CALCULATION OF THE EXPECTED COUNTING RATE AND DETECTION EFFICIENCY OF THE HALE DUMAND EXPERIMENT

Part 1. Description and Analysis of Apparatus.

by
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INTRODUCTION

This paper is intended to describe the equipment of the HALE Dumand experiment, and to lay the groundwork for calculating the expected counting rate of the experimental setup in detecting cosmic-ray muons. It does not attempt to predict the expected counting-rate, but lays the foundatioon on which that may be attempted. An accurate evaluation of this quantity is required to determine the detection efficiency to be expected from the DUMAND detector modules.

APPARATUS

Fig. 1 shows the detector set-up in elevation and end views. In brief, the detector is a Cerenkov counter, in which the radiator is a tank of water. It is unique in that there are three PMT detectors: two 8" PMT's and one 16". The two 8" PMT detectors are preceded by optical collimators (see Figs. 2,3) which require that the light reaching the PMT be accurately horizontal; and the 16" PMT is gated by a coincidence of the 8" PMT's (see Fig. 4). This insures that only those muons will be detected whose Cerenkov light cone contains a horizontal element. Since the nominal Cerenkov angle for relativistic muons in water is 41.5°, only muons making an angle of $41.5 \pm 0.58^{\circ}$ or less with the horizontal will be detected.

The muons are selected by a triggering system of three liquid scintillation counters, one above and two below the Cerenkov radiator; the bottom two are separated by several inches of lead to reduce the effect of showers and slow particles. Each of the scintillation counters is viewed by two PMT's at each end, and the amplitudes of the detected signals are used to locate the muon (along the counter axis) to an accuracy of about $\pm$ .... cm. Since the collimation of these counters is far less constrictive than the requirement for horizontal (and paraxial) Cerenkov light, the muon-produced coincidence counting rate of the two scintillators will greatly exceed that of the Cerenkov detectors. The optical collimator is shown in Fig. 2, and its installation in Fig. 3. Its effect is to admit only light within 0.58° of the horizontal (see Figs. 4 and 5).

The optical constraint of horizontal Cerenkov light requires only that the muon make a minimum angle with the vertical of 48.5°. It does not specify the orientation of the vertical plane containing the muon trajectory with respect to the horizontal axis of the Cerenkov detector, a quantity we can call the azimuthal angle. The maximum efficiency will be obtained when the muon trajectory lies in a vertical plane that intersects the Cerenkov radiator in a line parallel to the Cerenkov radiator longitudinal axis. For such a muon its horizontal Cerenkov light is parallel to the radiator axis, its azimuthal angle is zero, and even distant muons (i.e. those traversing the far end of the radiator) can be detected. Such a muon is shown in Fig. 6.

The location of the muon trajectory with three scintillation counters gives a reasonably accurate trajectory of the muon through the water Cerenkov counter. However, in the transverse direction the only constraint is the geometrical one that the trajectory intersect all three counters; this gives an angular limitation of $\pm 14^{o}$ in that direction (see Fig. 1). Along the axis the location error of $\pm$.... and the counter separation of 280 cm yield an error of $\pm$ .... in the vertical angle. Thus the optical collimator places a considerably stronger constraint on the vertical angle, and only a weak one on the horizontal (azimuthal) angle.

Several classes of muons meet these criteria for a detectable trajectory. They include muons incident in the plane defined by a vertical line and the horizontal axis of the Cernkov radiator. All such muons making an angle with the horizontal very close to the Cerenkov angle, nominally 41.5^{o}, may be detected. Muons making a smaller angle with the horizontal may also be detected, depending on their distance from the PMT. Such muons are illustrated in Figs 6 and 7. Also, some muons whose azimuthal coordinate is between zero and $\pm 14^{o}$ can be detected - see Fig. 8.

EFFECT OF DISPERSION.

To date, we have treated the Cerenkov radiation as though it were monochromatic, and could be described by a single index of refraction. This is of course not the case, and it appears that the design of the experiment is fortuitously such as to emphasize the deleterious effects of dispersion.

Fig. 9 shows the wavelength dependence of the PMT sensitivity; the curve is truncated at 3500 A by the transmission of the glass sphere. Fig. 10 shows the variation of the Cerenkov angle in water over the range of wavelengths (3500–5700A) for which the setup has usable sensitivity. The change in the Cerenkov cone angle of nearly a degree is large enough to give a very wide angular spread among rays of different colors – a spread that might be as much as 5 to 6^{o} over the usable wavelength range.

This angular spread is illustrated in several figures. Fig. 6, as we have seen, shows the nominal tangential Cerenkov cone for a muon incident at the Cerenkov angle. Since the angular acceptance of the optical collimator is $\pm$ 0.58^{o}, the range of Cerenkov angles of one degree produced by dispersion would be only partially accepted. In addition one should remember that since the acceptance curve of the collimator is triangular in shape (Fig. 5), there will be an additional loss of efficiency.

The real effect of the dispersion is best seen by comparing Figs. 6 and 7. Instead of a single direction, light from a real muon will came over a rather large spread of angles. From Fig. 11 we can begin to see how serious this effect is.

This extreme effect of dispersion is due to running at exactly the maximum Cerenkov angle, which is what varies most with the index of refraction. If the experiment could be redesigned to work further away from that critical angle, the effect of dispersion might be greatly decreased. It is by no means obvious how to do it.

Fig. 11, an end-on view of the Cerenkov cone, illustrates the deviation of each of the two horizontal rays produced by a 42-degree cone in a horizontal plane set for a 41.5° angle. From the figure, cos A = 41.5/42.=.988095. Thus A = 8.85°. But A= 90° corresponds to 42°, the Cerenkov cone angle, so the angular deviation of R1 is 42/90 x 8.85 = 4.13°. R2 has an equal deviation in the other direction, so there are two rays each at 4.13° to the longitudinal axis. Fig. 12 shows a plot of the divergence as a function of Cerenkov angle, for a Cerenkov angle of 41.24°. The divergence can reach 5°. That means that a ray starting with this divergence from the longitudinal axis will not be detected if it originates more than 5.8m away from the detector.

Muons whose trajectory lies in a plane defined by a vertical line and a horizontal line making an angle of not more than 14° with the longitudinal counter axis are also detectable; such muons we call skew. Neglecting dispersion for the moment, they must make an angle with the horizontal plane of not more than 41.5°. Such muons are shown in Fig. 8. For muons at 41.5° with the horizontal, the Cerenkov cone is tangential to the horizontal plane. and lies in the plane defined by the muon trajectory and the vertical.

Muons making an angle of 41.5° or less with the horizontal plane may or may not be detected, depending on their angles with the horizontal and their orientation with respect to the longitudinal Cerenkov counter axis. Figs. 7 and 8 illustrate such muons.

For all the above cases the vertical angles mentioned have a maximum tolerance of $\pm$ 0.58 degrees, as determined by the vertical acceptance of the horizontal collimator in the Cerenkov detector. From Fig. 12, we see that this is quite enought to admit the entire spectrum, but with an efficiency varying as shown in Fig. 5. The half-value points have a full width of 0.58 degree, so that the spectrum has to be carefully centered in the collimator aperture.

CALCULATION OF MUON DETECTION EFFICIENCY.

A major objectives is to calculate the expected rate of detection of muons by the equipment. For this we need to know the following:

1. The intensity and zenith angle dependence of muons at our location. This information is available.

2. A calculation of the range of angles, both zenith and azimuthal, in which detectable muons can be seen. This will be a function of distance from the detectors, and may well have to done graphically rather than analytically. The problem can also be subdivided into those muons in or close to the vertical plane containing the longitudinal counter axis, and skew muons.

3. Numerical calculation for a given angle of incidence and skew angle of the muon over:

a). Cerenkov angle vs. wavelength

b). Calculation of direction of horizontal Cerenkov rays vs. wavelength ($\pm$ 0.3 degrees to approximate collimator admittance)

c). Graphical determination of the area of the counter from which these rays can reach the detector.

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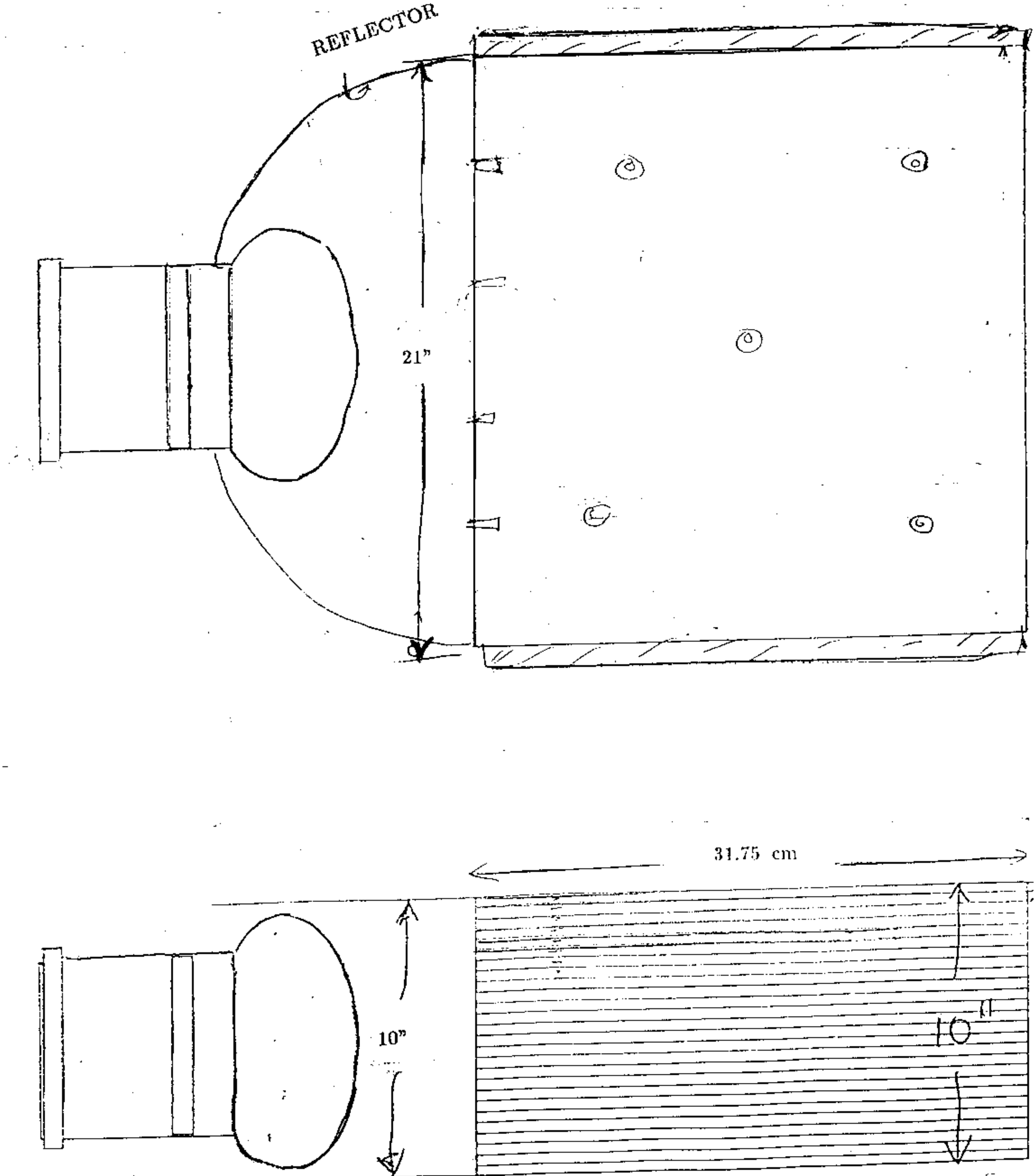

Fig. 2. The optical collimator consists of a stack of parallel plates 10" high, and 12.5" long. The plate separation is 1/8" = 3.18 mm; the optical aperture thus obtained is shown in Fig. 5.

CERENKOV COLLIMATOR

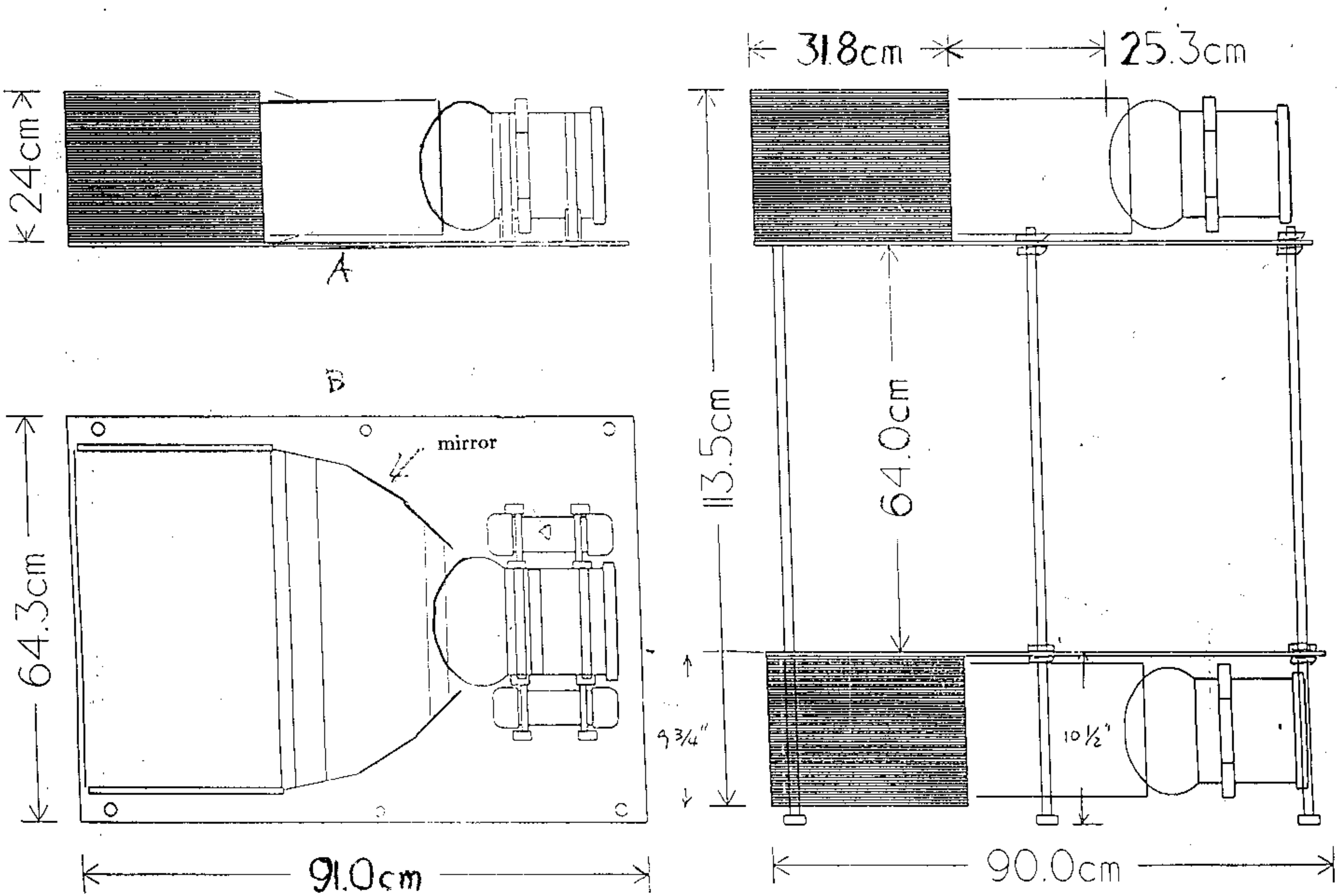

Fig. 3. This shows the mounting of optical collimators at the top and bottom of the water Cerenkov radiator. The 16" PMT DUMAND module under test can be mounted between them to view the selected horizontal light.

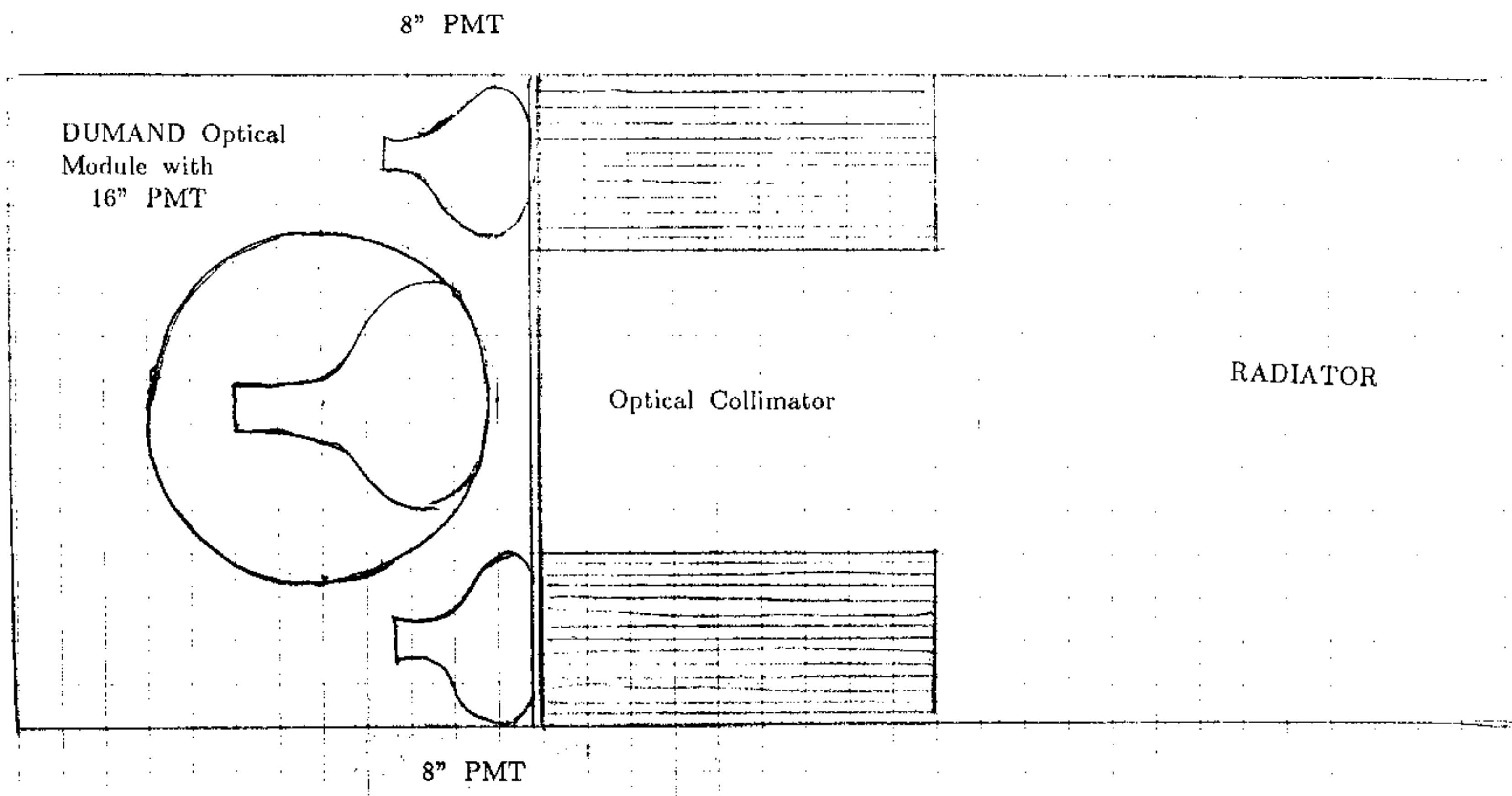

Fig. 4. Collimation and triggering of radiator detectors. The 8" PMT's in coincidence, receiving collimated light, trigger the 16" PMT, which thus responds only to light that is accurately horizontal.

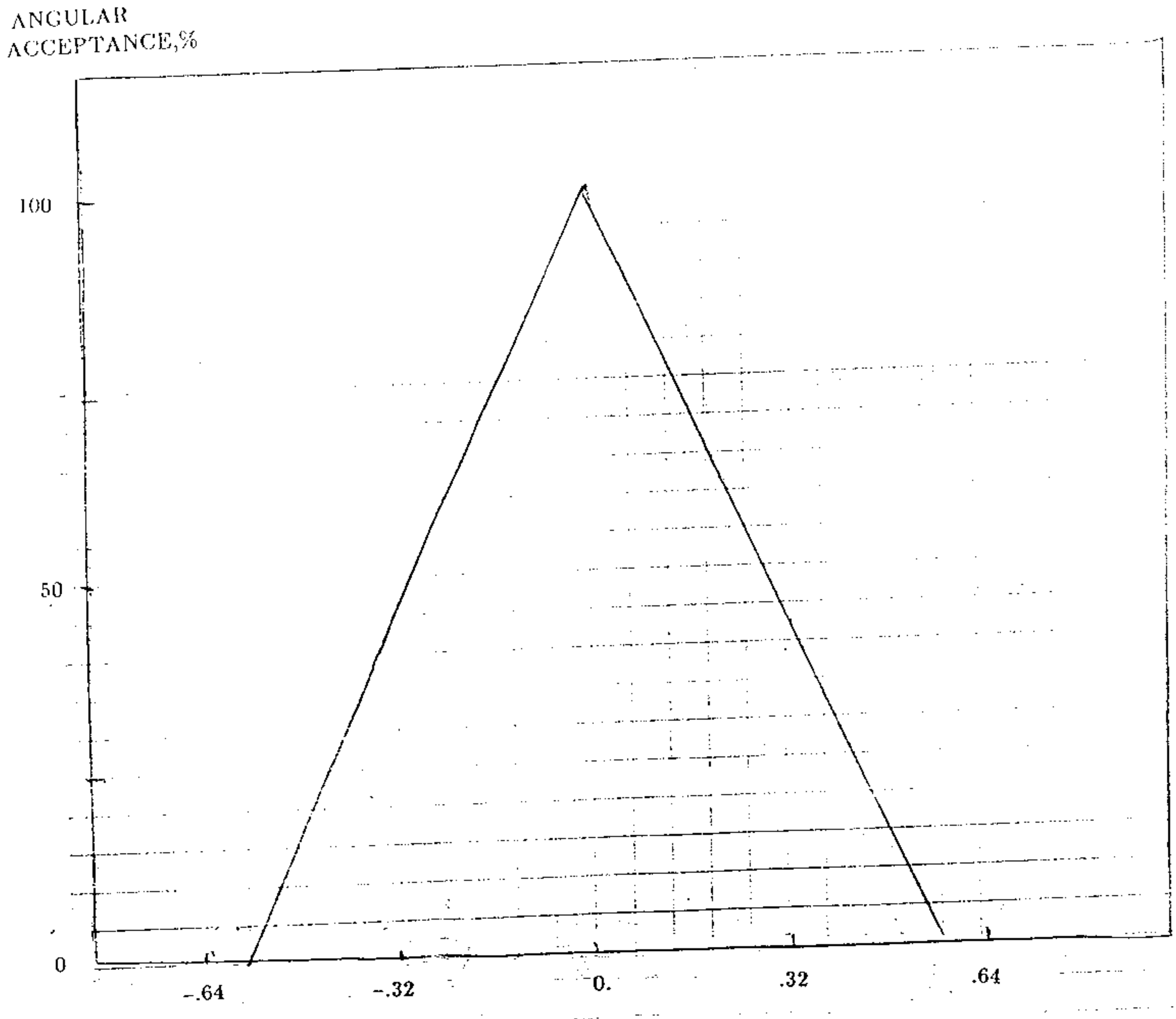

Fig. 5. Angular acceptance of the optical collimator along the vertical axis. The extreme angular deviation permissible is 0.57 degrees.

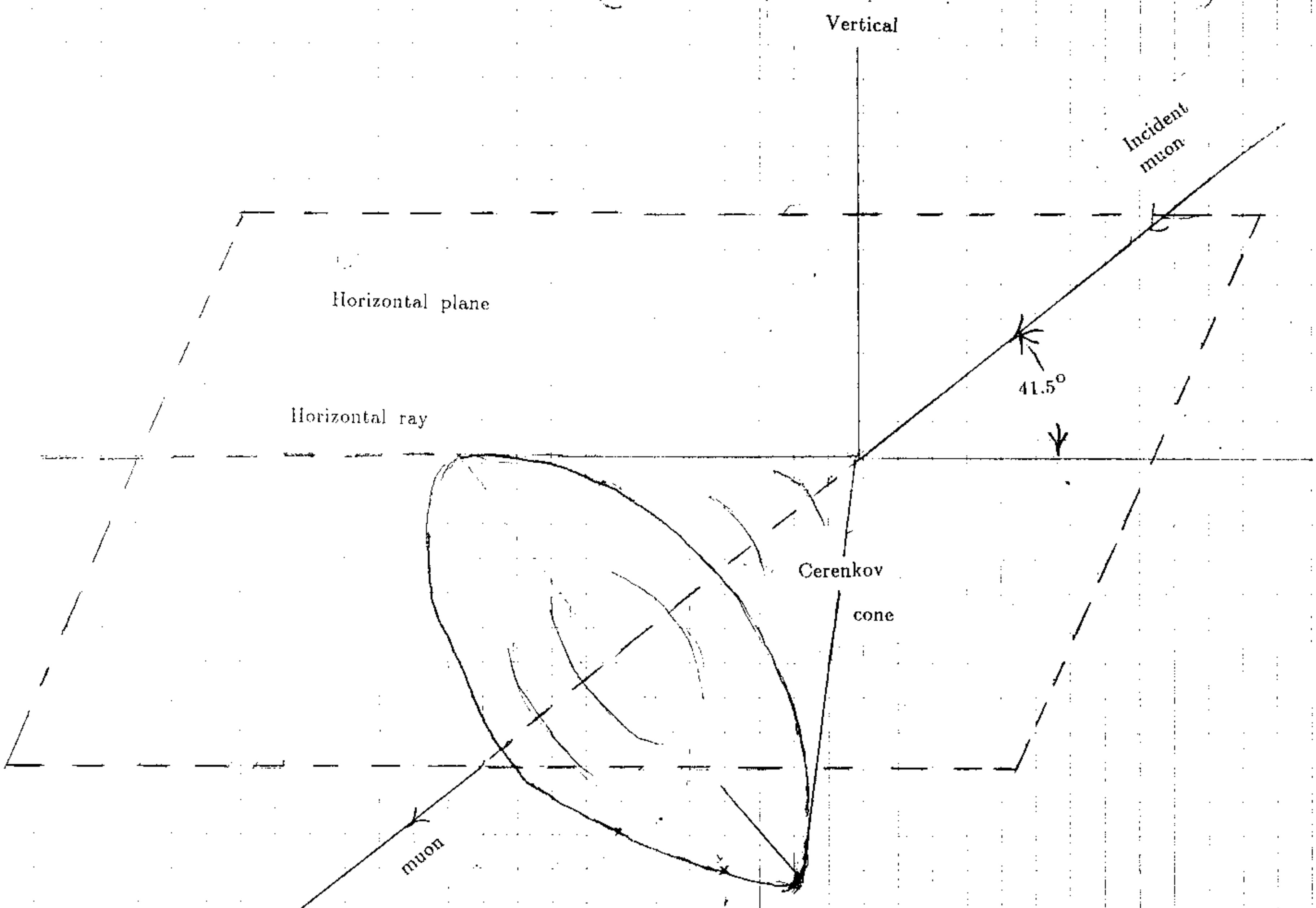

Fig. 6. The "standard" muon detection. A muon incident in a vertical plane parallel to the longitudinal radiator axis, and incident at the Cerenkov angle, produces a light cone whose uppermost ray lies in a horizontal plane, and is moreover parallel to the longitudinal axis of the radiator. and is therefore detectable.

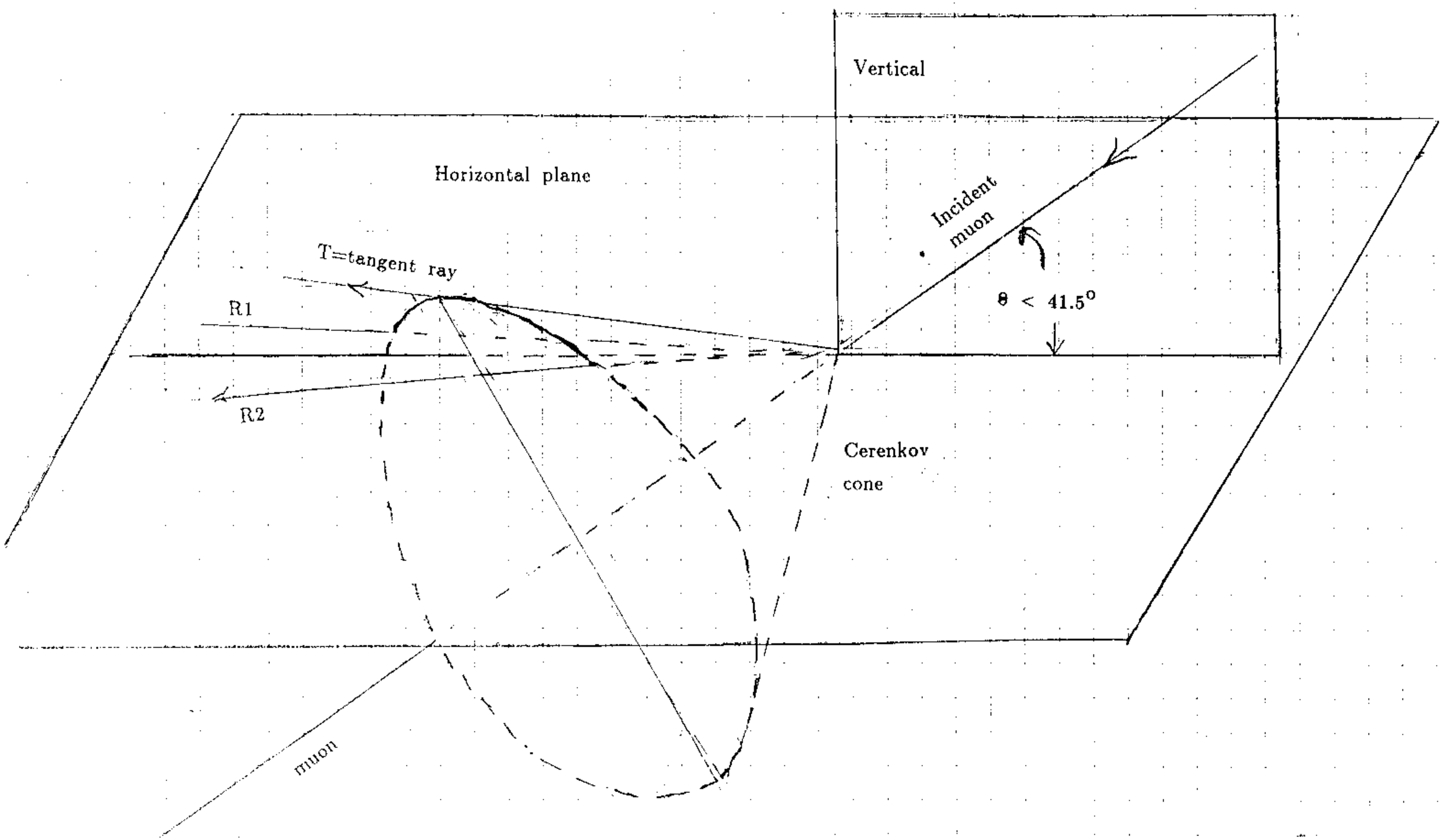

Fig. 7. A muon in the same vertical plane, but incident at a smaller angle with the horizontal. The uppermost Cernkov ray now lies above the horizontal plane and there are now two rays, R1 and R2, in the horizontal plane, but they are no longer parallel to the longitudinal axis. They will therefore be detected only if they do not originate too far away from the detector.

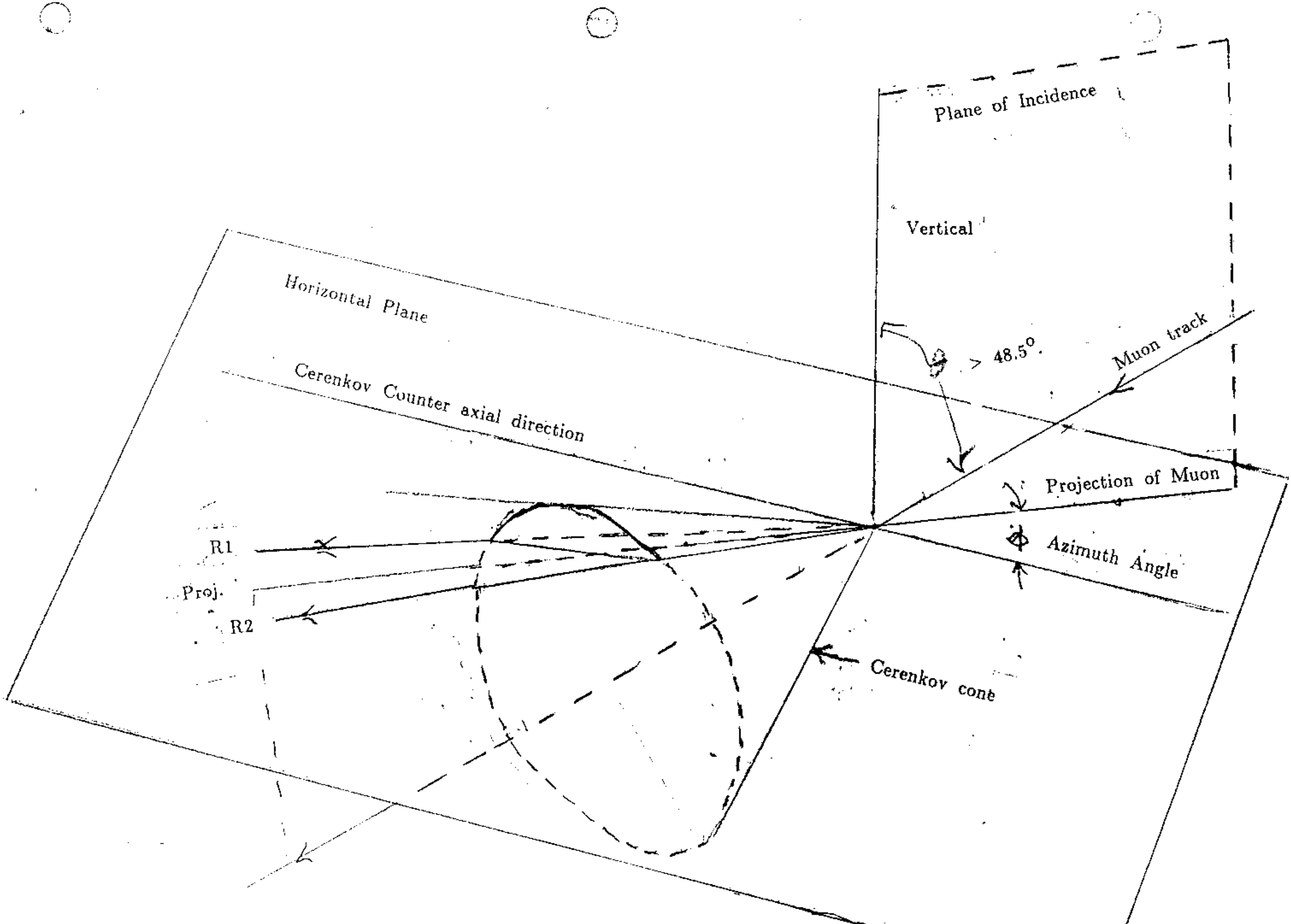

Fig. 8. Skew meson detection. The muon lies in a vertical plane of incidence which has an azimuth angle not more than 14^{o} with a vertical plane through the counter longitudinal axis. For incident angles with the horizontal less than the Cerenkov angle, the deviation of one of the acceptable horizontal rays (R1) from the plane of the incident muon is opposite in sense to the particle deviation from the longitudinal axis, and thus improves the detection efficiency.

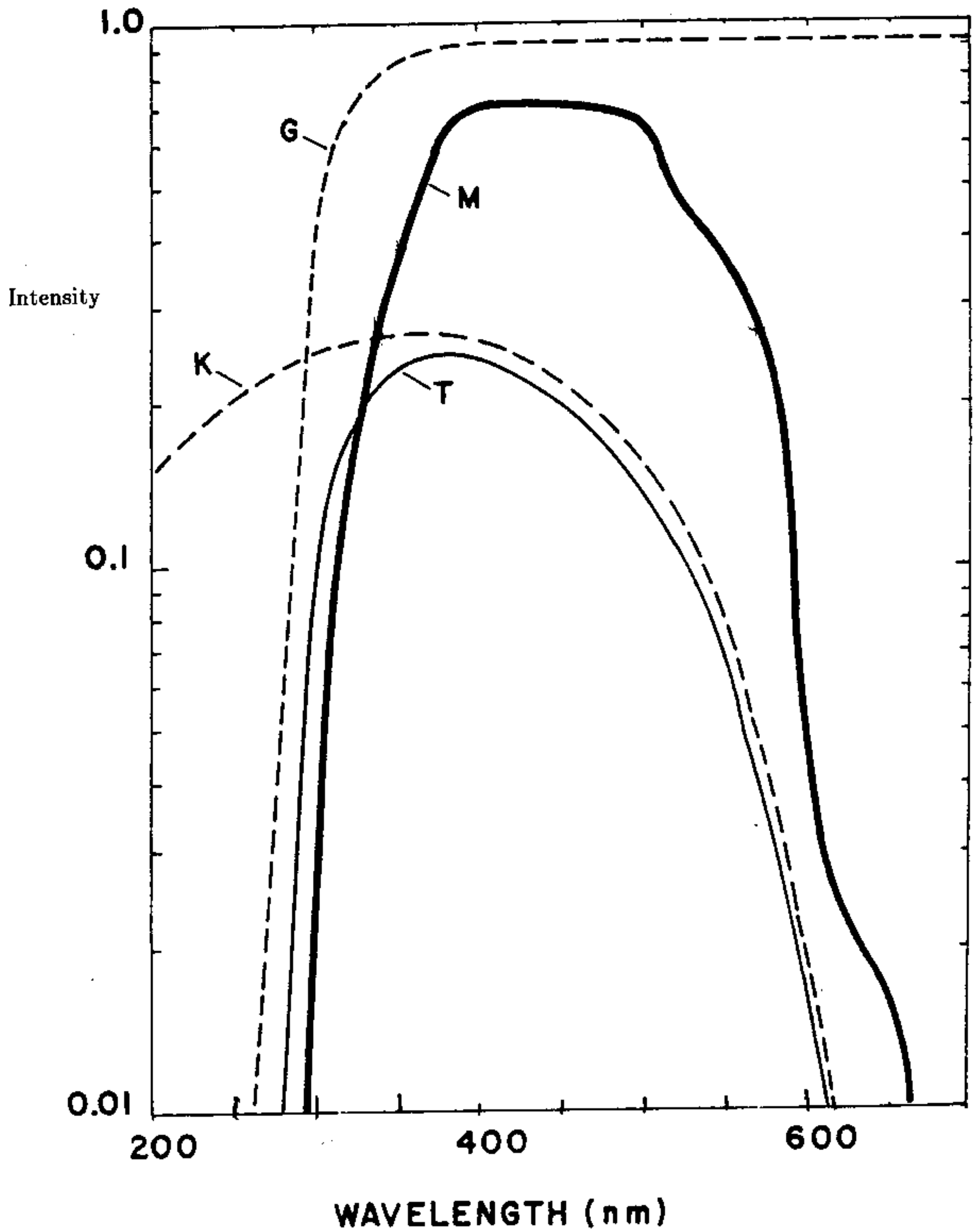

Fig. 9. DUMAND module sensitivity as a function of wavelength, in arbitrary units. Curve K is the photocathode response, G is the transmission of the PMT glass envelope, T the transmission of the ocean, and M the overall PMT. The short wavelength acceptance is truncated at 350 nm by the increasing opacity of the Benthos glass sphere; the effective sensitive range is thus 350–570 nm.