Dr. M. Jones,
Department of Physics,
Rutgers College,
New Brunswick,
New Jersey 08903,
U.S.A.

Dear Dr. Jones,

We are sorry to have taken so long to respond to your letter of 30 October 1974. Other excitement in physics have been taking our attention, and we could not sit down quietly to examine the situation. However, we now believe that everything is in order, as we shall try to explain.

Errors. First, there are two errors in our tables for $\Lambda$ states, which must be corrected. These are both in off-diagonal elements $<70\Lambda(8,2)|70\Lambda(1,2)>$.

For $(70, L^P)$ states, $<70\Lambda(8,2)|_{\frac{21}{105}}^{8,1} 70\Lambda(1,2)> = \frac{5}{2}$, (in place of $+1$)

$<70\Lambda(8,2)|_{\frac{15}{189}}^{8,1} 70 (1,2) > = \frac{5}{6}$ (in place of $-\frac{5}{6}$)

These corrections have been made in all our calculations discussed here.

Calculations with "Old Separate" Parameters. We have re-calculated the spectrum and states for $(70, 1-)$ using the separate parameters of Table 4 of paper II. The states are now in complete agreement with your last letter; our and your mixing matrices have identical elements, to two decimal places. The final mass values are given on the enclosed sheet. Of course, the significant changes are in the $\Lambda$ states. Now, the agreement between us is much better than it was before; generally, the values agree to 0.5 MeV, or better.

Nevertheless, the discrepancies are systematic. All of Horgan’s values lie below your values, and the discrepancy is typically 0.5 MeV. We think the discrepancy arises most probably from rounding errors in your calculations. To how many decimal places do you specify the $N_i$? Many of the contributions involve large numerical factors (e.g. $27/2$) and we have found it necessary to calculate them to 3 decimal places and to use them to 2 decimal places in calculating matrix-elements. We'll now discuss the comparison in more detail.

Comparison with Matrix-Elements and Energies given by Jones. Your last letter gives us explicitly (in terms of the $N_i$) your mass matrix for $S1\Sigma$, $S0\Lambda$ and $D0\Lambda$, as well as the energy values. The previous letter gave us explicit expressions in $N_i$ for $D1S\Omega$, $D1S\Sigma$, $D0S\Lambda$ and $D1S\Xi$, and for $D3S\Lambda$ and $D0S\Omega$.

First, we have taken the expressions you give for $N_i$ in terms of $<T>$ and substituted them in each of these expressions. In each case your matrix-element expressed in terms of $<T>$ agrees precisely with the expressions deduced from the tables of our paper I. This is a partial check on your algebra. Reassuring, at least.
Next we tried an internal check. Consider the energy matrix for D03A. This should have its trace invariant with respect to diagonalisation. However, your matrix elements give
\[ M(4^8, 4^8) + M(2^8, 2^8) + M(2^1, 2^1) = 1827.0 + 1726.4 + 1533.4 = 5086.8, \]
whereas your energies \( \Lambda \) give
\[ \Lambda_3 + \Lambda_2 + \Lambda_1 = 1887.7 + 1668.2 + 1528.6 = 5084.5, \]
a discrepancy of 2.3 MeV. Similar results hold for S11E and S01A. We think that this may be a fair estimate of your rounding errors (or of the uncertainty of your matrix-inversion).

Now, we take our analytic expressions for the matrix-elements in terms of the \( <T_0> \), which we have checked above to be algebraically equivalent with yours, and evaluate them by hand from the \( <T_0> \) in Table 4 of Part II. We find
\[ M(4^8, 4^8) = 1831.96, \quad M(2^8, 2^8) = 1546.27, \quad M(2^1, 2^1) = 1541.37. \]
These deviate appreciably from the values you give, namely, 1832.4, 1549.1 and 1541.9, in turn. With our values, we have
\[ M(4^8, 4^8) + M(2^8, 2^8) + M(2^1, 2^1) = 4919.60. \]
This is in excellent agreement with our value \( (\Lambda_3 + \Lambda_2 + \Lambda_1) \ d = 4919.6 \) deduced from the enclosed table of final mass values for the S01A states. Hence our expressions and numbers do satisfy these consistency checks.

The discrepancy in \( M(2^8, 2^8) \) seemed particularly large. I believe that this is a simple arithmetical error. My addition of your numbers gives 1546.7 not 1549.1. Using the \( N_i \) to two decimal places, my addition gives 1546.51.

We also took the expressions for the one-dimensional matrices in terms of \( <T_0> \), which are algebraically equivalent with your expressions in terms of \( N_i \), and made a hand calculation for each of them.

Your hand evaluation:
\begin{align*}
D_{15N} & \quad D_{15E} & \quad D_{05A} & \quad D_{15A} & \quad D_{33A} & \quad D_{03A} \\
1690.3 & \quad 1765.0 & \quad 1817.8 & \quad 1918.4 & \quad 1649.8 & \quad 2048.2
\end{align*}
(after the corrections explained our letter dated 21 October 1974).

Our hand calculation:
\begin{align*}
D_{15N} & \quad D_{15E} & \quad D_{05A} & \quad D_{15A} & \quad D_{33A} & \quad D_{03A} \\
1691.4 & \quad 1764.3 & \quad 1818.0 & \quad 1917.7 & \quad 1649.4 & \quad 2047.5
\end{align*}
This latter now agrees precisely with our computer calculation, as it should. We suspect that it is your rounding-off the values for \( N_i \) calculated from \( <T_0> \) which causes errors in the sums \( (\Sigma c_i N_i) \) which you evaluated to give the hand \( ^2 \) calculated values. Many of the coefficients \( c_i \) are of order 10. However, it is not obvious that this can account for the systematic difference between your computer-calculated mass values and ours.

The \( N_i \) we have computed to 2-decimal places are as follows:
\begin{align*}
N_0 & \quad N_1 & \quad N_2 & \quad N_3 & \quad N_4 & \quad N_5 & \quad N_6 & \quad N_7 & \quad N_8 & \quad N_9 & \quad N_{10} \\
972.19 & \quad -166.20 & \quad 21.11 & \quad 3.20 & \quad -83.70 & \quad -8.73 & \quad 0.35 & \quad -11.4 & \quad -0.71 & \quad -15.96 & \quad 29.92
\end{align*}
Using these in place of your (1-decimal place) \( N_i \), I calculate D03o (for example) to lie at 2047.63, to be compared with your value of 2048.2. However, this still doesn't quite agree with our computer value 2047.55 (which I reproduce correctly by hand calculation from our \( <T_0> \) ). Your expressions in
N. are algebraically equivalent to our expressions in $<T_0>$, as we have checked. The small difference is readily accounted for as being due to contributions from the 3rd-decimal place in the $N_1$.

We made a detailed spot check on $M(^28, ^8)$ for SOIA. Using your expression and the above values for the $N_1$, we obtain $M(^28, ^8) = 1546.36$, in reasonable agreement with our hand calculation (giving 1546.27) from the $<T_0>$. Before we mail this letter, we will print out the energy matrices from within our computer calculation, as a further check. As remarked above, there is a numerical error in your evaluation by hand for this matrix element.

Final Fit: (New (70,1) Separate) Using the same input data as was used for the combined fit ($L = 0, 1$ and 2) in Paper II, and the corrected table of matrix elements, Horgan has repeated the "Best Combined Fit (BCF)" calculation. The BCF parameters are given on the first line of the following table:

<table>
<thead>
<tr>
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<th>21</th>
<th></th>
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<th>15</th>
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<td>$T_{1,1}$</td>
<td>$T_{1,1}$</td>
<td>$T_{8,1}$</td>
<td>$T_{8,1}$</td>
<td>$T_{8,1}$</td>
</tr>
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<td>5</td>
<td>189</td>
<td>35</td>
<td>189</td>
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<td>1320.0</td>
<td>67.0</td>
<td>228.2</td>
<td>-12.7</td>
<td>1270.6</td>
</tr>
<tr>
<td>1320.0</td>
<td>77.9</td>
<td>208.5</td>
<td>-8.7</td>
<td>2215.2</td>
</tr>
</tbody>
</table>

As was done in Paper II, the BCF parameters were used as starting values for the fit to the same (70,1-) levels as were considered for the corresponding fit in Paper II. This leads to a separate fit which we will call the "New Separate Fit". The parameters reached in this second stage are given on the last line of the table above. The energy levels and states for this "New Separate Fit" are given on the enclosed sheet. Note that the lowest $\Xi^*$ is now at 1677.8; obviously this can be shifted around quite a bit. It is interesting that this $\Xi^*$ mass remains low even when the $S_{11}$ mass rises to 1560 MeV. We shall enclose the print-out of the energy matrix for this "New Separate Fit", which now replaces the fit to which you referred in your October 30th letter as "The Combined Fit". You can then use this for checking purposes.

We hope (and believe) that all the important discrepancies (that we know about) are all now resolved. However, we would encourage you to try to understand why your mass values are systematically lower than ours for the "Old Separate" parameters, and to see how well your program reproduces our energies and states for the "New Separate Fit" parameters. We would be glad to hear from you how you get on, and what the final situation is with your program.

With best greetings,

Yours sincerely,

R. H. Dalitz  
R. R. Horgan