

Physics 170 - *Mechanics*

Lecture 11

Newton's Laws - part 2

Newton's Second Law of Motion

An object may have several forces acting on it; the acceleration is due to the net force:

$$\sum \vec{F} = m\vec{a}$$

TABLE 5-2 Units of Mass, Acceleration, and Force

System of units	Mass	Acceleration	Force
SI	kilogram (kg)	m/s ²	newton (N)
cgs	gram (g)	cm/s ²	dyne (dyn)
British	slug	ft/s ²	pound (lb)

(Note: 1 N = 10⁵ dyne = 0.225 lb.)

Newton's Second Law of Motion

Newton's 2nd Law:

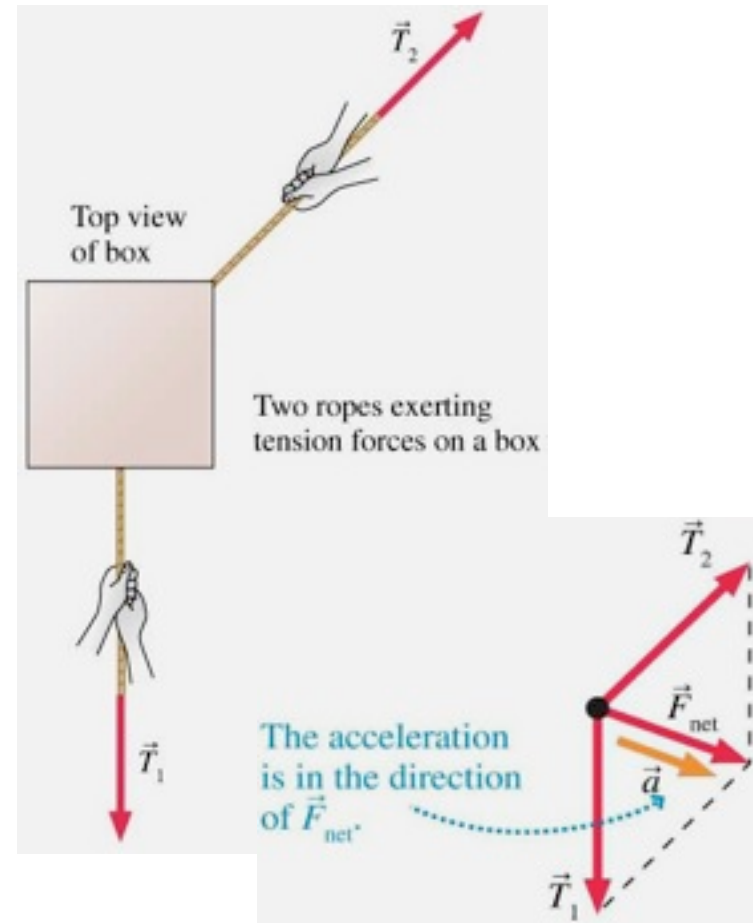
An object of a given mass m subjected to forces \vec{F}_1 , \vec{F}_2 , \vec{F}_3 , ... will undergo an acceleration \vec{a} given by:

$$\vec{a} = \vec{F}_{net}/m$$

where

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots$$

The mass m is positive, force and acceleration are in the same direction.



Newton's Third Law of Motion

The harder one sumo wrestler pushes, the harder the other pushes back. Newton's third law describes how two objects interact with each other.

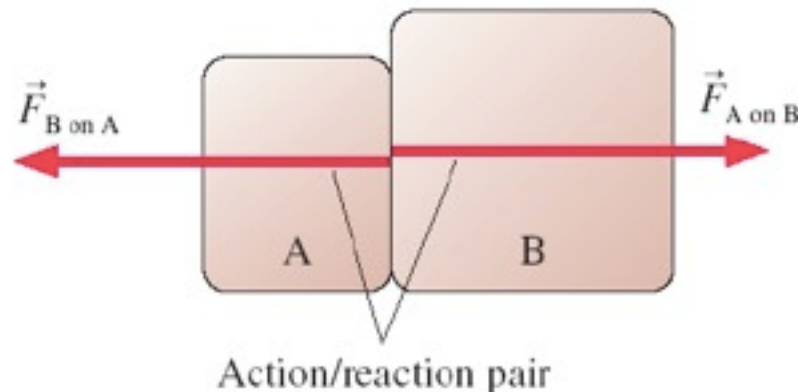


Newton's Third Law of Motion

Forces always come in pairs, acting on different objects:

If Object A exerts a force F on Object B, then Object B exerts a force $-F$ on Object A.

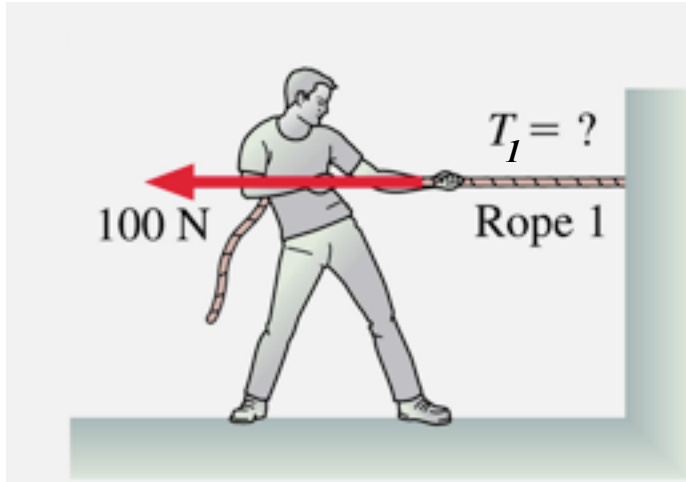
These forces are called action-reaction pairs.



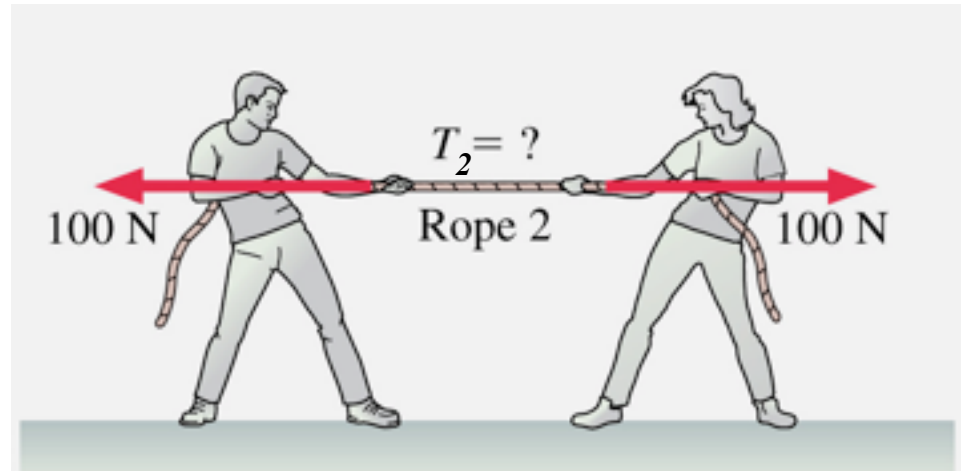
$$\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A}$$

Alternate Wording: "For every action there is an equal and opposite reaction."

Example: Pulling a Rope (1)



A student pulls horizontally with a force of 100 N on a rope that is attached to a wall.



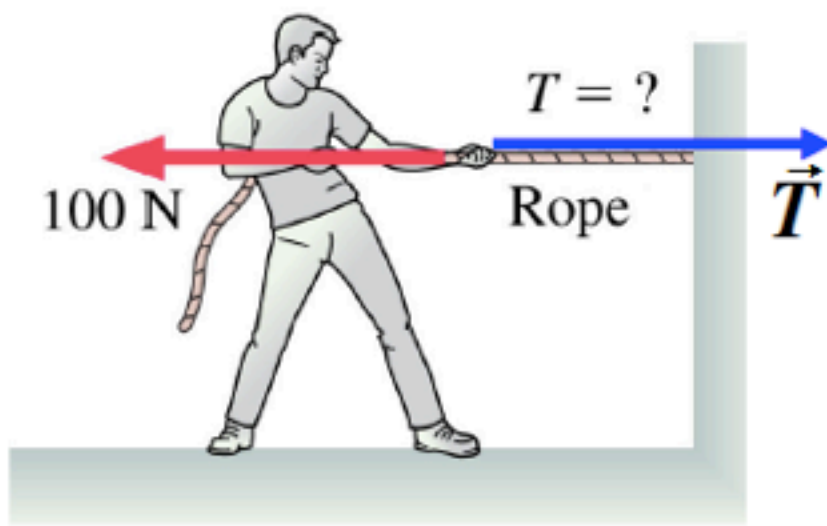
Two students pull on opposite ends of a rope with forces of 100 N each.

Which tension is larger?

a. $T_1 > T_2$

b. $T_1 = T_2$

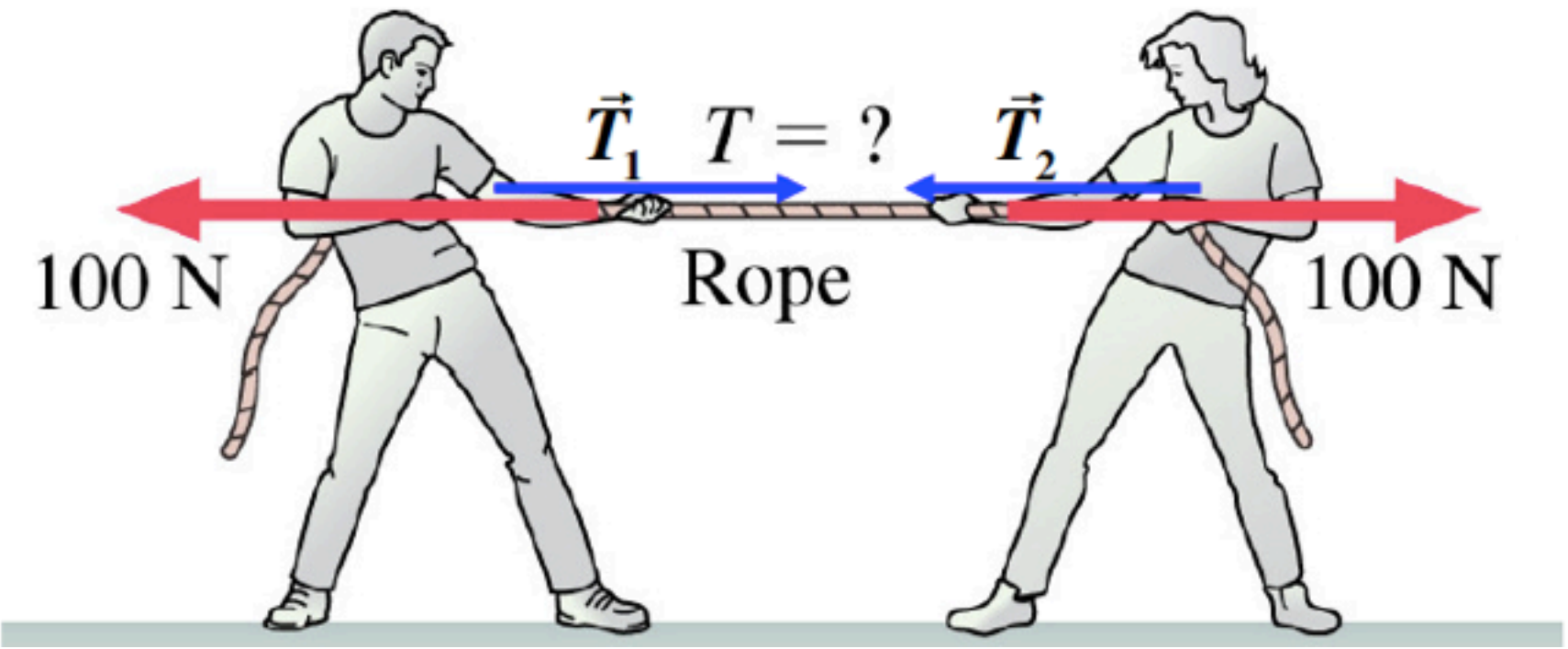
c. $T_1 < T_2$



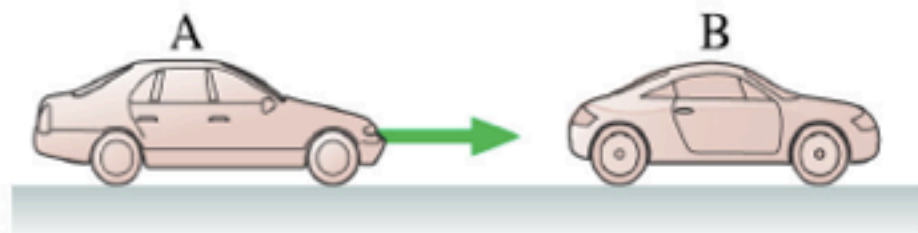
Newton's Third Law:

$$T = 100\text{ N}$$

$$T_1 = T_2 = 100\text{ N}$$

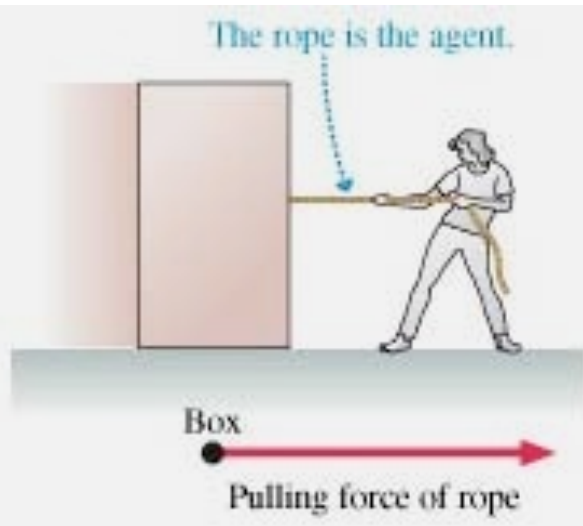


Car B is stopped for a red light. Car A, which has the same mass as car B, doesn't see the red light and runs into the back of B. Which of the following statements is true?

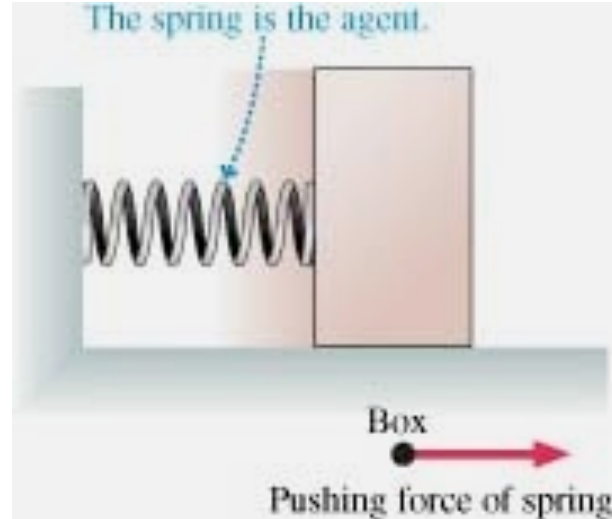


- A. A exerts a larger force on B than B exerts on A.
- B. A exerts a force on B but B doesn't exert a force on A.
- C. B exerts a force on A but A doesn't exert a force on B.
- D. B exerts a larger force on A than A exerts on B.
- E. **B exerts the same amount of force on A as A exerts on B.**

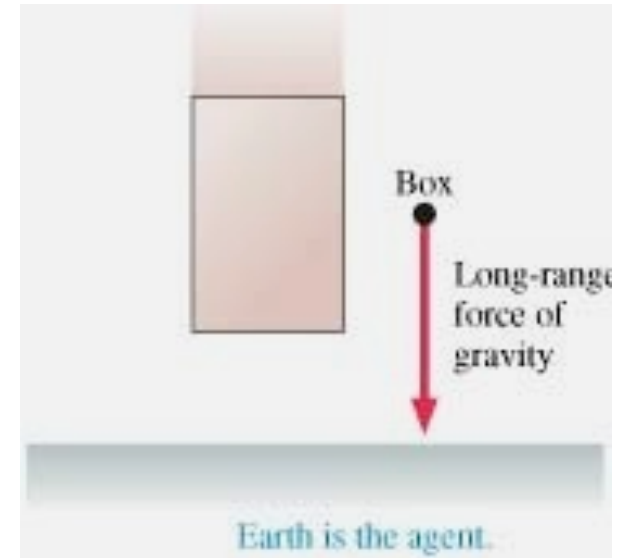
Examples of Force Vectors



Pull
(contact force)

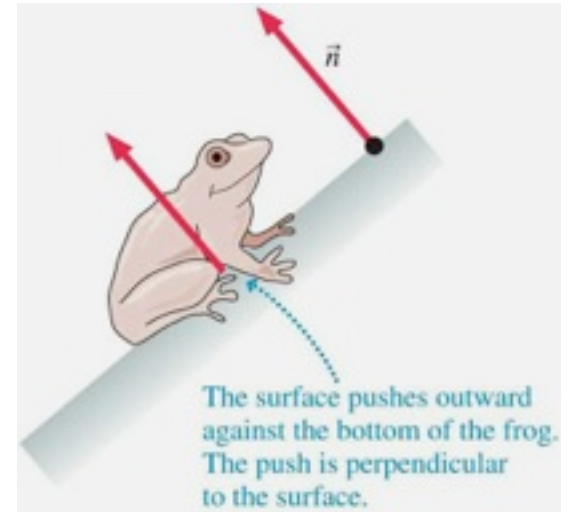
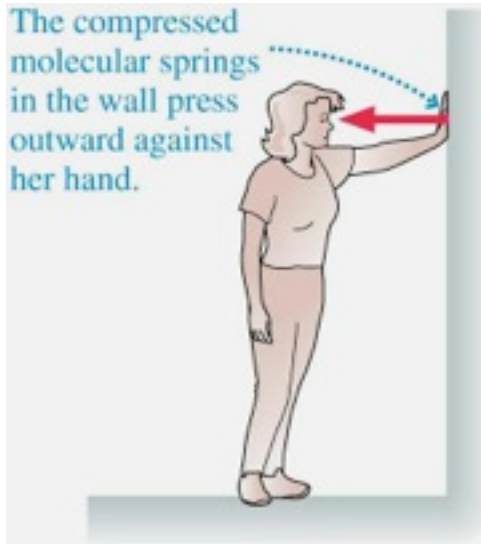
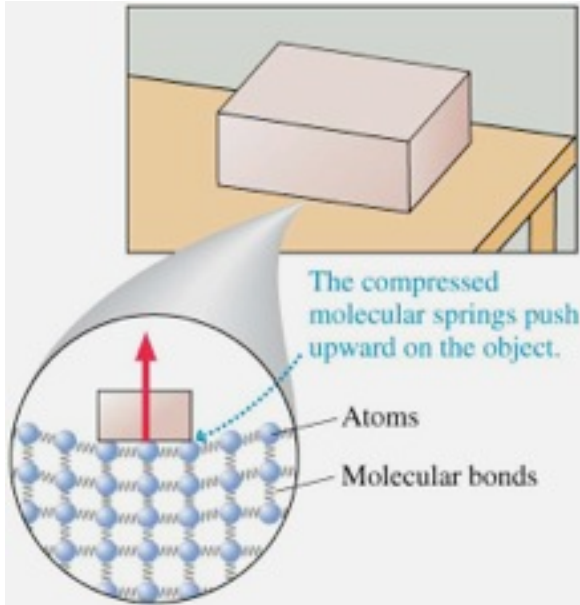


Push
(contact force)



Gravity
(long-range force)

A Catalog of Forces: (1. Normal)



Normal Force

A solid object resists the action of another force which compresses it with what we call the **normal force**. The normal force always acts outward and perpendicular to the surface of the compressed object. The symbol for the normal force is \vec{n} .

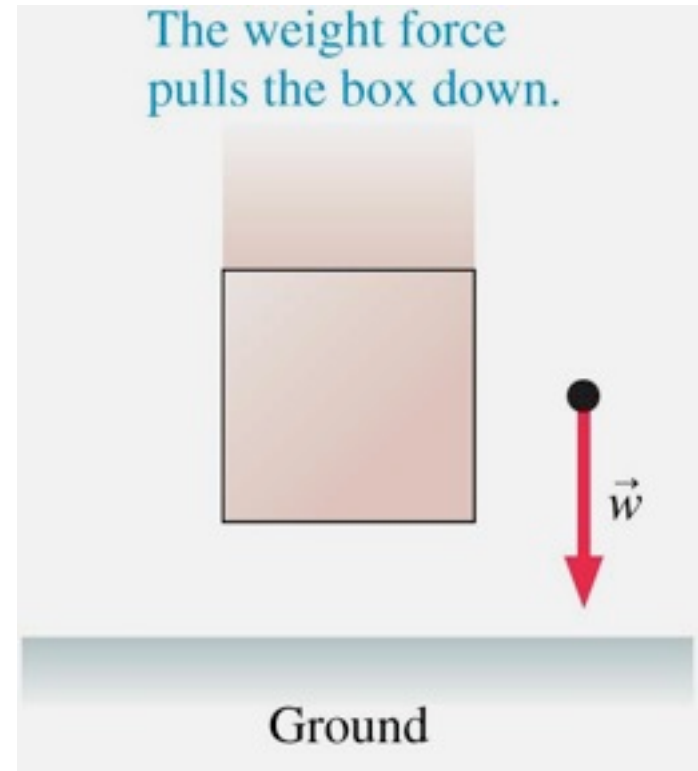
A Catalog of Forces: (2. Weight)

Weight

The falling box is pulled toward the Earth by the long-range force of gravity. The gravitational pull on a object on or near the surface of the Earth is called **weight**, for which we use the symbol \vec{w} .

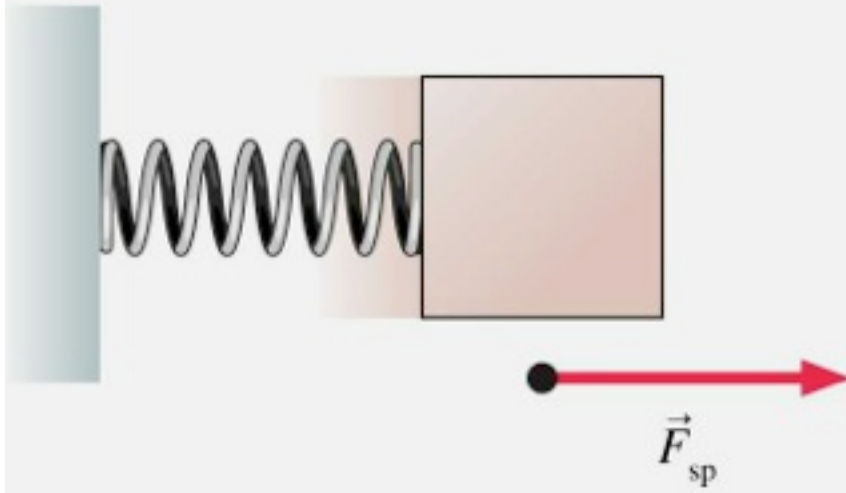
Weight is the only long range force that we will consider in this course. The agent for the weight force is the entire Earth pulling on the object. Weight acts equally on objects at rest or in motion. The weight vector always points vertically downward, and it can be considered to act at the center of mass of the object.

$$g = 9.81 \text{ m/s}^2; \quad \vec{w} = m\vec{g}$$

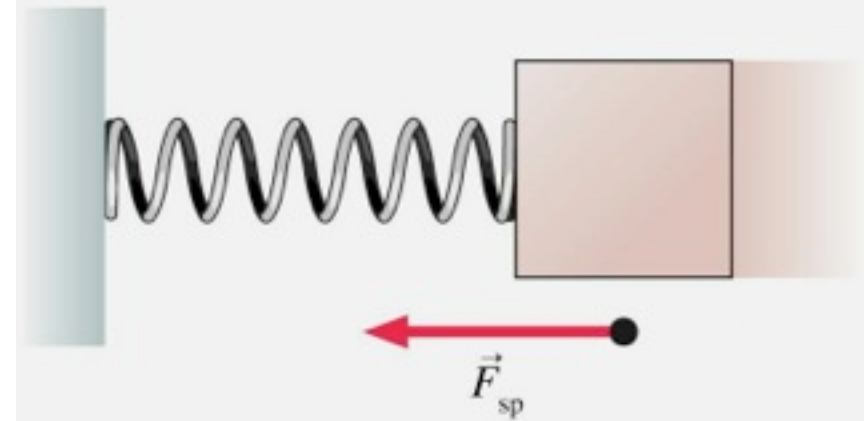


A Catalog of Forces: (3. Spring)

A compressed spring exerts a pushing force on an object.



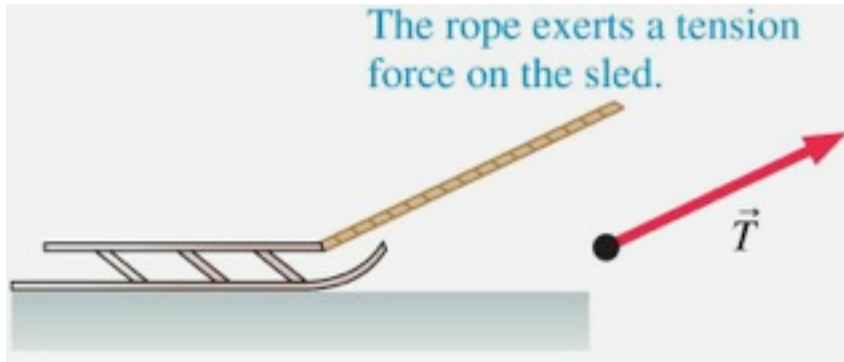
A stretched spring exerts a pulling force on an object.



Spring Force

A stretched or compressed spring exerts one of the most common contact forces. A spring can either push (when compressed) or pull (when stretched). In either case, the tail of the vector force is attached to the contact point. There is no special symbol for the spring force, but we can use \mathbf{F}_{sp} .

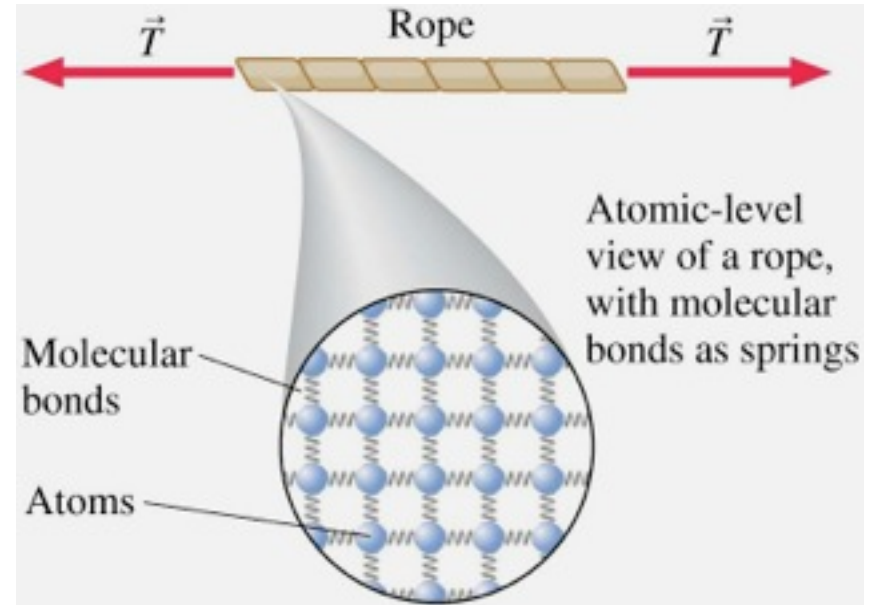
A Catalog of Forces: (4. Tension)



Tension Force

A string or rope exerts a contact force on an object when it pulls on it. We call this a tension force, represented by the symbol \vec{T} .

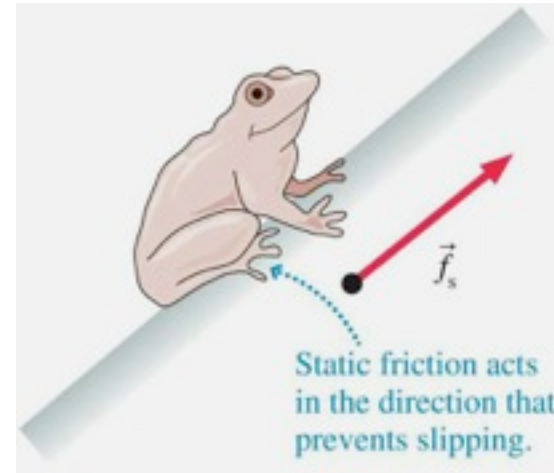
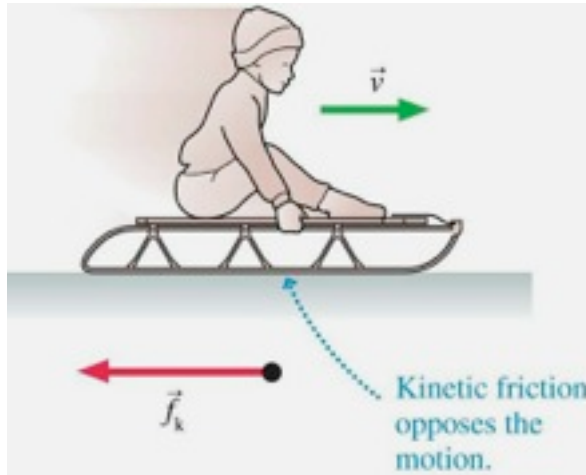
Tension is always directed **along the line of the rope** or string, with no component perpendicular to it.



Microscopic View

A powerful microscope would see that the string was made up of atoms connected by molecular bonds, that can be thought of as tiny springs holding the atoms together.

A Catalog of Forces: (5. Friction)



Friction

At the molecular level, surfaces tend to stick together, impeding motion. This produces the force we call **friction**. It comes in two varieties:

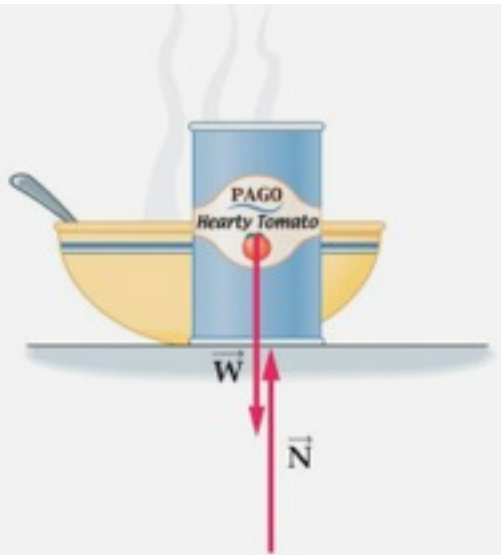
Kinetic friction, denoted by the symbol \vec{f}_k , appears when an object slides across a surface. This is a force that opposes the motion and points in the opposite direction from the velocity.

Static friction denoted by the symbol \vec{f}_s , is a force that keeps an object at rest stuck to a surface and prevents its motion. It points in the direction that prevents motion. Typically, it is larger than the kinetic friction that appears after the object begins to move.

Identifying Forces

- **Identify “the system” and “the environment.”** The system is the object whose motion you wish to study; the environment is everything else.
- **Draw a picture of the situation.** Show the object—the system—and everything in the environment that touches the system. Ropes, springs, and surfaces are all parts of the environment.
- **Consider a system at the time.**
- **Locate every point where the environment exerts contact forces on the object.**
- **Name and label each contact force acting on the object.** There is at least one force at each point of contact; there may be more than one. When necessary, use subscripts to distinguish forces of the same type.
- **Name and label each long-range force acting on the object.** For now, the only long-range force is weight.

Normal Force

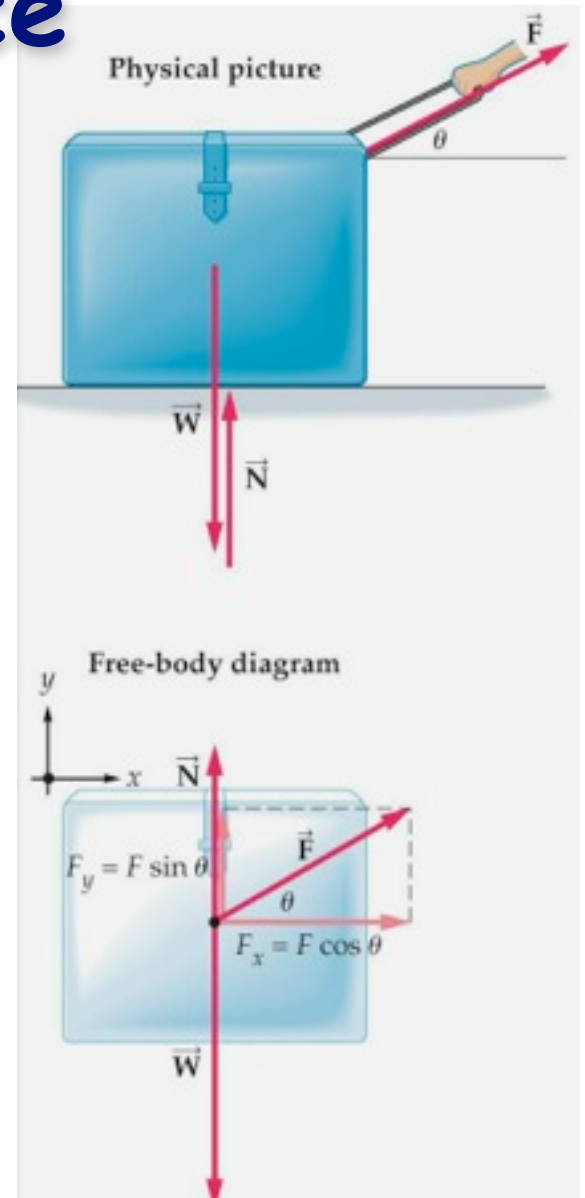


Physical picture



Free-body diagram

The normal force is the force exerted by a surface on an object.



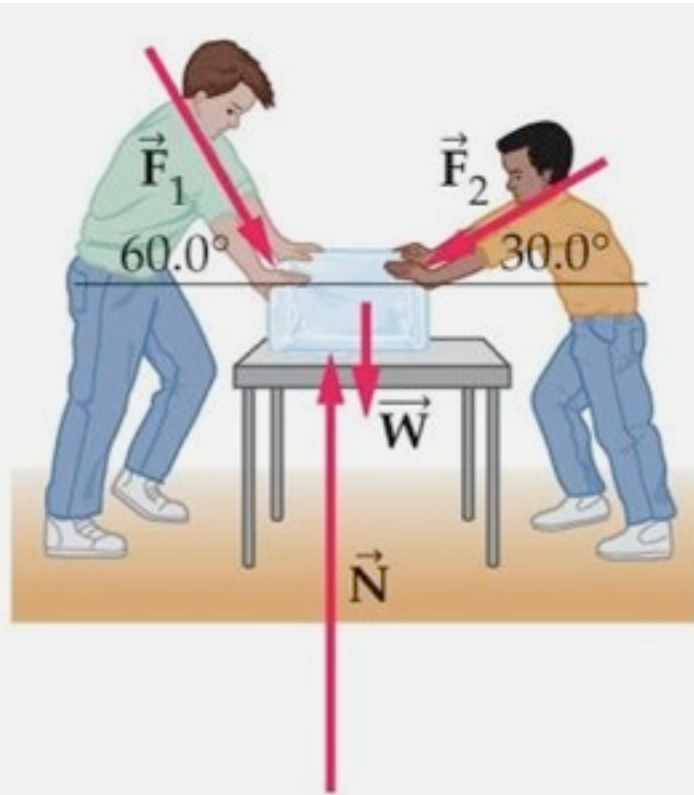
Normal Force

The normal force is always perpendicular to the surface that produces it.

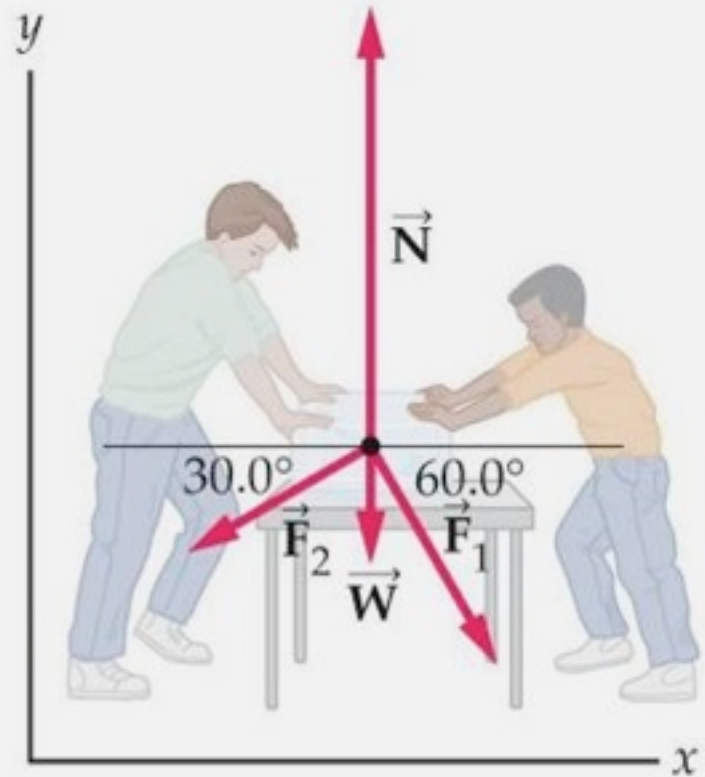


Normal Force

The normal force may be equal to, greater than, or less than the weight.



Physical picture

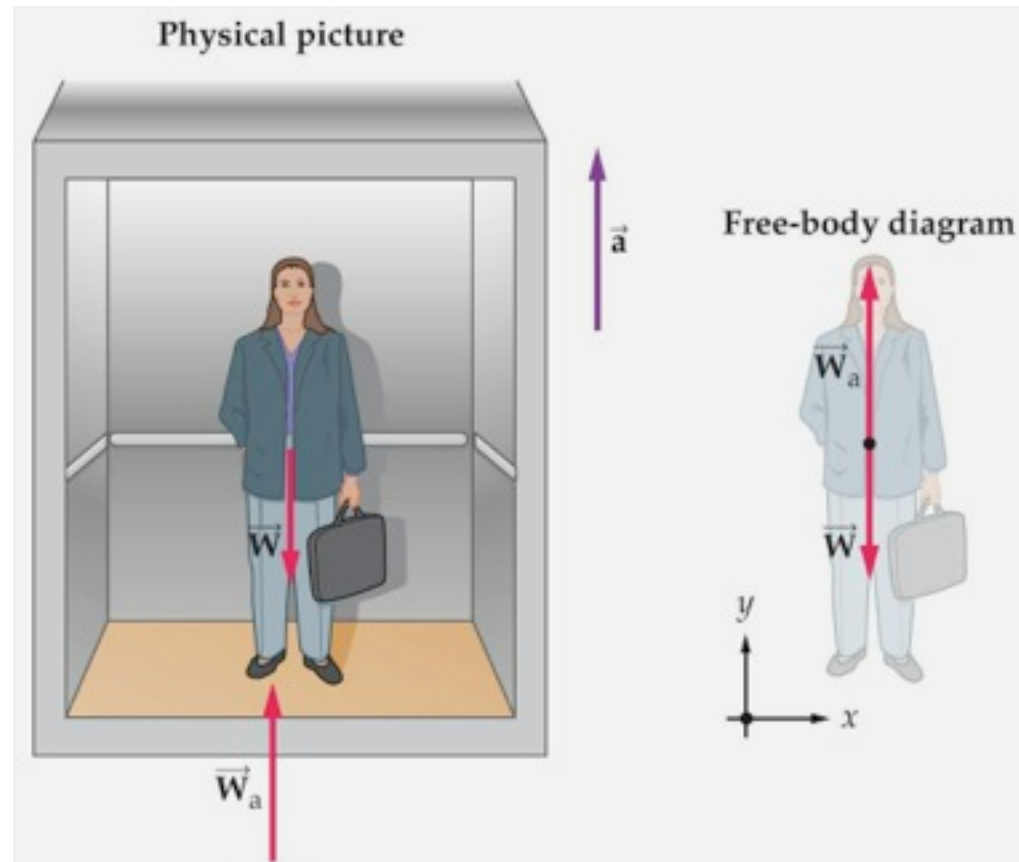


Free-body diagram

Apparent Weight

Your perception of your weight is based on the contact forces between your body and your surroundings.

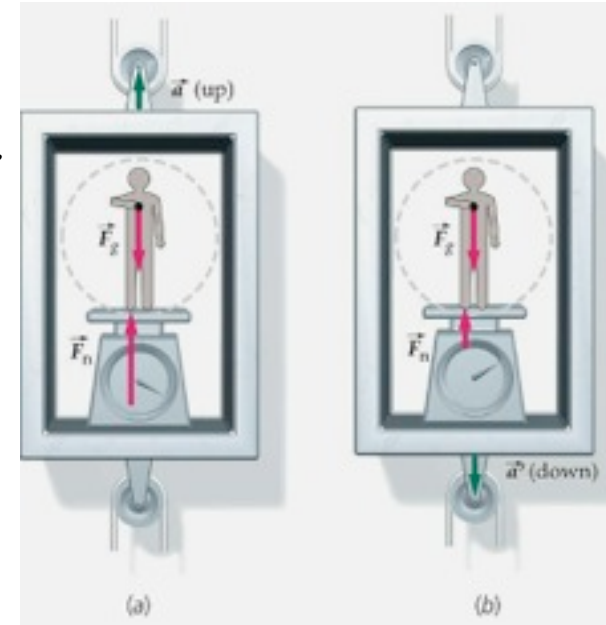
If your surroundings are **accelerating**, your apparent weight may be more or less than your actual weight.



Example: Weighing Yourself in an Elevator

Suppose that your mass is 80 kg, and you are standing on a scale fastened to the floor of an elevator. The scale measures force and is calibrated in newtons.

- (a) What does the scale read when the elevator is rising with acceleration \vec{a} ?
- (b) When it is descending with acceleration \vec{a}' ?

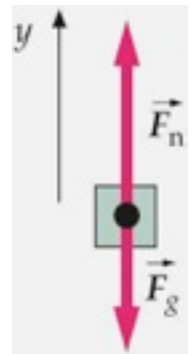


$$\vec{F}_n + \vec{F}_g = m\vec{a}$$

$$F_n - mg = ma$$

$$F_{na} = m(g + a) \quad \text{the } \vec{a} \text{ push up so the apparent weight is more}$$

$$F_{nb} = m(g - a') \quad \text{the } \vec{a} \text{ push down so the apparent weight is less}$$

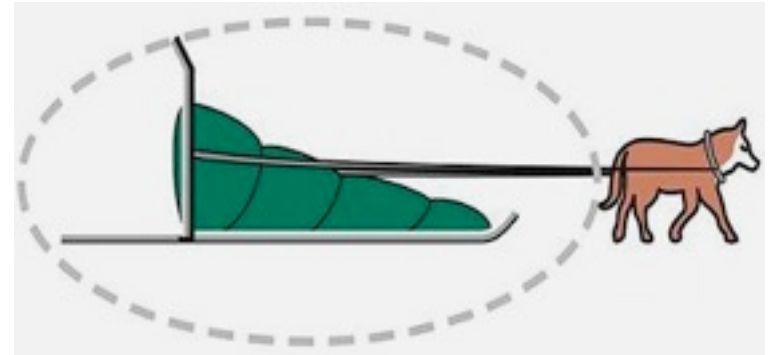


Using Free-Body Diagrams

$$\sum F_x = F_{nx} + F_{gx} + F_x = ma_x$$

$$0 + 0 + F_x = ma_x$$

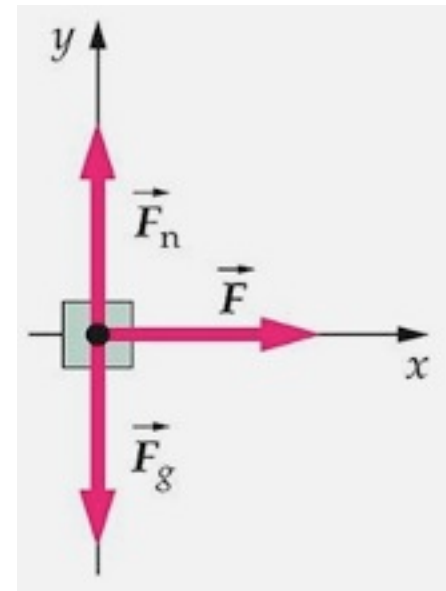
$$a_x = \frac{F_x}{m}$$



$$\sum F_y = F_{ny} + F_{gy} + F_y = ma_y$$

$$F_n - F_g + 0 = ma_y = 0$$

$$F_n = F_g$$



Example: A sled

During your winter break, you pull a rope attached to the sled with a force of 150 N at 25° above the horizontal. The mass of the sled-rope system is 80 kg, and there is negligible friction between the sled runners and the ice.

(a) Find the acceleration of the sled

(b) Find the magnitude of the normal force exerted on the surface by the sled.

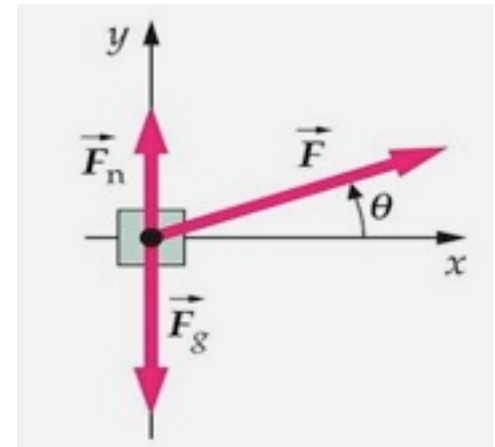
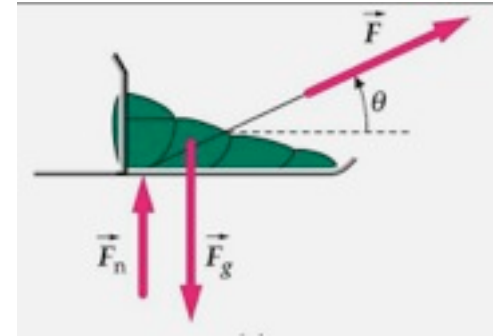
$$\vec{F}_n + \vec{F}_g + \vec{F} = m\vec{a}$$

$$\mathbf{x}: F_x = F \cos\theta = ma_x$$

$$a_x = F \cos\theta / m = (150 \text{ N})(\cos 25^\circ) / (80 \text{ kg}) = 1.7 \text{ m/s}^2$$

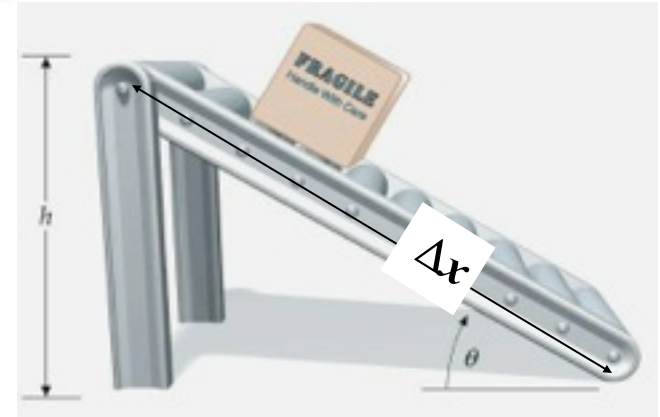
$$\mathbf{y}: F_y = F_n - mg + F \sin\theta = ma_y = 0$$

$$F_n = mg - F \sin\theta = (80 \text{ kg})(9.81 \text{ m/s}^2) - (150 \text{ N})(\sin 25^\circ) = 720 \text{ N}$$



Example: Unloading A Truck

You are working for a big delivery company, and you must unload a large, fragile package from your truck, using a 1 m high frictionless delivery ramp. If the downward component of velocity of the package is greater than 2.50 m/s when it reaches the bottom of the ramp, the package will break.



What is the greatest angle θ at which you can safely unload?

$$F_{nx} + F_{gx} = 0 + \cancel{mg \sin \theta} = \cancel{ma_x}$$

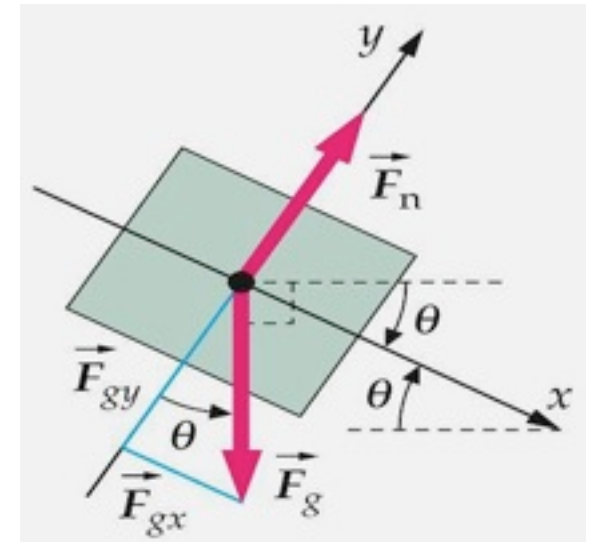
$$a_x = g \sin \theta \quad \Delta x = h / \sin \theta$$

$$v_x = \sqrt{2a_x \Delta x} = \sqrt{2(\cancel{g \sin \theta})(\cancel{h / \sin \theta})} = \sqrt{2gh}$$

$$v_d = v_x \sin \theta \quad \theta_{\max} = \arcsin[v_d / v_x]$$

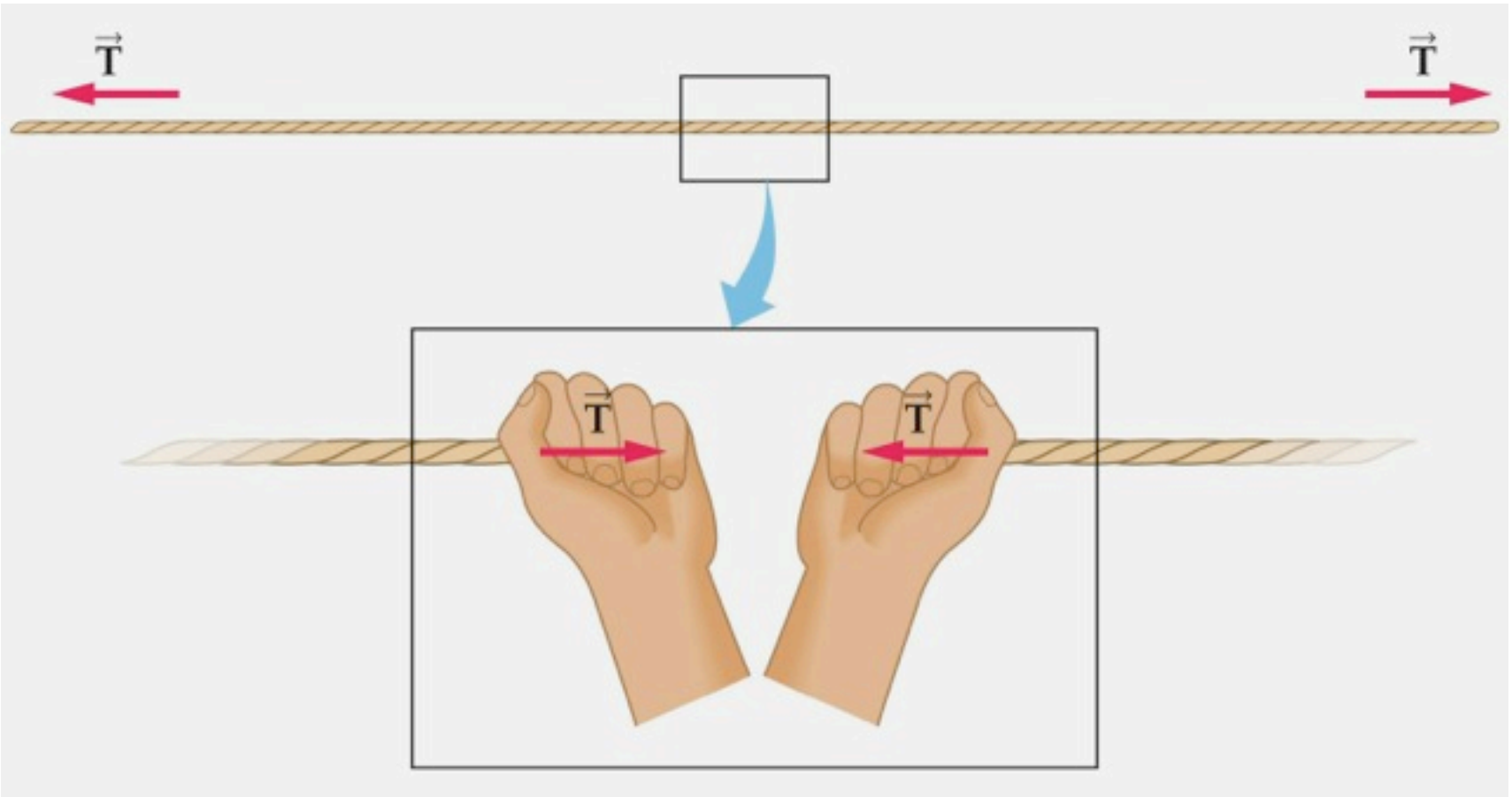
$$= \arcsin \left[(2.5 \text{ m}) / \sqrt{2(9.81 \text{ m/s}^2)(1.0 \text{ m})} \right]$$

$$= 34.4^\circ$$



Strings

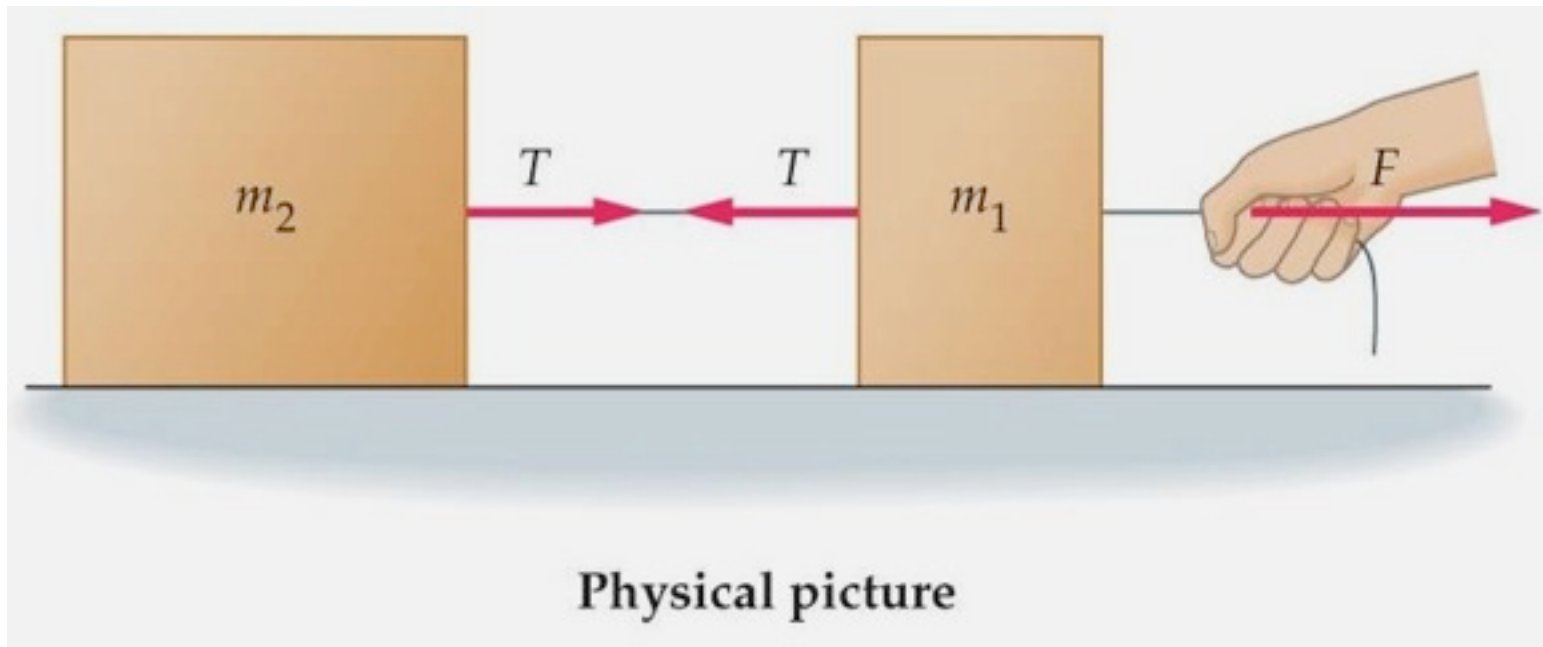
When you pull on a string or rope, it becomes taut. We say that there is **tension** in the string.



Connected Objects

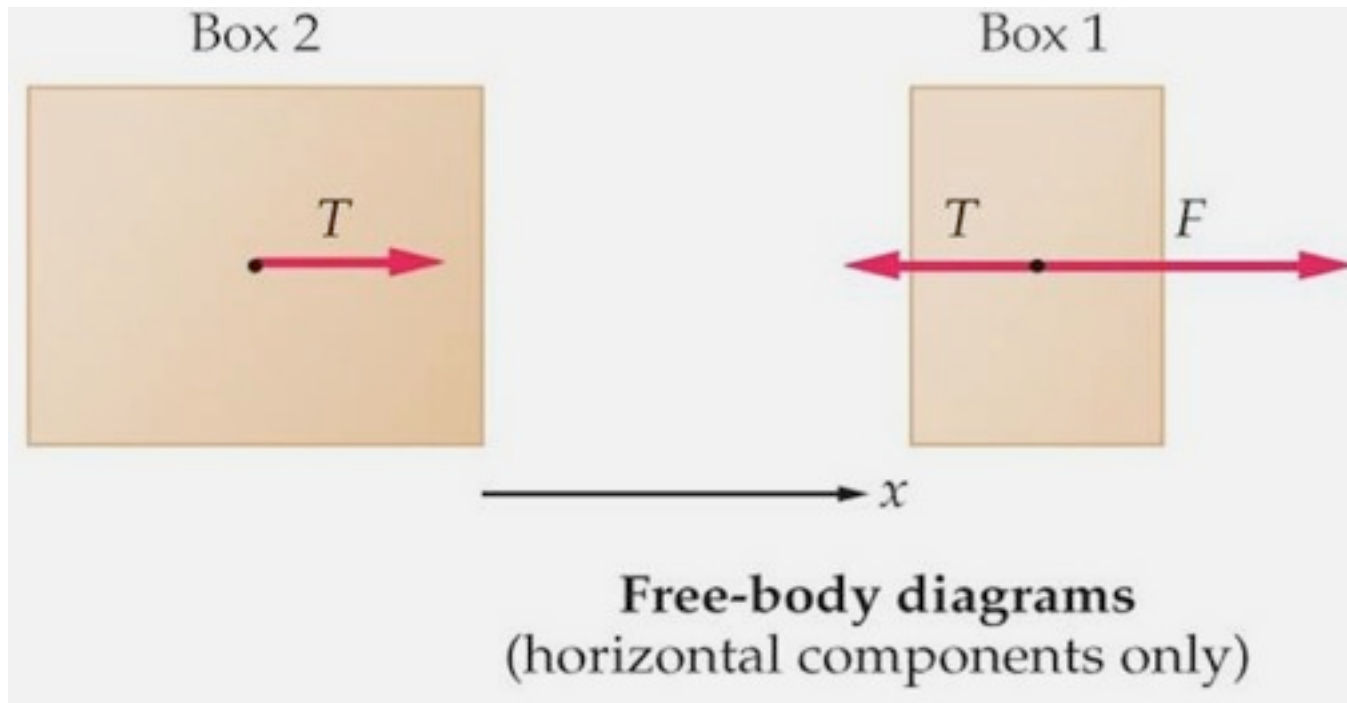
When forces are exerted on connected objects, their accelerations are the same.

If there are two objects connected by a string, and we know the force and the masses, we can find the acceleration and the tension:

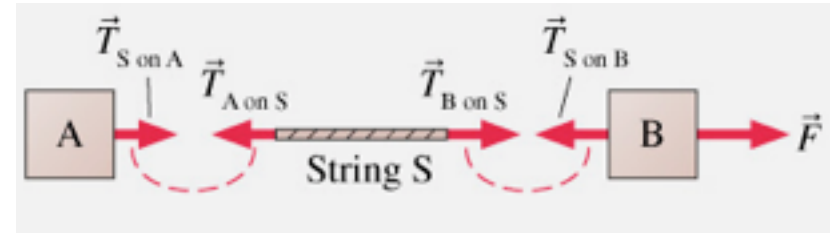
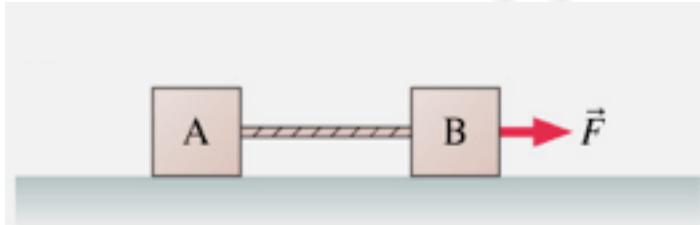


Connected Objects

We treat each box as a separate system. We write separate equations for each and solve them together.



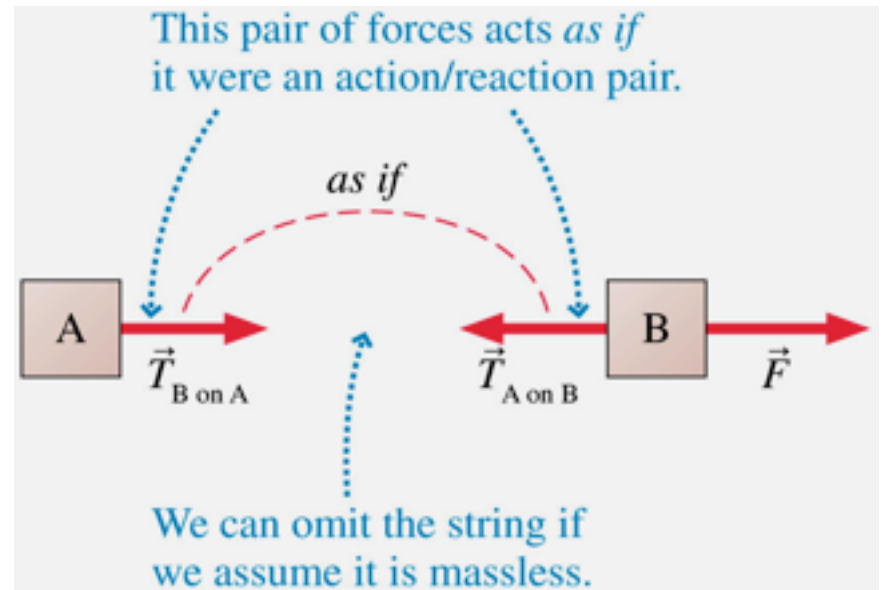
The Massless String Approximation



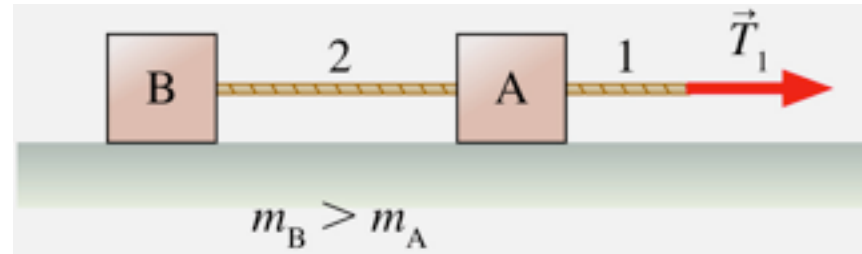
A horizontal force F acts on a block that is connected to another block by a string. Consider the constraints and the forces.

Massless String Approximation:

$$\mathbf{T}_{A \text{ on } S} = \mathbf{T}_{B \text{ on } S}$$



Example: Comparing Tensions



Blocks A and B are connected by massless String 2 and pulled across a frictionless surface by massless String 1. The mass of B is larger than the mass of A.

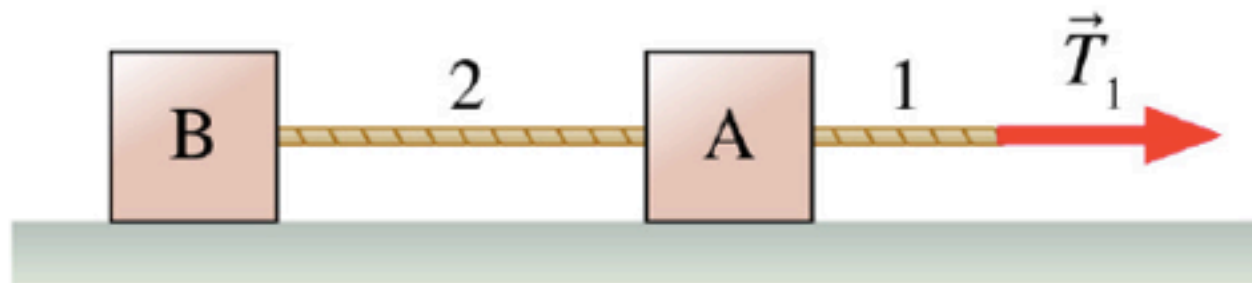
Is the tension in String 2 smaller, equal, or larger than the tension in String 1?

Find tension of rope 2 $T_2 = ?$

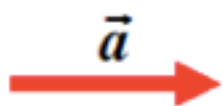
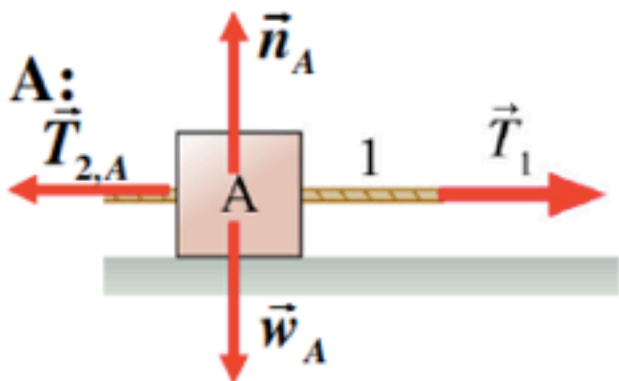
$$T_1 = 100\text{ N}$$

$$m_A = 1\text{ kg}$$

$$m_B = 4\text{ kg}$$



System A:



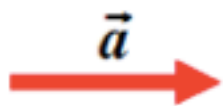
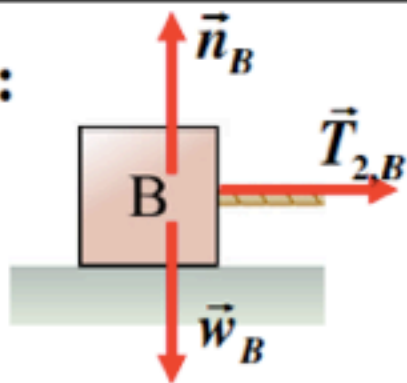
No friction

x

$$\vec{F}_{A,net} = \vec{T}_{2,A} + \vec{n}_A + \vec{w}_A + \vec{T}_1 = m_A \vec{a}$$

$$T_1 - T_{2,A} = m_A a$$

System B:



x

$$\vec{F}_{B,net} = \vec{T}_{2,B} + \vec{n}_B + \vec{w}_B = m_B \vec{a}$$

$$T_{2,B} = m_B a$$

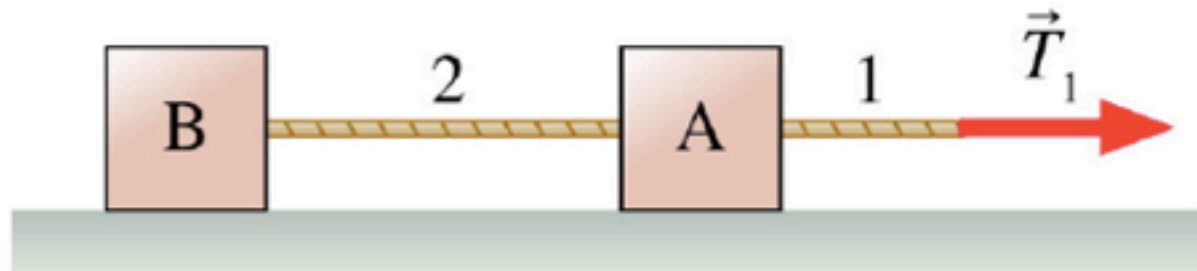
Find tension of rope 2 $T_2 = ?$

$$T_1 = 100\text{ N}$$

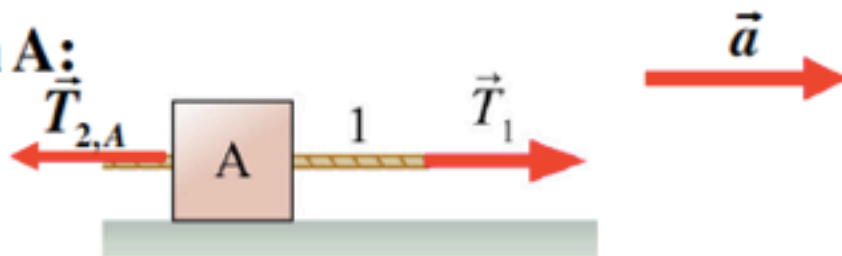
$$m_A = 1\text{ kg}$$

$$m_B = 4\text{ kg}$$

No friction

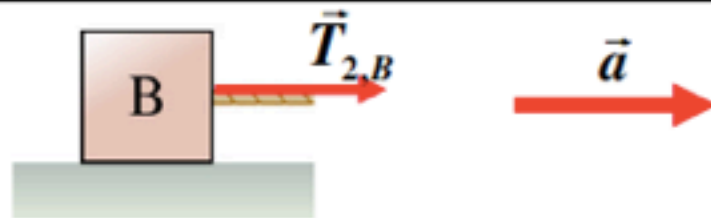


System A:



$$T_1 - T_{2,A} = m_A a$$

System B:



$$T_{2,B} = m_B a$$

Newton's Third Law: $T_{2,B} = T_{2,A} = T_2$

$$T_1 - T_2 = m_A a$$

$$T_2 = m_B a$$

$$a = \frac{T_1}{m_A + m_B}$$

$$T_2 = \frac{m_B T_1}{m_A + m_B} = \frac{4 \cdot 100}{5} = 80\text{ N}$$

Translational Equilibrium

When an object is in **translational equilibrium**,
the net force on it is zero:

$$\sum \vec{F} = 0$$

An object in translational equilibrium is either at rest or moving with a constant velocity.

Its acceleration is zero.

Example: Picture Hanging

A picture weighing 8.0 N is supported by two wires with tensions T_1 and T_2 .

Find each tension.

$$\vec{T}_1 + \vec{T}_2 + \vec{F}_g = m\vec{a} = 0$$

$$\sin 30^\circ = \cos 60^\circ = \frac{1}{2}$$

$$\cos 30^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

x: $T_1 \cos 30^\circ - T_2 \cos 60^\circ + 0 = 0$

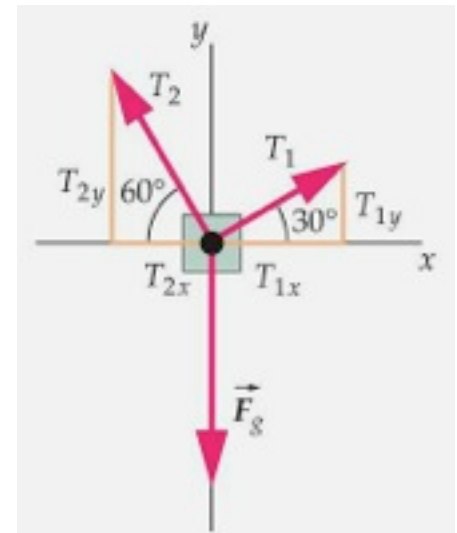
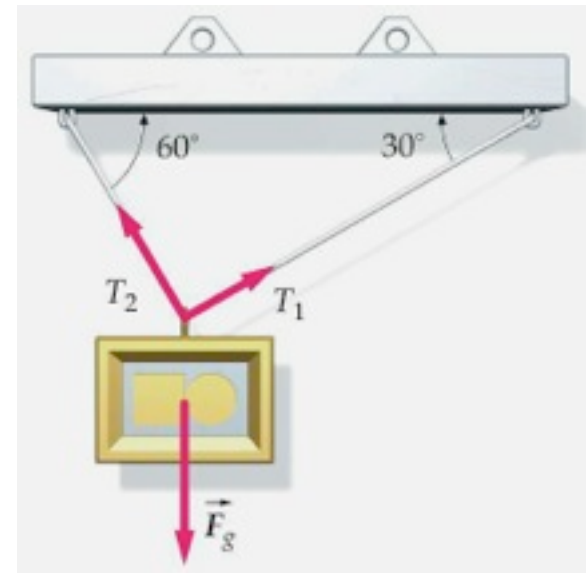
$$T_2 = T_1 (\cos 30^\circ / \cos 60^\circ) = T_1 \sqrt{3}$$

y: $T_1 \sin 30^\circ + T_2 \sin 60^\circ - F_g = 0$

$$F_g = T_1 \sin 30^\circ + T_2 \sin 60^\circ = T_1 \left(\frac{1}{2} + \sqrt{3} \frac{\sqrt{3}}{2} \right) = 2T_1$$

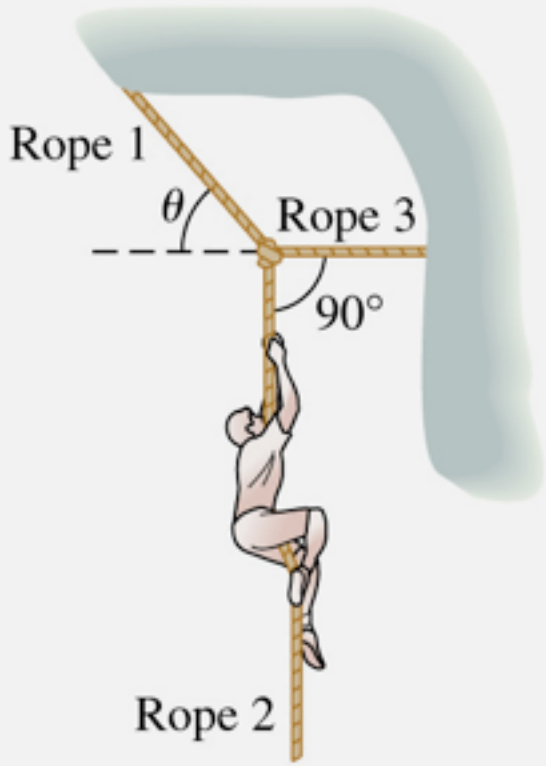
$$T_1 = \frac{1}{2} F_g = \frac{1}{2} (8.0 \text{ N}) = 4.00 \text{ N}$$

$$T_2 = T_1 \sqrt{3} = 6.93 \text{ N}$$



Example: Mountain Climbing (1)

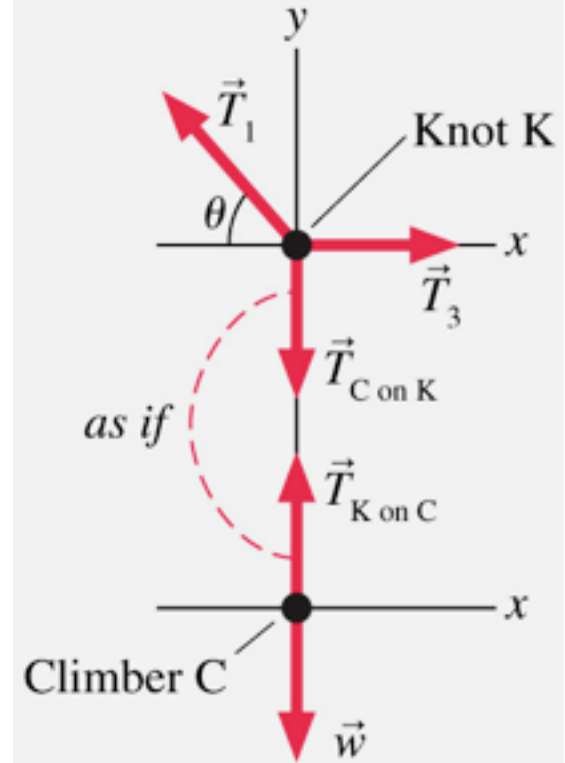
Pictorial representation



A 90 kg mountain climber is suspended from ropes as shown. Rope 3 can sustain a maximum tension of 1500 N before breaking.

What is the smallest that angle θ can become before the rope breaks?

Physical representation



Example: Mountain Climbing (2)

Climber:

$$\sum (F_{\text{on M}})_y = T_{\text{K on C}} - w = T_{\text{K on C}} - mg = 0$$

Knot:

$$\sum (F_{\text{on K}})_x = T_3 - T_1 \cos\theta = 0$$

$$\sum (F_{\text{on K}})_y = T_1 \sin\theta - T_{\text{C on K}} = 0$$

$$T_{\text{K on C}} = T_{\text{C on K}} = mg$$

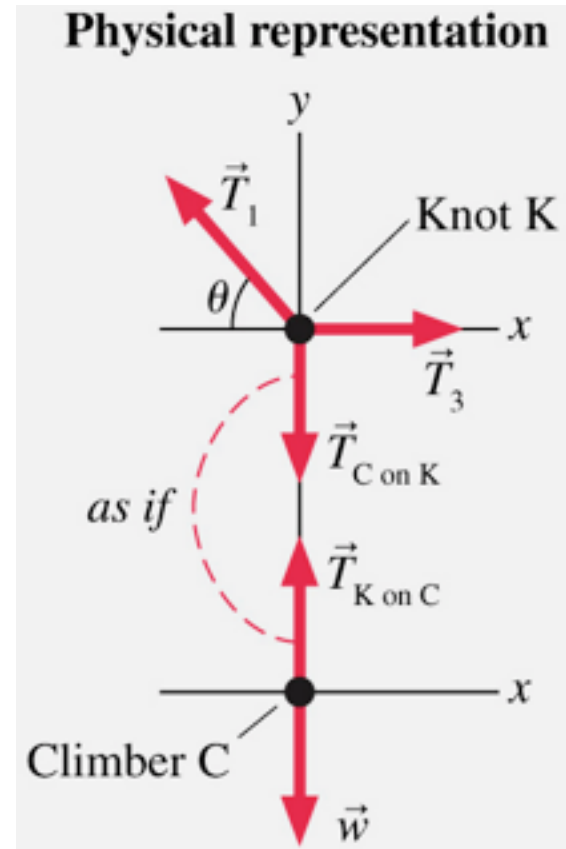
$$T_1 \cos\theta = T_3$$

$$T_1 \sin\theta = mg$$

$$\text{so } \tan\theta = mg / T_3$$

$$\theta_{\text{max}} = \tan^{-1}(mg / T_{3\text{max}})$$

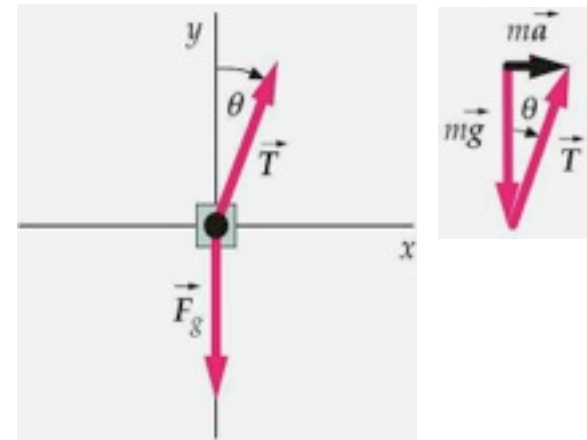
$$= \tan^{-1}[(90 \text{ kg})(9.81 \text{ m/s}^2) / (1500 \text{ N})] = 30.5^\circ$$



Example: An Accelerating Jet Plane

As your jet plane speeds along the runway on takeoff, you decide to determine its acceleration, so you take out your yo-yo and note that when you suspend it, the string makes an angle of 22° with the vertical.

- (a) What is the acceleration of the airplane?
- (b) If the mass of the yo-yo is 40.0 g, what is the tension in the string?



$$\vec{T} + m\vec{g} = m\vec{a}$$
$$y: T \cos\theta = mg \qquad \tan\theta = a/g$$
$$x: T \sin\theta = ma$$
$$a = g \tan\theta = (9.81 \text{ m/s}^2)(\tan 22^\circ) = 3.96 \text{ m/s}^2$$

$$T = mg / \cos\theta = (0.040 \text{ kg})(9.81 \text{ m/s}^2) / (\cos 22^\circ) = 0.423 \text{ N}$$