Physics 272 – Lecture 16

- Faraday’s Law
- Magnetic Induction
- Magnetic Materials
Faraday's Law

\[ \Phi_B \equiv \int \vec{B} \cdot d\vec{S} \]

\[ \varepsilon = -\frac{d \Phi_B}{dt} \]
So far we have studied electric fields created from charges and constant magnetic fields created by moving charges.

NOW we investigate effects of time varying magnetic fields on loops and we will find electric fields are induced in loops, which create EMFs and cause currents to flow.

Electric generators are based on the physics of electromagnetic induction and Faraday’s law.
Faraday's Law

Changing magnetic fields create electric fields

Stationary magnet has fixed magnetic field in the loop and creates NO current in loop

Moving magnet causes changing magnetic field in loop and creates current in loop
Coil fixed in an electromagnet; constant or changing magnetic field

- Magnetic field constant
  - NO current in loop

- Magnetic field increasing
  - Current in loop
History of the Discovery of Electromagnetic Induction

In 1830, Joseph Henry of Princeton (invented telegraph) observed induction, “conversion of magnetism to electricity”, but did not publish and did not recognize the vast importance of this observation. Later however, he observed and published evidence for self induction.

In 1831, Michael Faraday observed and published evidence of electromagnetic induction and also invented the homopolar generator that converts continuous DC.

Faraday at the Royal Society

Faraday’s Laboratory
Recall the definition of magnetic flux is \( \Phi_B = \int \vec{B} \cdot d\vec{A} \)

Faraday’s Law is the induced EMF in a closed loop equal the negative of the time derivative of magnetic flux change in the loop,

\[
\mathcal{E} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A} = -\frac{d\Phi_B}{dt}
\]

Constant B field, no induced EMF in loop

changing B field, causes induced EMF in loop

Tricky part is figuring out the EMF direction
Define the loop and an area vector, $\mathbf{A}$, whose magnitude is the area and whose direction is normal to the surface.

The choice of vector $\mathbf{A}$ direction defines the direction of EMF by the right hand rule. Your thumb in $\mathbf{A}$ direction and then your fingers curl in the direction of positive EMF.
(a) Positive flux ($\Phi_B > 0$)
Flux becoming more positive ($\frac{d\Phi_B}{dt} > 0$)
Induced emf is negative ($\mathcal{E} < 0$)

(b) Positive flux ($\Phi_B > 0$)
Flux becoming less positive ($\frac{d\Phi_B}{dt} < 0$)
Induced emf is positive ($\mathcal{E} > 0$)
Lenz's Law - easier way!

Lenz's Law: the magnetic field of any induced current opposes the change that induces it.

⇒ Convenient method to determine \( I \) direction

Example if an external magnetic field on a loop is increasing, the induced current creates a field opposite that reduces the net field.

Example if an external magnetic field on a loop is decreasing, the induced current creates a field parallel to the that tends to increase the net field.

Lenz's Law is a consequence of conservation of energy
Lenz’s Law; consider moving magnet towards a wire loop

No current and no field

stationary magnet

Induced field is opposite

moving N side of magnet into loop

moving S side of magnet into loop

Repulsive force, like pushing two S poles together.
A copper loop is placed in a non-uniform magnetic field. The magnetic field does not change in time. You are looking from the right.

2) Initially the loop is stationary. What is the induced current in the loop?
   a) zero                    b) clockwise           c) counter-clockwise

3) Now the loop is moving to the right, the field is still constant. What is the induced current in the loop?
   a) zero                        b) clockwise                  c) counter-clockwise
When the loop is stationary: the flux through the ring does not change!!!  
\[ \Rightarrow \frac{d\Phi}{dt} = 0 \quad \Rightarrow \text{there is no emf induced and no current.} \]

When the loop is moving to the right: the magnetic field at the position of the loop is increasing in magnitude.  
\[ \Rightarrow |\frac{d\Phi}{dt}| > 0 \]
\[ \Rightarrow \text{there is an emf induced and a current flows through the ring.} \]

Use Lenz’ Law to determine the direction: The induced emf (current) opposes the change!  
The induced current creates a B field at the ring which opposes the increasing external B field.
Clicker

- A conducting rectangular loop moves with constant velocity $v$ in the $+x$ direction through a region of constant magnetic field $B$ in the $-z$ direction as shown.

- What is the direction of the induced current in the loop?
  
  (a) ccw  
  (b) cw  
  (c) no induced current

- A conducting rectangular loop moves with constant velocity $v$ in the $-y$ direction and a constant current $I$ flows in the $+x$ direction as shown.

- What is the direction of the induced current in the loop?
  
  (a) ccw  
  (b) cw  
  (c) no induced current
Clicker

• A conducting rectangular loop moves with constant velocity $v$ in the $+x$ direction through a region of constant magnetic field $B$ in the -$z$ direction as shown.

**2A** What is the direction of the induced current in the loop?

(a) ccw  (b) cw  (c) no induced current

• There is a non-zero flux $\Phi_B$ passing through the loop since $B$ is perpendicular to the area of the loop.

• Since the velocity of the loop and the magnetic field are CONSTANT, however, this flux DOES NOT CHANGE IN TIME.

• Therefore, there is NO emf induced in the loop; NO current will flow!!
Clicker

- A conducting rectangular loop moves with constant velocity \( v \) in the -\( y \) direction and a constant current \( I \) flows in the +\( x \) direction as shown.

- What is the direction of the induced current in the loop?

(a) ccw  (b) cw  (c) no induced current

- The flux through this loop DOES change in time since the loop is moving from a region of higher magnetic field to a region of lower field.

- Therefore, by Lenz’ Law, an emf will be induced which will oppose the change in flux.

- Current is induced in the clockwise direction to restore the flux.
The area is $LS(t)$ and $dS/dt$ equals velocity of slider

$$
\mathcal{E} = -\frac{d}{dt} \Phi_B = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A} = -\frac{d}{dt} \int B \, dA
$$

$$
= -\frac{d}{dt} BA = -\frac{d}{dt} BLS = -BL \frac{d}{dt} S = -BLv
$$
we now ask what is power dissipated, $P_d$, and power applied, $P_a$.

We assume total resistance is $R$, then $I = \varepsilon / R$ and $P_d = I^2 R = B^2 L^2 v^2 / R$.

The power applied, $P_a = Fv$, is the rate of work to the bar to the right against the force, $F$, which is pointed to the left. $F = ILB = (\varepsilon / R)LB = B^2 L^2 v / R$. Thus $P_a = Fv = B^2 L^2 v^2 / R = P_d$ and we find the power put into the moving the bar equals the heat dissipated in the circuit.

$\Rightarrow$ Power needed to move bar = Power dissipated in circuit
Slide Wire Generator; use Lenz's Law to get I direction

Lenz’s Law says direction creates field that opposes change in magnetic flux.

If we pull bar to right, the net magnetic flux in rectangle increases into screen, hence the I direction must induce opposite B field which is out of screen and is correct in drawing.

Suppose Lenz’s law were reversed, then I would be reversed and F would go right and the bar would be accelerated to the right, w/o need of external positive work and heat would be dissipated at the same time. This violates cons. of Energy, so Lenz’s law is correct.
Motional Electromotive Force

\[ \mathcal{E} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A} = -\frac{d\Phi_B}{dt} \]

In Faraday’s Law, we can induce EMF in the loop when the magnetic flux, \( \Phi_B \), changes as a function of time. There are two Cases when \( \Phi_B \) is changing,
1) Change the magnetic field (non-constant over time)
2) Change or move the loop in a constant magnetic field

The slide wire generator is an example of #2 and the induction of EMF via moving parts of the loop is called, motional EMF.
Wire loop area $A$ rotates with respect to constant magnetic field.

\[ \Phi_B = \int \vec{B} \cdot d\vec{A} = BA \cos \phi \]

If the angular frequency is $\omega$, then

\[ \Phi_B = BA \cos(\omega t) \]

\[ \frac{d}{dt} \Phi_B = -BA \omega \sin(\omega t) \]

and EMF in the loop is

\[ \mathcal{E} = -\frac{d}{dt} \Phi_B = BA \omega \sin(\omega t) \]
In a physics laboratory experiment, a coil with 200 turns enclosing an area of 122 cm$^2$ is rotated in a time interval of 0.04 s from a position where its plane is perpendicular to the earth's magnetic field to one where its plane is parallel to the field. The earth's magnetic field at the lab location is 6x10$^{-5}$ T.

What is the total magnetic flux through the coil before it is rotated?
What is the total magnetic flux through the coil after it is rotated?
What is the average emf induced in the coil?

\[
\Phi_i = N \int \vec{B} \cdot d\vec{A} = NB \cos 0^\circ \int dA = NBA \\
= 200(6 \times 10^{-5} T)(0.0122 m^2) = 1.46 \times 10^{-4} Tm^2 \\
\Phi_f = NB \cos 90^\circ A = 0 \\
\bar{\varepsilon} = -\frac{(\Phi_f - \Phi_i)}{\Delta t} = \frac{7.31 \times 10^{-7} Tm^2}{0.04 s} = 3.66 \times 10^{-3} V
\]
Strategy for Faraday's Law Problems

B is time-varying but area of current loop is fixed

\[ \varepsilon = -\frac{d}{dt} \oint B \cdot d\vec{A} = -AdB / dt \]

Area of current loop is varying but B is fixed

\[ \varepsilon = -\frac{d}{dt} \oint B \cdot d\vec{A} = -BdA / dt \]

Orientation of current loop relative to B is time-varying but B and area A are fixed

\[ \varepsilon = -\frac{d}{dt} \oint B \cdot d\vec{A} = -BAd(\cos(\theta)) / dt \]
Applications of Magnetic Induction

• AC Generator
  - Water turns wheel
    → rotates magnet
    → changes flux
    → induces emf
    → drives current

• “Dynamic” Microphones
  (E.g., some telephones)
  - Sound
    → oscillating pressure waves
    → oscillating [diaphragm + coil]
    → oscillating magnetic flux
    → oscillating induced emf
    → oscillating current in wire

Question: Do dynamic microphones need a battery?
More Applications of Magnetic Induction

• Tape / Hard Drive / ZIP Readout
  – Tiny coil responds to change in flux as the magnetic domains (encoding 0’s or 1’s) go by.
    Related to 2007 Nobel Prize!!

• Credit Card Reader
  – Must swipe card
    \(\rightarrow\) generates changing flux
  – Faster swipe \(\rightarrow\) larger signal
More Applications of Magnetic Induction

- **Magnetic Levitation (Maglev) Trains**
  - Induced surface (“eddy”) currents produce field in opposite direction
    - Repels magnet
    - Levitates train
  - Maglev trains today can travel up to 310 mph
    - Twice the speed of Amtrak’s fastest conventional train!
  - May eventually use superconducting loops to produce B-field
    - No power dissipation in resistance of wires!
Induction cooking

Stove top does not become hot !!

Special induction cookware
Summary

• Faraday’s Law
  – a changing magnetic flux through a loop induces a current in that loop

\[
\varepsilon = - \frac{d\Phi_B}{dt}
\]

\[
\Phi_B \equiv \int \vec{B} \cdot d\vec{S}
\]

negative sign indicates that the induced EMF opposes the change in flux

• Lenz’s Law

The direction of any magnetic induction effect is such as to oppose the cause of the effect.
Classification of Magnetic Materials

Paramagnetism
Small, positive $\chi$

Diamagnetism
Small, negative $\chi$

Ferromagnetism
Huge positive $\chi$

$\vec{M} = \vec{\mu}_{total} / V$

$M = \chi_M H$

$B = B_o + \mu_o M = \mu_o H + \mu_o M$

$B = \mu_o H + \mu_o M = \mu_o (1 + \chi_M) H$

Insert in a solenoid
**Paramagnetism**

\[ M = C \cdot \frac{B}{T}, \]

---

**Dipoles orientation randomized by thermal fluctuations**

<table>
<thead>
<tr>
<th>Material</th>
<th>( x_m = K_m - 1 ) (x (10^{-5}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iron oxide (FeO)</td>
<td>720</td>
</tr>
<tr>
<td>Iron ammonium alum</td>
<td>66</td>
</tr>
<tr>
<td>Uranium</td>
<td>40</td>
</tr>
<tr>
<td>Platinum</td>
<td>26</td>
</tr>
<tr>
<td>Tungsten</td>
<td>6.8</td>
</tr>
<tr>
<td>Cesium</td>
<td>5.1</td>
</tr>
<tr>
<td>Aluminum</td>
<td>2.2</td>
</tr>
<tr>
<td>Lithium</td>
<td>1.4</td>
</tr>
<tr>
<td>Magnesium</td>
<td>1.2</td>
</tr>
<tr>
<td>Sodium</td>
<td>0.72</td>
</tr>
<tr>
<td>Oxygen gas</td>
<td>0.19</td>
</tr>
</tbody>
</table>
Diamagnetism

Diamagnetic ($\chi \times 10^{-6}$)

- Ammonia: -0.26
- Bismuth: -16.6
- Mercury: -2.9
- Silver: -2.6
- Carbon (diamond): -2.1
- Carbon (graphite): -1.6
- Lead: -1.8
- Sodium chloride: -1.4
- Copper: -1.0
- Water: -0.91

Diamagnetism is the result of Lenz’s Law at the atomic level.
For permanent magnets, the material retains its magnetization even after the external B field is removed. Important for magnetic storage.
Superconductor

Also a perfect diamagnet
## Properties of Superconductors

<table>
<thead>
<tr>
<th>Material</th>
<th>Type</th>
<th>$T_c$(K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zinc</td>
<td>metal</td>
<td>0.88</td>
</tr>
<tr>
<td>Aluminum</td>
<td>metal</td>
<td>1.19</td>
</tr>
<tr>
<td>Tin</td>
<td>metal</td>
<td>3.72</td>
</tr>
<tr>
<td>Mercury</td>
<td>metal</td>
<td>4.15</td>
</tr>
<tr>
<td>YBa$_2$Cu$_3$O$_6$</td>
<td>ceramic</td>
<td>90</td>
</tr>
<tr>
<td>TlBaCaCuO</td>
<td>ceramic</td>
<td>125</td>
</tr>
</tbody>
</table>
A magnetic material has a magnetic susceptibility $\chi = 10^{-5}$

It is a (a) ferromagnet (b) paramagnet (c) superconductor (d) diamagnet

A magnet hard disk (memory device) should use a (a) ferromagnet (b) paramagnet (c) superconductor (d) diamagnet