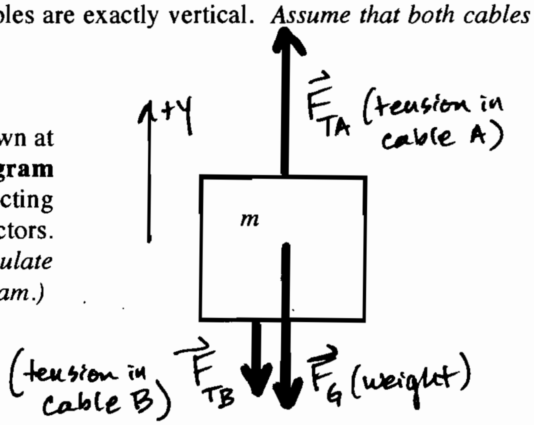
Midterm Exam #2, Part B

Part B: Show your work on all free-response questions. Be sure to use **proper units** and **significant figures** in your final answers. For any multiple-choice questions, circle the letter of the one best answer (unless more than one answer is asked for).

1. A large, heavy crate (m = 250.0 kg) is suspended on cable *A* from a crane. Also, a worker pulls downward on cable *B* with 380. N of force, to help guide and steady the crate. Both cables are exactly vertical. *Assume that both cables are massless and inelastic.*



a. (2 pts.) Using the crate shown at right, create a **free-body diagram** of m, showing **ALL forces** acting on it. **LABEL** ALL force vectors. *(You do NOT need to calculate their magnitudes for this diagram.)*



inventory of forces:

$\vec{F}_{TA} = (0, F_{TA})$

$\vec{F}_{TB} = (0, -F_{TB})$

$\vec{F}_G = (0, -mg)$

b. (2 pts.) If the crate is at rest, the **magnitude** of the **tension** in **cable** *A* is: **2830 N**

You do NOT need to show your work for part (b).

Newton's 2nd Law: $\Sigma F_y = m \cdot a_y$ and $a_y = 0 \Rightarrow F_{TA} - F_{TB} - mg = 0$

$F_{TA} = mg + F_{TB}$

$F_{TA} = (250.0\,kg)(9.80\,m/s^2) + 380.\,N = 2450\,N + 380.\,N = \underline{\underline{2830\,N}}.$

c. (5 pts.) Later, while the crate is moving, the tension in cable *A* is measured to be 2750. N. (The worker is still applying 380. N of downward force on cable *B*.) Find the **magnitude** and **direction** of the crate's **acceleration**. *Show your work completely.*

Newton's 2nd Law:

$$\Sigma F_y = m \cdot a_y$$

$$F_{TA} - F_{TB} - mg = m \cdot a_y$$

$$2750\,N - 380.\,N - (250.0\,kg)(9.80\,m/s^2) = (250.0\,kg)\,a_y$$

$$2750\,N - 380.\,N - 2450\,N = (250.0\,kg) \cdot a_y$$

$$-80\,N = (250.0\,kg) \cdot a_y$$

(note loss of sig figs)

$$\Rightarrow \boxed{a_y = -0.32\,m/s^2}$$

or, even 1 sig fig: $\boxed{-0.3\,m/s^2}$

+y was chosen to be upward, so $\underline{\underline{a_y \text{ is downward}}}$ (negative).

2. In the not-too-distant future, astronauts may use Mars's larger moon, Phobos, as a location for a lunar base and way-station to Mars. Throughout this question, assume that Phobos is a uniform-density, perfectly smooth sphere with radius 1.11×10^4 m and mass 1.07×10^{16} kg. *(Ignore the presence of Mars or any other astronomical bodies.)*

Two astronauts, Adam and Beverly, are having a friendly argument: Adam bets Bev that he can throw a 145-gram baseball horizontally (tangent to the ground) fast enough to put it into a *circular orbit* just barely above the surface of Phobos. Bev is skeptical, so she does a quick calculation…

a. (5 pts.) Find the **linear speed** necessary for the baseball. *Show your work.* (*Thought question:* Could a human indeed throw a baseball this fast? *Recall:* 1 m/s ≈ 2.24 miles/hour)

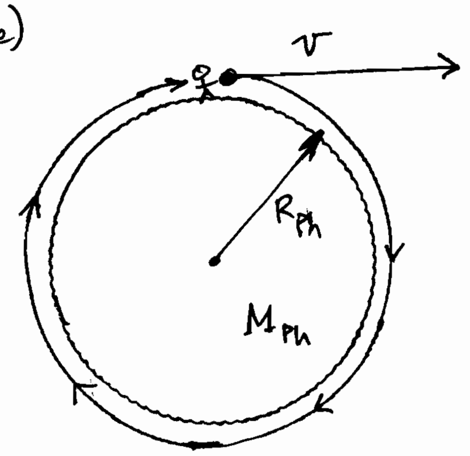
F_G (gravitational force) $= \Sigma F_{rad}$ (centripetal force) for uniform circular motion

$$\frac{G M_{Ph} \cdot m}{R_{Ph}^2} = \frac{m v^2}{R_{Ph}}$$

$$\Rightarrow \quad v = \sqrt{\frac{G M_{Ph}}{R_{Ph}}}$$

$$v = \left[\frac{(6.67\times10^{-11}\ \frac{N m^2}{kg^2})(1.07\times10^{16}\ kg)}{1.11\times10^{4}\ m}\right]^{1/2}$$

$$\boxed{v = 8.02\ m/s}$$



To prove his point, Adam does it: he throws the baseball at just the right speed, and away it goes in a circular orbit. While Adam stands grinning at Bev, the baseball circles Phobos completely and smacks him right in the helmet. Bev decides that it was worth the extremely long wait.

b. (5 pts.) How much **time** is needed for the baseball to complete one full orbit of Phobos? Convert your final answer to **hours**. *Show your work.* (*Hint: Your final answer will be between 1 and 3 hours.*)

Uniform circular motion:

$$v = \frac{2\pi r}{T}$$ T ← period of one cycle

$$\Rightarrow \quad T = \frac{2\pi r}{v} \quad \text{where } r = R_{Ph}$$

$$= \frac{2\pi\,(1.11\times10^{4}\ m)}{8.02\ m/s}$$ ← from part (a)

converting to hours

$$T = 8.70\times10^{3}\ s \cdot \left(\frac{1\ h}{3600\ s}\right)$$

$$\boxed{T = 2.42\ h}$$

OR Kepler's 3rd Law:

$$T^2 = \frac{4\pi^2}{G M_{Ph}} \cdot r^3, \quad \text{where } r = R_{Ph}$$

orbital period (T); (orbiting just above surface)

$$\Rightarrow \quad T = \left[\frac{4\pi^2\,(1.11\times10^{4}\ m)^3}{(6.67\times10^{-11}\ \frac{N m^2}{kg^2})(1.07\times10^{16}\ kg)}\right]^{1/2}$$

$$T = 8.70\times10^{3}\ s \cdot \left(\frac{1\ h}{3600\ s}\right)$$ ← converting to hours

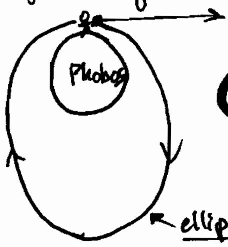
$$\boxed{T = 2.42\ h}$$

continued on next page…

2. continued:

Repeat of earlier information: Assume that Phobos is a uniform-density, perfectly smooth sphere with radius 1.11×10^4 m and mass 1.07×10^{16} kg. *(Ignore the presence of Mars or any other astronomical bodies.)*

c. (1 pt.) If Adam had thrown the baseball horizontally with ***slightly*** **greater speed** than the speed calculated in part (a), what would have happened to the baseball's **orbit**?



A. There would be no change: the baseball would still execute an identical circular orbit just barely above the surface of Phobos, just with a faster speed.

B. The baseball would have ascended to a larger radius, then orbited Phobos in a larger circular orbit at that new radius, high above the surface.

C. The baseball would have orbited Phobos in a large ellipse: ascending for the first half of the orbit, then descending for the second half (striking Adam in the head as it grazes Phobos's surface on its return).

One of the challenging things about working and living on Phobos is the very weak surface gravity.

d. (5 pts.) Find the **acceleration due to gravity** on the surface of Phobos, and convert your final answer to **Earth "gees."** *Show your work.*

gravitational force = weight, for any object m on surface of Phobos.

$$F_G = m \cdot g_{Ph}$$

$$\frac{G M_{Ph} \cdot \cancel{m}}{R_{Ph}^2} = \cancel{m} \cdot g_{Ph}$$

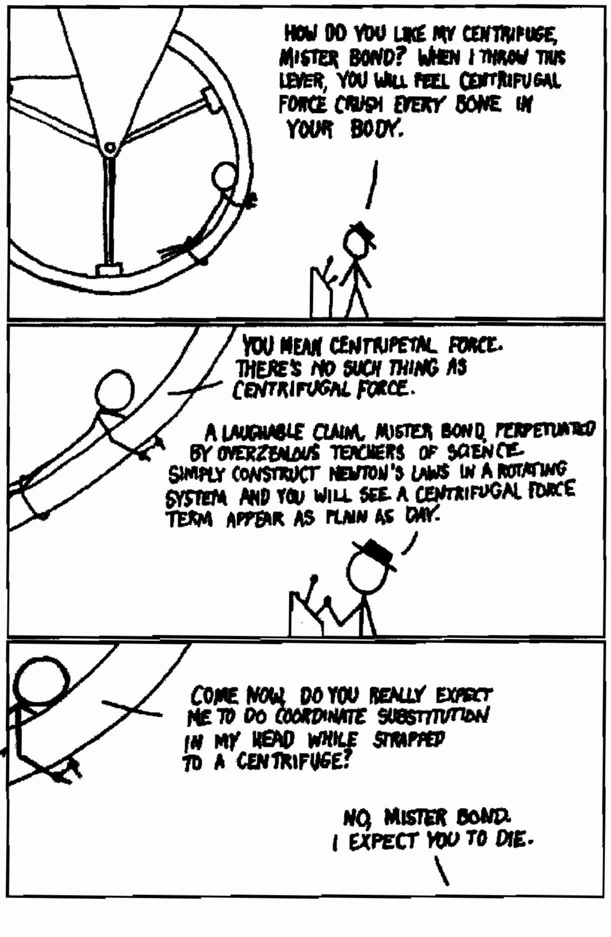
$$\Rightarrow \quad g_{Ph} = \frac{G M_{Ph}}{R_{Ph}^2}$$

$$g_{Ph} = \frac{(6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2})(1.07 \times 10^{16} kg)}{(1.11 \times 10^4 m)^2}$$

converting to gees

$$g_{Ph} = 5.79 \times 10^{-3} \frac{m}{s^2} \cdot \left(\frac{1 \text{ gee}}{9.80 \text{ m/s}^2}\right)$$

$$\boxed{g_{Ph} = 5.91 \times 10^{-4} \text{ gee}}$$



OR: For baseball in circular orbit just above surface of Phobos,

$$F_G = \Sigma F_{rad} \Rightarrow m \cdot g = m \frac{v^2}{r} \quad (= m \cdot a_{rad})$$

centripetal force

centripetal acceleration

$$\Rightarrow \quad g = \frac{v^2}{r}$$

$$\Rightarrow g = \frac{(8.02 \text{ m/s})^2}{1.11 \times 10^4 m} = 5.79 \times 10^{-3} \text{ m/s}^2.$$

(Then convert to gees, as above.)