

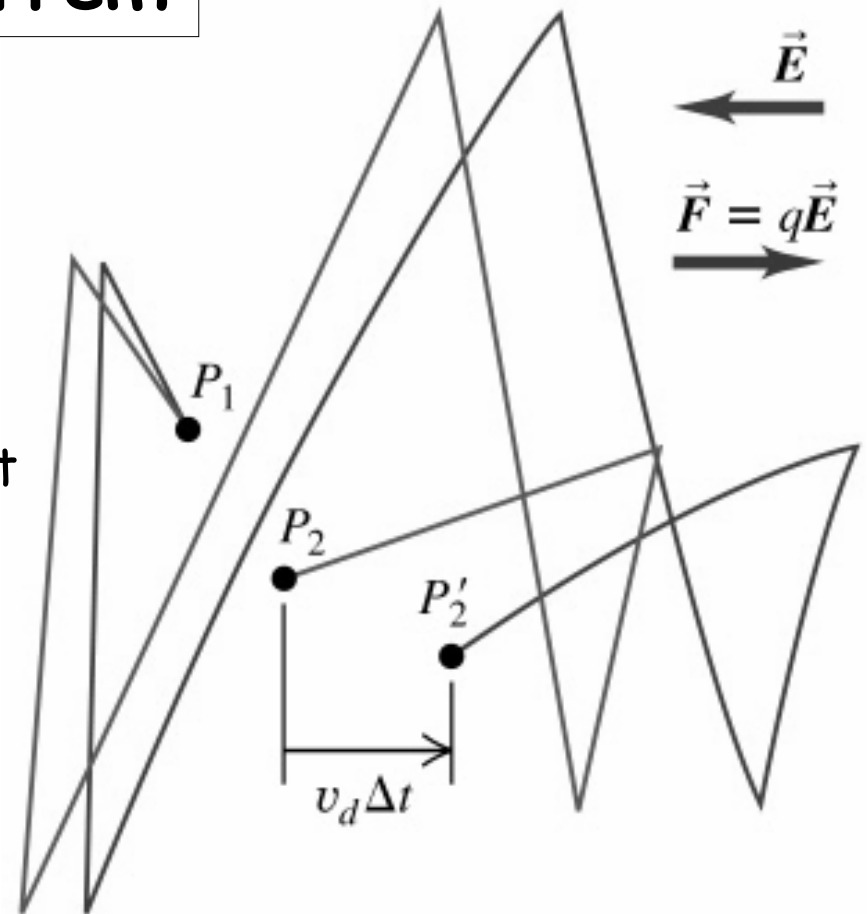
Lecture 8 -- Physics 272
Electric Currents
Resistance & Resistivity

Electric Current

NO longer electrostatics!
Can have \vec{E} in conductor and a flow of charge.

Consider free electron motion without any external electric field. The net motion (blue) is random and the average displacement is zero. Electron moves from $P_1 \rightarrow P_2$

Consider electron motion with an external electric field. The net motion (red) has a drift and the average displacement is opposite to the electric direction. Electron moves $P_1 \rightarrow P_2'$. The net drift velocity v_d is $\sim 10^{-4}$ m/s. The drift velocity is very small !



Apparent Paradox: Light turns on immediately when you flip the switch despite the tiny drift velocity of charge carriers ($\sim 10^{-4}$ m/s)

Think of water in a garden hose. If the hose is full the water will flow out of the end immediately.

Current in Wire; charge carriers

Current, I , is the rate of charge flow through cross section of wire or charge per unit time:

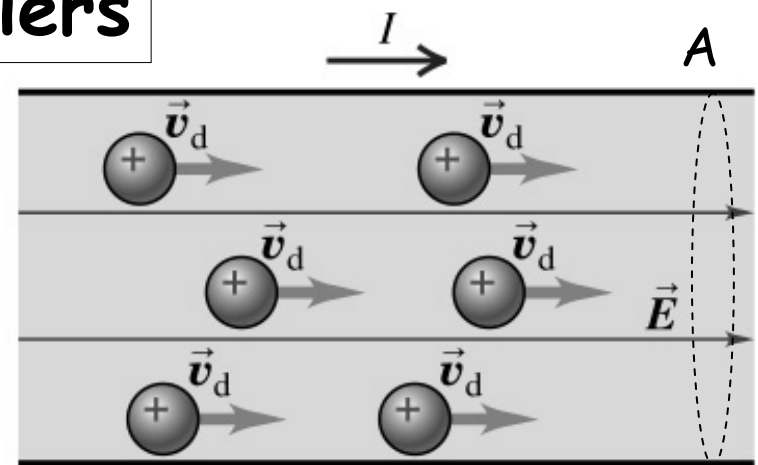
units of $C/s = 1$ Ampere

$$I = \frac{dQ}{dt}$$

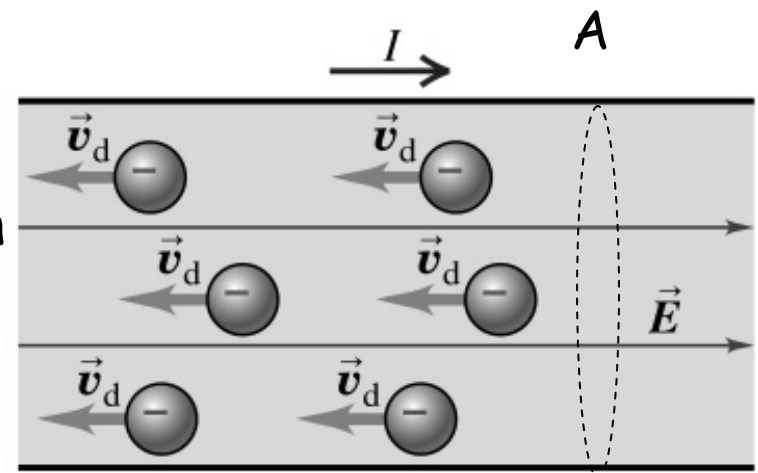
Current can be formed with positive charges moving in positive direction or with negative charges moving in the opposite direction.

Electron flow is opposite to the direction of current flow. This is a historical convention due to Benjamin Franklin.

House outlets are fused at 15Amps
Whole house circuit breaker is around 200Amps



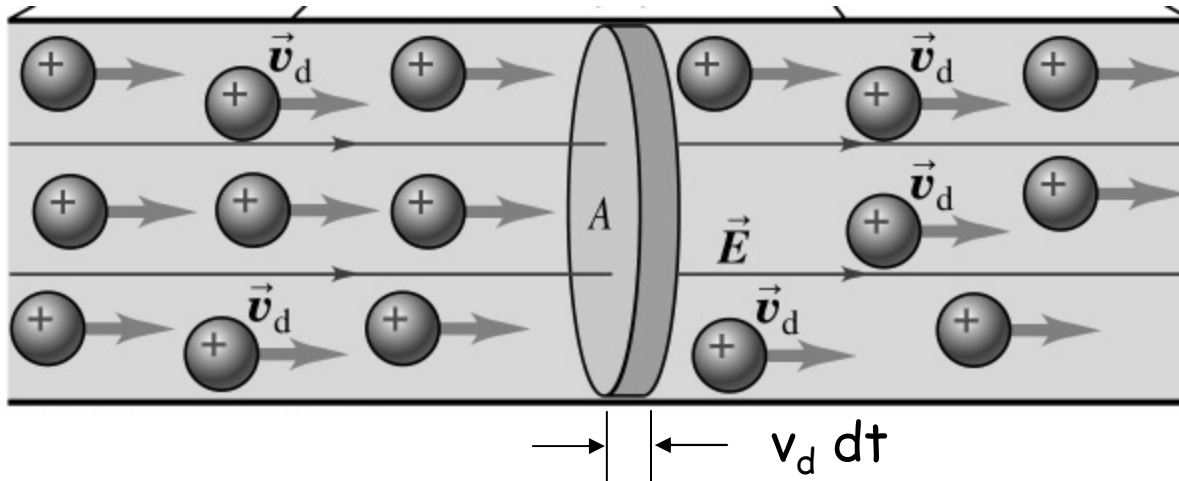
effective picture



(b) Y&F fig 25.2

Current Density, J

Flow of + charges in a wire, through a surface area A



Y&F fig 25.3

Current Density, J , is the current flow per unit area (amp/m^2)

$$J = \frac{I}{A} = \frac{1}{A} \frac{dQ}{dt}$$

If + charges, q , have velocity v_d and a volume density, n ($\#/\text{volume}$). Then in a time dt , a volume, $A v_d dt$, is swept out and the differential amount of charge is

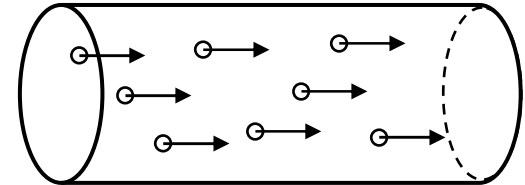
$$dQ = q n A v_d dt$$

We can write current as, $I = \frac{dQ}{dt} = nqv_d A$

Example

A wire is made of copper and has a radius 0.815 mm. Calculate the drift velocity assuming 1 free electron per atom for $I = 1A$.

$$I = \frac{dQ}{dt} = nqv_d A \qquad v_d = \frac{I}{nqA}$$



$$n = n_a = \frac{\rho N_A}{M} = \frac{(8.93 \text{ g/cm}^3)(6.02 \times 10^{23} \text{ atoms/mole})}{63.5 \text{ g/mole}}$$

$$= 8.47 \times 10^{28} \text{ atoms/m}^3$$

n_a = number density of copper atoms

ρ = density of copper

N_A = Avogadro's number

M = atomic mass of Cu

$$v_d = \frac{I}{nqA} = \frac{1A}{(8.47 \times 10^{28} \text{ m}^{-3})(1.6 \times 10^{-19} \text{ C}) \pi (8.15 \times 10^{-4} \text{ m})^2}$$

$$= 3.54 \times 10^{-5} \text{ m/s}$$

Very slow.

7.8 hours to travel 1 m!

Why does the light come on so quickly when switch is thrown?

Current Density, J

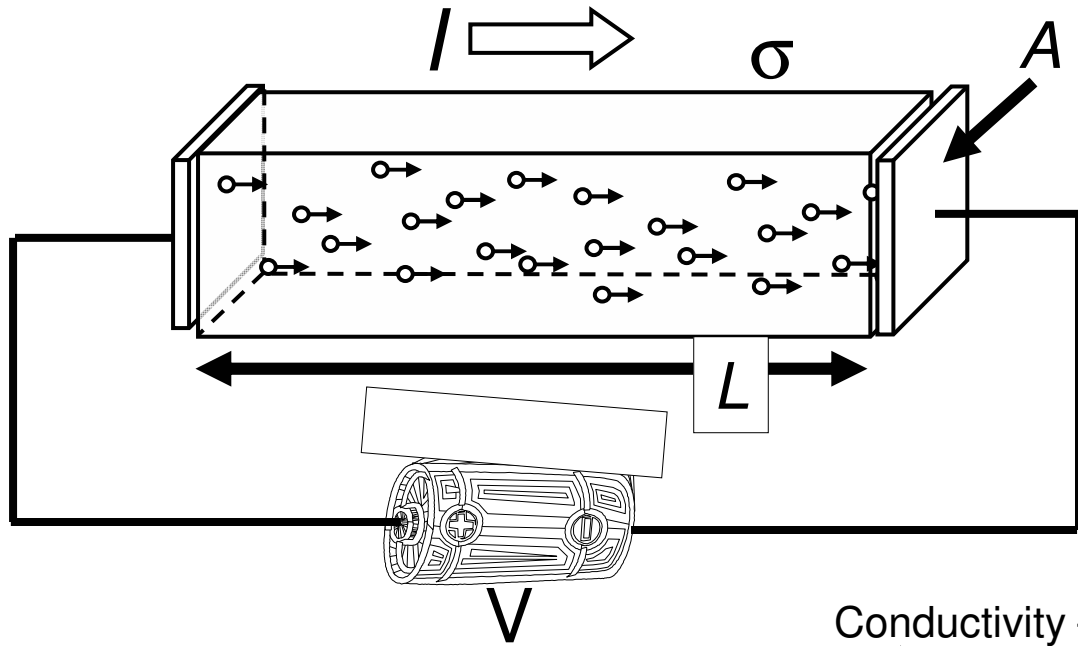
Since the current is, $I = \frac{dQ}{dt} = nqv_d A$

We can write the current density as, $J = \frac{I}{A} = nqv_d$

Technically, current density is a vector quantity since velocity is a vector quantity,

$$\vec{J} = nq\vec{v}_d$$

Review:



Conductivity – high for good conductors.

Ohm's Law: $J = \sigma E$

Observables:

$V = EL$
 $I = JA$

$\Rightarrow I/A = \sigma V/L \Rightarrow I = V/(L/\sigma A)$

$R = \text{Resistance}$
 $\rho = 1/\sigma$

$I = V/R$

\uparrow
 $R = \frac{L}{\sigma A}$

This is just like plumbing!

I is like flow rate of water

V is like pressure

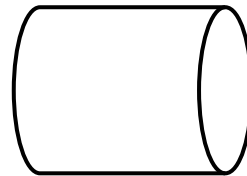
R is how hard it is for water to flow in a pipe

$$R = \frac{L}{\sigma A}$$

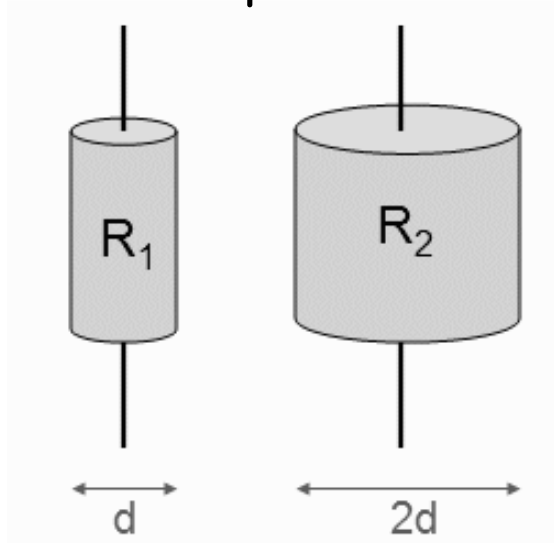
To make R big, make L long or A small



To make R small, make L short or A big



Clicker problem I



Same current through both resistors

Compare voltages across resistors

$V_1 > V_2$ $V_1 = V_2$ $V_1 < V_2$

A)

B)

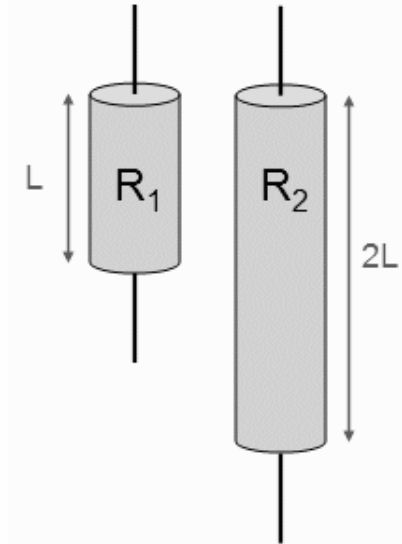
C)

$$R \propto \frac{L}{A}$$

$$V = IR \propto \frac{L}{A}$$

$$A_2 = 4 A_1 \rightarrow V_2 = 1/4 V_1$$

Clicker problem II



$V_1 > V_2$ $V_1 = V_2$ $V_1 < V_2$

A)

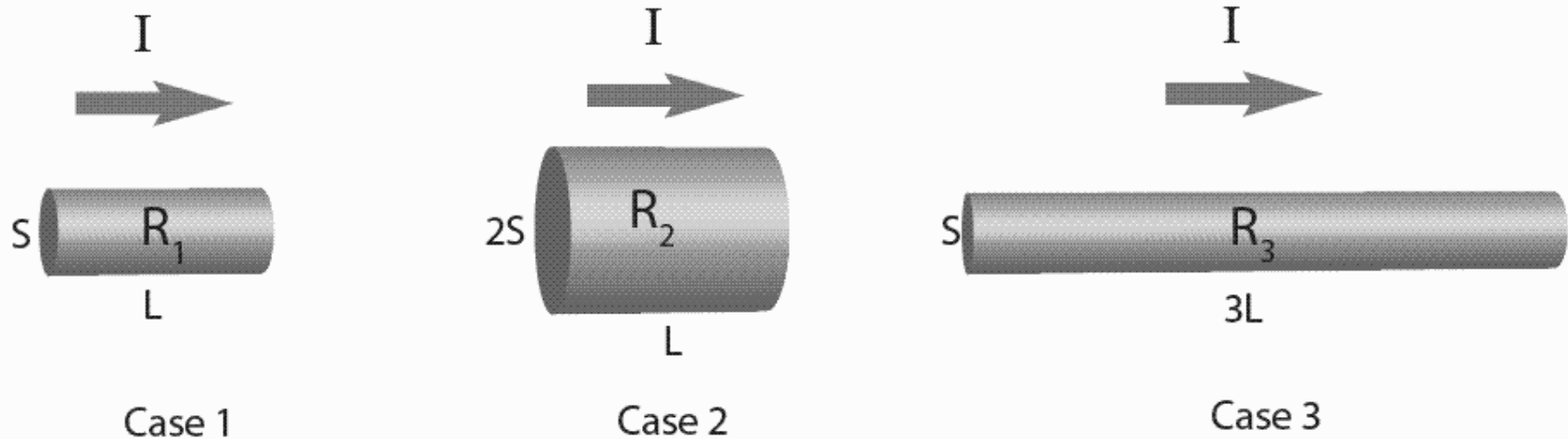
B)

C)

$$L_2 = 2 L_1 \rightarrow V_2 = 2 V_1$$

Clicker problem

12) The SAME amount of current I passes through three different resistors. R_2 has twice the cross-sectional area and the same length as R_1 , and R_3 is three times as long as R_1 but has the same cross-sectional area as R_1 .



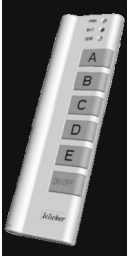
In which case is the CURRENT DENSITY through the resistor the smallest?

- Case 1 Case 2 Case 3

A) B) C)

$$J \equiv \frac{I}{A} \quad \Rightarrow \quad J_1 = J_3 = 2J_2$$

Same Current $\Rightarrow J \propto \frac{1}{A}$



BB

Let's review:

CLICKER QUESTION

Suppose we have a wire, with cross section area, A , length L , and a charge dQ flows through the wire in a time dt , what is the Current Density, J , defined as??

a) $\frac{dQ}{dt}$

d) $\frac{1}{LA} \frac{dQ}{dt}$

b) $\frac{1}{A} \frac{dQ}{dt}$

e) *none of these*

c) $A \frac{dQ}{dt}$

Y&F Problem 25.1 (Example, not a clicker question)

A current of 3.60 A flows through an automobile headlight. How many coulombs of charge flow through the headlight in a time of 2.60 hrs?

$$I = dQ / dt$$

$$dQ = Idt$$

$$Q = \int I dt \quad I \text{ constant}$$

$$Q = It = (3.6A)(2.6 \text{ hr})(3600 \text{ s / hr}) \quad (A=C/s)$$

$$= 3.37 \times 10^4 \text{ C}$$

Resistivity ρ

The current density at a point in a material depends on the material and on E . For some materials, J is proportional to E at a given temperature. These materials (metals for example) are ohmic and are said to obey "Ohm's Law".

Other materials (e.g. semiconductors) are non Ohmic.

Define resistivity as the ratio of electric field to current density ($V \cdot m / \text{Amp}$). The symbol for resistivity is the Greek letter ρ .

$$\rho = \frac{|\vec{E}|}{|\vec{J}|} \quad \text{OR} \quad \vec{E} = \rho \vec{J}$$

For ohmic materials, ρ at a given temperature is nearly constant.

New notation for V/Amp is unit, Ohm, represented by Greek letter, Ω . Units of ρ are $\Omega \cdot m$. Insulators have *large* values of ρ .

For Glass, $\rho > 10^{10} \Omega \cdot m$.

Resistivity ρ

The inverse of resistivity is called conductivity. Conductors have *large* values of *conductivity* or very small values of ρ .

$$\sigma = \frac{1}{\rho}$$

For copper $\rho = 2.44 \times 10^{-8} \Omega \cdot \text{m}$.

Temperature dependence of ρ :

For ohmic materials:

$$\rho(T) = \rho_0 [1 + \alpha(T - T_0)]$$

ρ_0 = resistivity at room temp (20° C)

$\rho(T)$ = resistivity at T

α = temperature coefficient of resistivity

Other materials (non-ohmic) more complicated.

CLICKER QUESTION

What is the definition of resistivity??

a) $\frac{\text{Voltage}}{\text{Current}}$

d) Voltage Current

b) $\frac{\text{Amps}}{\text{Area}}$

e) $\frac{\text{electric field}}{\text{current density}}$

c) $\frac{\text{current density}}{\text{electric field}}$

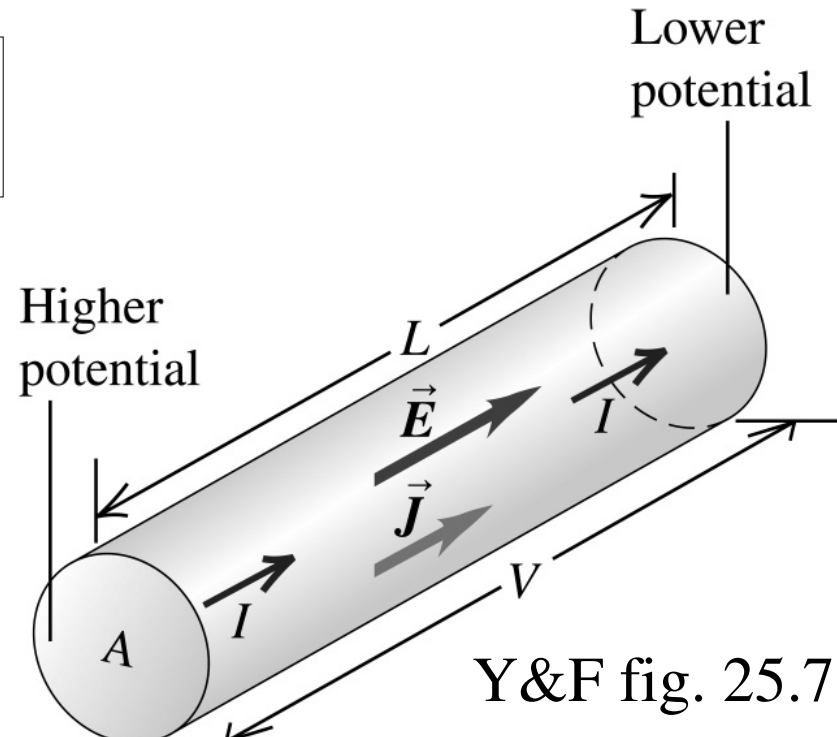
Resistance R

Consider a uniform, straight section of wire of length L and cross section A and with current I .

$$\vec{E} = \rho \vec{J}$$

Multiply by length L

$$|\vec{E}|L = V = \rho |\vec{J}| L = \rho \frac{I}{A} L = \left(\rho \frac{L}{A} \right) I = R I$$



We define resistance, R as $R = \rho \frac{L}{A}$

And we have for ohmic materials Ohm's Law: $V = I R$

Resistance increases with bigger L and decreases with bigger A

Units of R : Ω

V, R, I easier to measure than $E, \rho,$ and J .

Resistance R

We define resistance, R as $R = \rho \frac{L}{A}$

And we have for ohmic materials Ohm's Law:

$$V = I R$$

Units of R: Ω

Temperature dependence (ohmic materials):

$$R(T) = R_0 [1 + \alpha(T - T_0)]$$

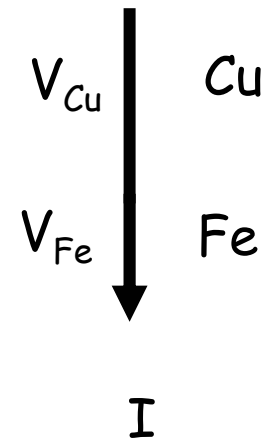
Example: A 80.0 m Cu wire 1.0 mm in diameter is joined to a 49.0 m iron wire of the same diameter. The current in each is 2.0 A.
 a.) Find V in each wire.

$$R_{Cu} = \frac{\rho_{Cu} L_{Cu}}{A} = \frac{(1.7 \times 10^{-8} \Omega m)(80.0 m)}{\pi(0.5 \times 10^{-3} m)^2} = 1.73 \Omega$$

$$R_{Fe} = \frac{\rho_{Fe} L_{Fe}}{A} = \frac{(10 \times 10^{-8} \Omega m)(49.0 m)}{\pi(0.5 \times 10^{-3} m)^2} = 6.24 \Omega$$

$$V_{Cu} = IR_{Cu} = (2.0 A)(1.73 \Omega) = 3.46 V$$

$$V_{Fe} = IR_{Fe} = 12.5 V$$



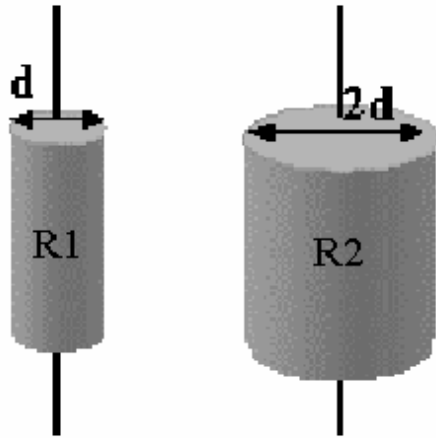
b.) Find E in each wire.

$$V = EL$$

$$E_{Cu} = \frac{V_{Cu}}{L_{Cu}} = 0.43 \frac{V}{m}$$

$$E_{Fe} = \frac{V_{Fe}}{L_{Fe}} = 0.255 \frac{V}{m}$$

Two part clicker problem:



Two cylindrical resistors are made from the same material, and they are equal in length. The first resistor has diameter d , and the second resistor has diameter $2d$.

2) Compare the resistance of the two cylinders.

a) $R_1 > R_2$

b) $R_1 = R_2$

c) $R_1 < R_2$

3) If the same current flows through both resistors, compare the average velocities of the electrons in the two resistors:

a) $v_1 > v_2$

b) $v_1 = v_2$

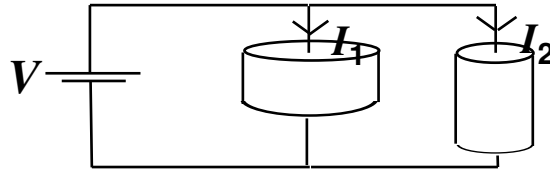
c) $v_1 < v_2$

Clicker problem

1A

Two cylindrical resistors, R_1 and R_2 , are made of the same material. R_2 has twice the length of R_1 but half the radius of R_1 .

– These resistors are then connected to a battery V as shown:



– What is the relation between I_1 , the current flowing in R_1 , and I_2 , the current flowing in R_2 ?

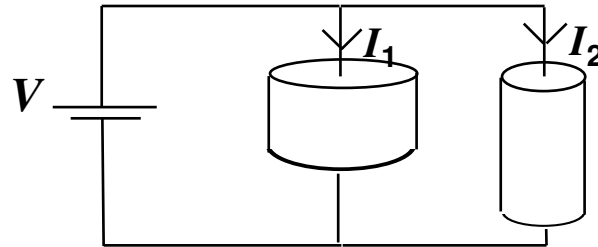
(a) $I_1 < I_2$

(b) $I_1 = I_2$

(c) $I_1 > I_2$

Clicker problem

- Two cylindrical resistors, R_1 and R_2 , are made of identical material. R_2 has twice the length of R_1 but half the radius of R_1 .
 - These resistors are then connected to a battery V as shown:



- What is the relation between I_1 , the current flowing in R_1 , and I_2 , the current flowing in R_2 ?

(a) $I_1 < I_2$

(b) $I_1 = I_2$

(c) $I_1 > I_2$

- The resistivity of both resistors is the same (ρ).
- Therefore the resistances are related as:

$$R_2 = \rho \frac{L_2}{A_2} = \rho \frac{2L_1}{(A_1/4)} = 8\rho \frac{L_1}{A_1} = 8R_1$$

- The resistors have the same voltage across them; therefore

$$I_2 = \frac{V}{R_2} = \frac{V}{8R_1} = \frac{1}{8} I_1$$