Physics 170 - *Mechanics*

Lecture 20

Momentum Conservation & Inelastic collisions

Conservation of Linear Momentum

The net force acting on an object is the rate of change of its momentum:

$$\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$

If the net force is zero, the momentum does not change:

Conservation of Momentum

If the net force acting on an object is zero, its momentum is conserved; that is, $\vec{p}_f = \vec{p}_i$

Conservation of Linear Momentum

Internal Versus External Forces:

Internal forces act between objects within the system.

As with all forces, they occur in action-reaction pairs. As all pairs act between objects in the system, the internal forces always sum to zero:

$$\sum \vec{F}_{int} = 0$$

Therefore, the net force acting on a system is the sum of the external forces acting on it.

Conservation of Linear Momentum

Furthermore, internal forces cannot change the momentum of a system.

Conservation of Momentum for a System of Objects

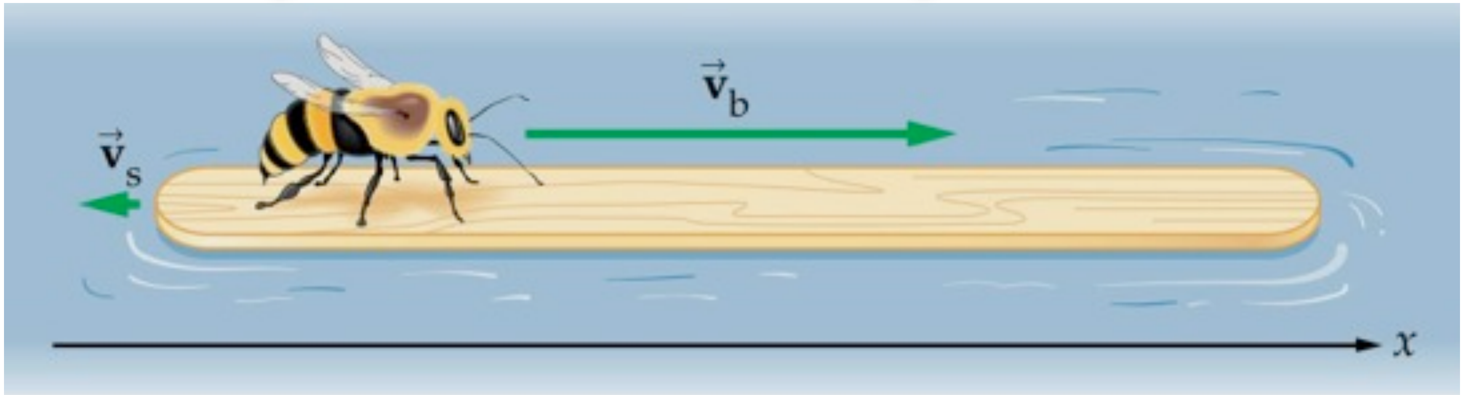
Internal forces have absolutely no effect on the net momentum of a system.

- If the *net external* force acting on a system is zero, its net momentum is conserved. That is,

$$\vec{p}_{1,f} + \vec{p}_{2,f} + \vec{p}_{3,f} + \dots = \vec{p}_{1,i} + \vec{p}_{2,i} + \vec{p}_{3,i} + \dots$$

However, the momenta of components of the system may change.

Example: Velocity of a Bee



A honeybee with a mass of 0.150 g lands on one end of a floating 4.75 g popsicle stick. After sitting at rest for a moment, it runs to the other end of the stick with a velocity v_b relative to still water. The stick moves in the opposite direction with a velocity of 0.120 cm/s.

Find the velocity v_b of the bee.

$$p_s = m_s v_s$$

Conservation of x-axis momentum: $p_s + p_b = 0$

$$p_b = m_b v_b = -p_s = -m_s v_s$$

$$v_b = -\frac{m_s}{m_b} v_s = -\frac{(4.75 \text{ g})}{(0.150 \text{ g})}(-0.120 \text{ cm/s}) = 3.80 \text{ cm/s}$$

Collision Types

Collision: two objects striking one another

[Time of collision is short enough that external forces may be ignored.]

- **Completely inelastic:** bodies stick together (bullet in a tree) $p_f = p_{i1} + p_{i2}$ but $K_f \neq K_i$.

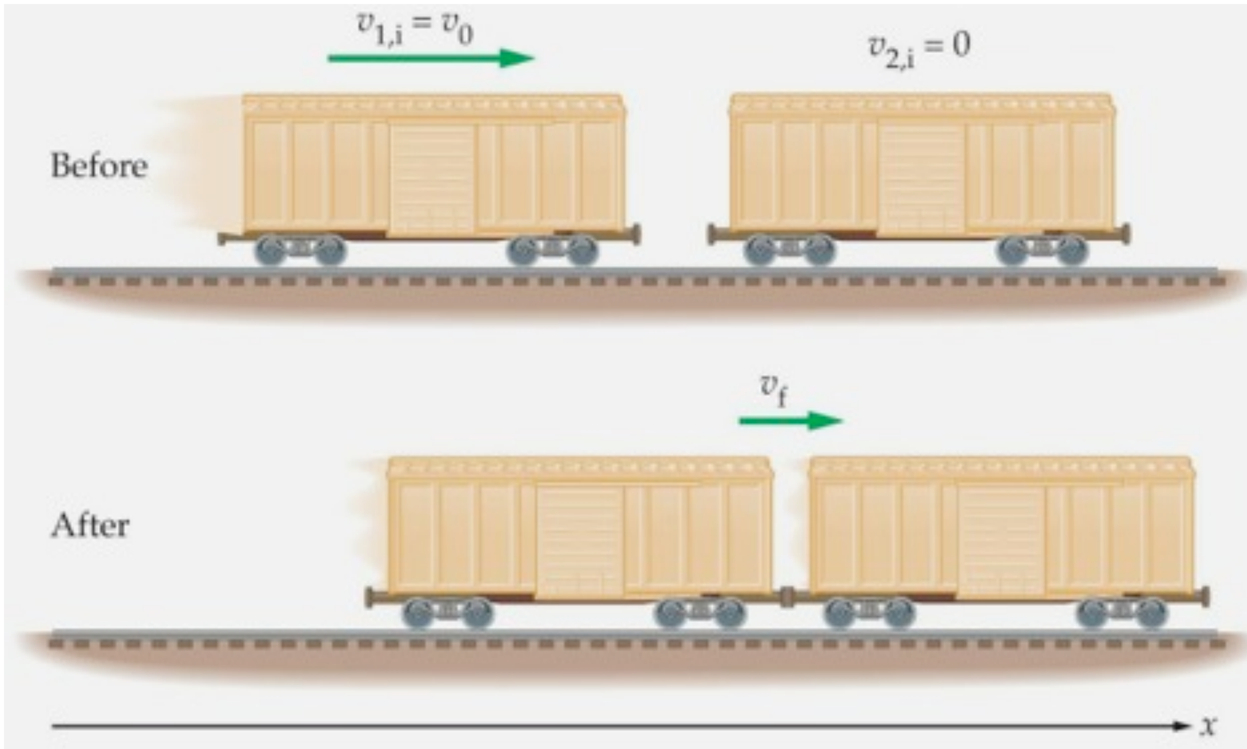
- **Partially inelastic collision:** bodies separate but work is done. Momentum is conserved but kinetic energy is not (Car accident). $p_f = p_i$ but $K_f \neq K_i$.

- **Elastic collision:** bodies separate, momentum and energy are both conserved: (billiard balls)

$p_f = p_i$ and $K_f = K_i$.

Inelastic Collisions

A completely inelastic collision:



$$m_1 v_{1,i} + m_2 v_{2,i} = (m_1 + m_2) v_f$$

Inelastic Collisions

Solving for the final momentum in terms of the initial momenta and masses:

$$p_i = m_1 v_{1,i} + m_2 v_{2,i}$$

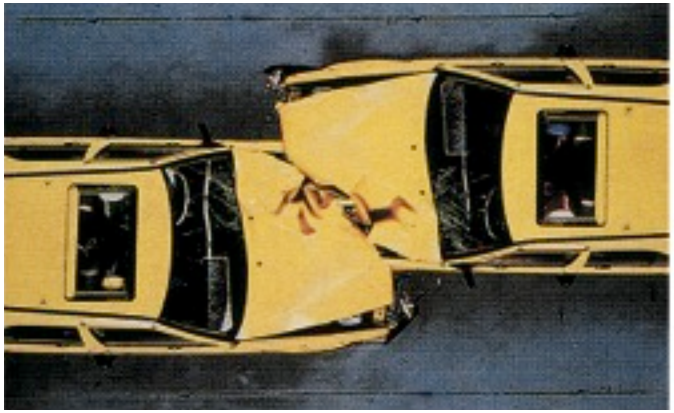
$$p_f = (m_1 + m_2) v_f$$

$$v_f = \frac{m_1 v_{1,i} + m_2 v_{2,i}}{m_1 + m_2}$$

Inelastic Collisions in 1D

$$m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + m_2 v_{2i}$$

Inelastic collision: $v_{1f} = v_{2f} = v_{cm}$



$$(m_1 + m_2) v_{cm} = m_1 v_{1i} + m_2 v_{2i}$$

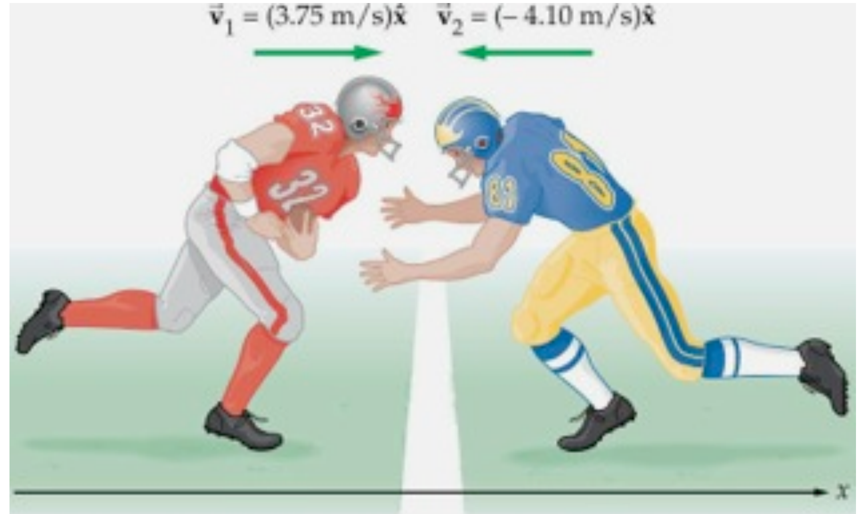
$$P_{sys} = p_{1i} = m_1 v_{1i}$$

$$K = \tfrac{1}{2} m v^2 = \frac{p^2}{2m}, \text{ so } K_i = \frac{P_{sys}^2}{2m_1} \text{ and } K_f = \frac{P_{sys}^2}{2(m_1 + m_2)}$$

Therefore, $K_f < K_i$ and there is a net loss of energy in an inelastic collision.

Example: Goal-Line Stand

On a touchdown attempt, a 95.0 kg running back runs toward the end zone at 3.75 m/s. A 111 kg line backer moving at 4.10 m/s meets the runner in a head-on collision and locks his arms around the runner.



(a) Find their velocity immediately after the collisions.

(b) Find the initial and final kinetic energies and the energy ΔK lost in the collision.

$$v_f = \frac{m_1 v_{1,i} + m_2 v_{2,i}}{m_1 + m_2} = \frac{(95.0\text{ kg})(3.75\text{ m/s}) + (111.0\text{ kg})(-4.10\text{ m/s})}{(95.0\text{ kg}) + (111.0\text{ kg})} = -0.480\text{ m/s}$$

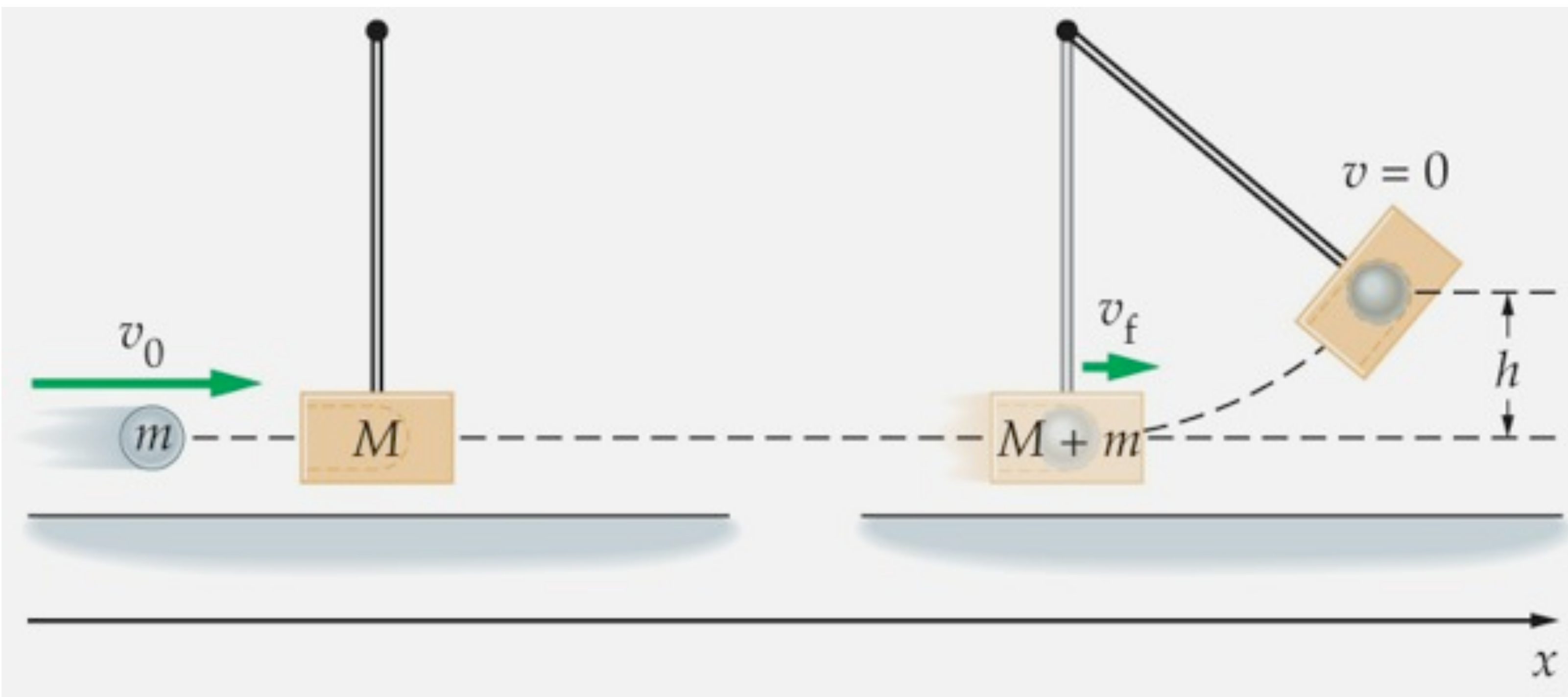
$$K_i = \tfrac{1}{2} m_1 v_{1,i}^2 + \tfrac{1}{2} m_2 v_{2,i}^2 = \tfrac{1}{2}(95.0\text{ kg})(3.75\text{ m/s})^2 + \tfrac{1}{2}(111.0\text{ kg})(-4.10\text{ m/s})^2 = 1600\text{ J}$$

$$K_f = \tfrac{1}{2}\left(m_1 + m_2\right) v_f^2 = \tfrac{1}{2}\left[(95.0\text{ kg}) + (111.0\text{ kg})\right](-0.480\text{ m/s})^2 = 23.7\text{ J}$$

$$\Delta K = -1576\text{ J}$$

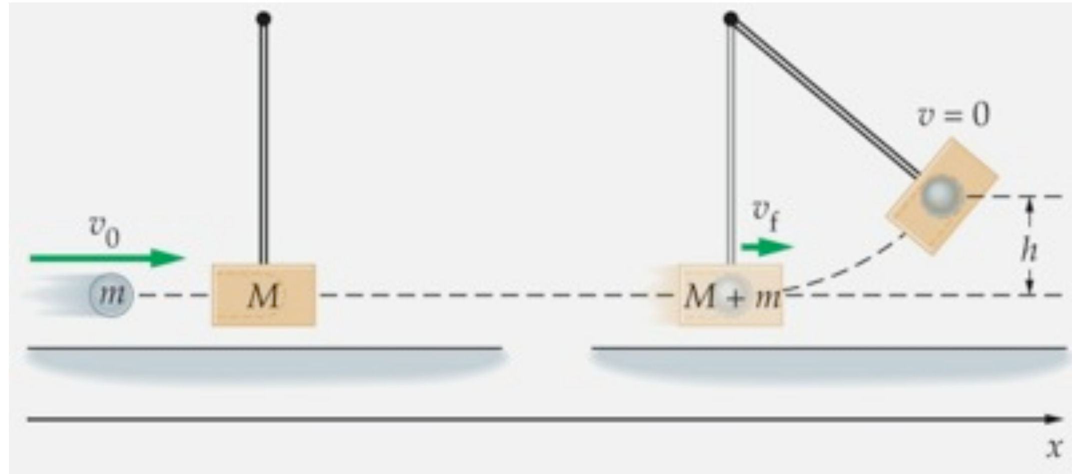
Inelastic Collisions

Ballistic pendulum: the height h can be found using conservation of mechanical energy after the object is embedded in the block.



Example: Ballistic Pendulum

A projectile of mass m is fired with an initial speed v_0 at the bob of a pendulum. The bob has mass M and is suspended by a rod of negligible mass.



After the collision the projectile and bob stick together and swing at speed v_f through an arc reaching height h.

Find the height h.

Momentum Conservation: $v_f = \frac{m}{m+M} v_0$

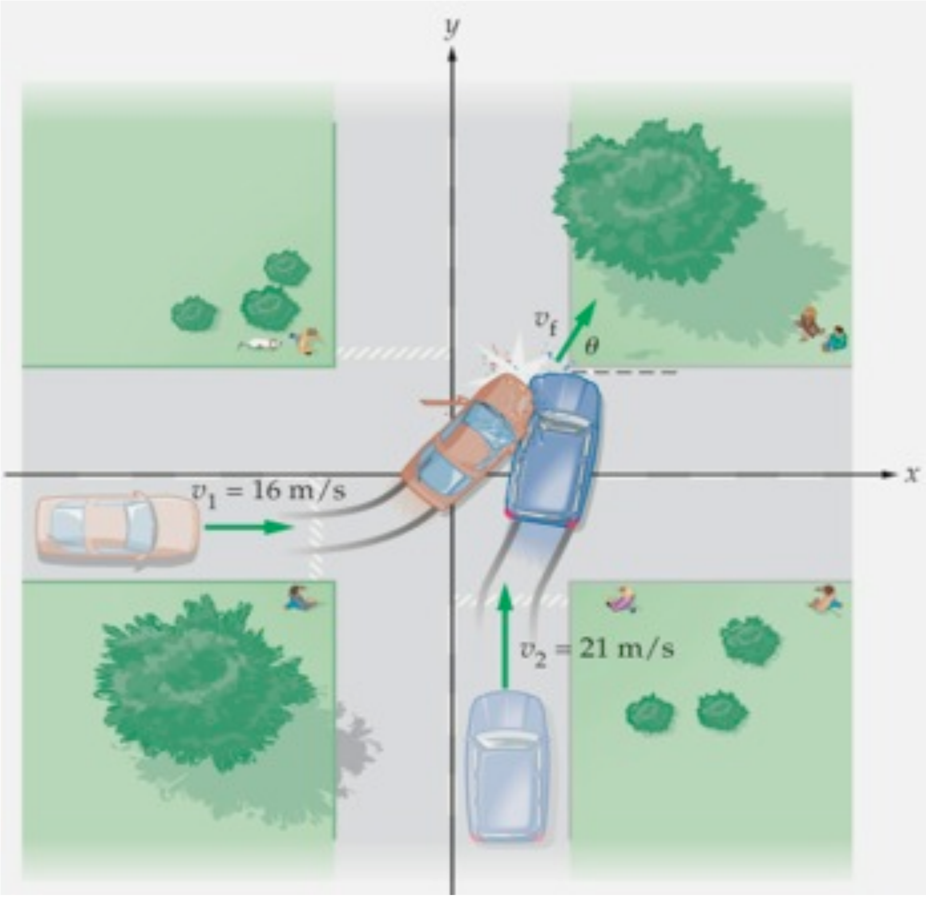
Energy Conservation: $E = \frac{1}{2}(m+M)v_f^2 = (m+M)gh$

$$h = \frac{v_f^2}{2g} = \left(\frac{m}{m+M}\right)^2 \frac{v_0^2}{2g}$$

$$v_0 = \left(\frac{m+M}{m}\right)\sqrt{2gh}$$

Inelastic Collisions

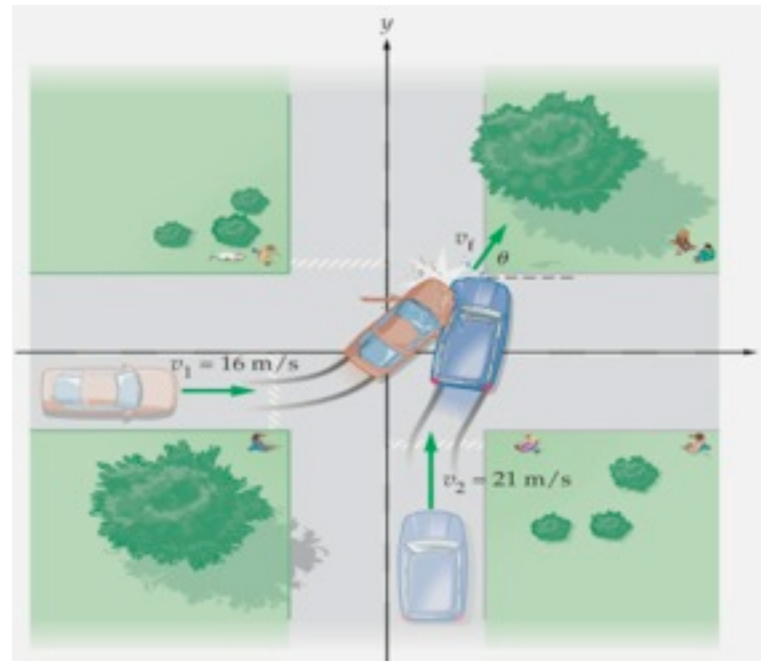
For collisions in two dimensions, conservation of momentum is applied separately along each axis:



Example: A Traffic Accident

A car of mass $m_1 = 950$ kg and a speed $v_{1,i} = 16$ m/s approaches an intersection. A minivan of mass $m_2 = 1300$ kg and speed $v_{2,i} = 21$ m/s enters the same intersection. The cars collide and stick together.

Find the direction θ and final speed v_f of the wrecked vehicles just after the collision.



x-momentum: $m_1 v_1 = (m_1 + m_2) v_f \cos\theta$

y-momentum: $m_2 v_2 = (m_1 + m_2) v_f \sin\theta$

$$\frac{m_2 v_2}{m_1 v_1} = \frac{(m_1 + m_2) v_f \sin\theta}{(m_1 + m_2) v_f \cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\theta = \arctan\frac{m_2 v_2}{m_1 v_1} = \arctan\frac{(1300 \text{ kg})(21 \text{ m/s})}{(950 \text{ kg})(16 \text{ m/s})} = 61^\circ$$

$$v_f = \frac{m_1 v_1}{(m_1 + m_2)\cos\theta} = \frac{(950 \text{ kg})(16 \text{ m/s})}{\left[(950 \text{ kg}) + (1300 \text{ kg})\right]\cos 61^\circ} = 14 \text{ m/s}$$