

How to Calculate the Undersea Muon Flux Produced by Very High Energy Neutrinos

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The muon flux above E_μ entering a detector at a depth d from the zenith direction θ is given by

$$F_\mu(>E_\mu) = N_A \int_{E_\mu}^{\infty} dE_\nu \frac{dF_\nu}{dE_\nu} e^{-N_A \sigma_T L(\theta)} \int_0^{1 - \frac{E_\mu}{E_\nu}} dy R_\mu[E_\mu, (1-y)E_\nu] \frac{d\sigma_\nu}{dy}$$

where F_ν is the muon neutrino flux entering the earth at that zenith, $d\sigma_\nu/dy$ is the muon neutrino charged current differential cross section, $\sigma_T(E_\nu)$ is the total muon neutrino cross section, and

$$L(\theta) = \int_0^{s(\theta)} \rho(r) dx$$

is the integrated column density from the surface of the earth to the detector along the path s shown in Fig. 1. The mass density $\rho(r)$ of the earth, as a function of the distance from the center of the earth, is shown in Fig. 2.

The total path length from the surface to the detector is given by

$$s(\theta) = [(R - d)^2 \cos^2 \theta + 2Rd - d^2]^{1/2} - (R - d) \cos \theta$$

where R is the radius of the earth. The distance x from the detector to a point along that path corresponds to a distance r from the center of the

earth given by

$$r^2 = (R - d)^2 + x^2 + 2(R - d)x \cos\theta$$

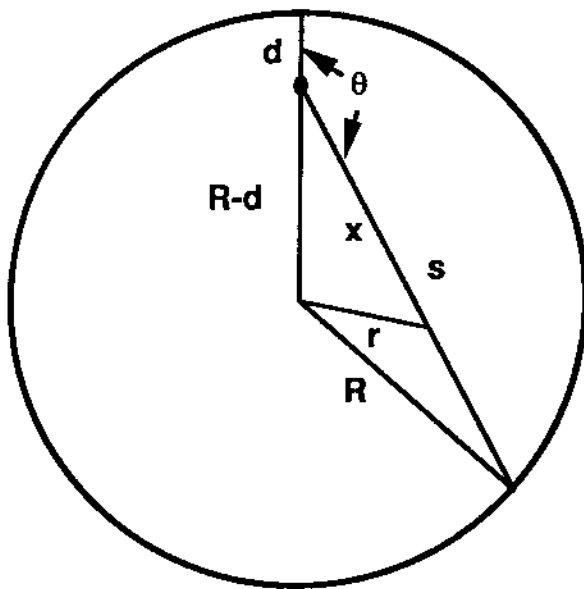


Fig. 1. A neutrino entering the earth at a zenith angle θ reaches a detector at a depth d after traversing a path s . A point along the path a distance x from the detector is a distance r from the center of the earth.

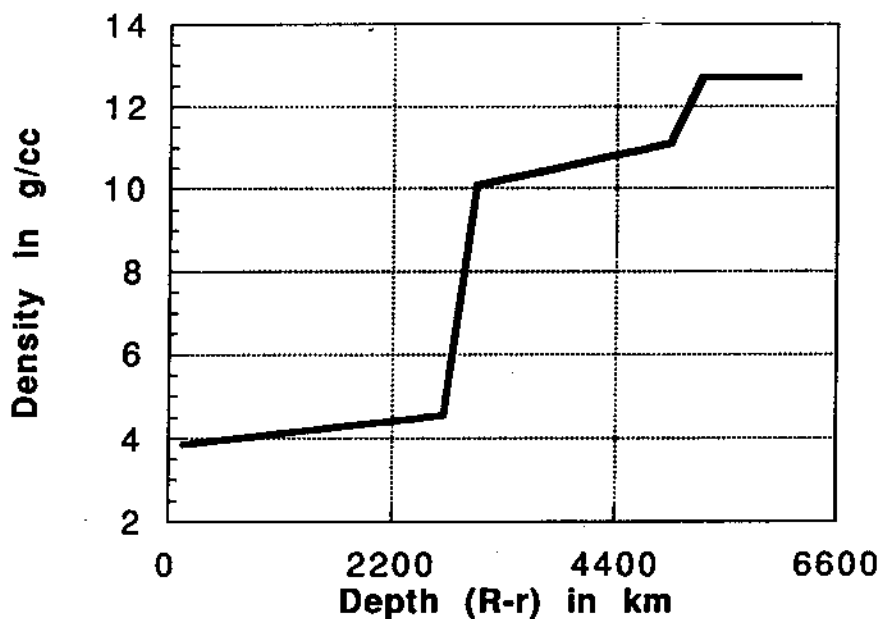


Fig. 2. The density of the earth as a function of depth below the surface.¹

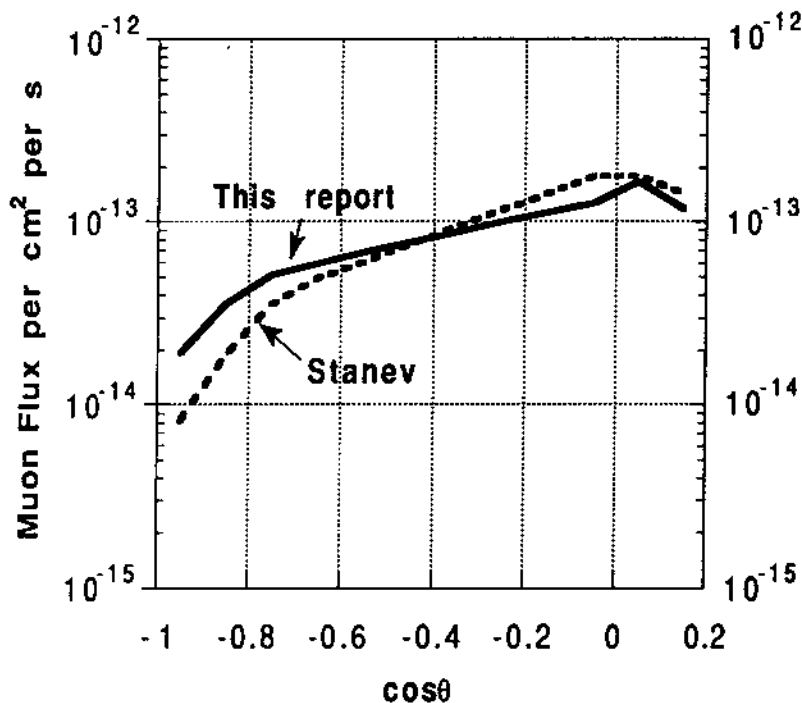


Fig. 3. A comparison of the muon flux calculation of Stanev with that reported here.

References

1. Frank P. Stacey, 1976. "Physics of the Earth." (New York:John Wiley).
2. L.B. Bezrukov and E.V. Bugaev, Proc. 1979 DUMAND Summer Workshop, p. 218.
3. F.W. Stecker, C. Done, M.H. Salamon and P. Sommers, PRL 66, 2657 (1991).
4. J. G. Learned and T. Stanev, submitted to the 3rd Venice meeting on Neutrino Telescopes (1991).

$R_\mu(E, E_0)$ is the range of a muon of initial energy E_0 and final energy E . This is calculated by assuming $-dE/dx = a(E) + b(E)E$, where a and b are energy dependent.² For the results presented here, the track was divided into 50 energy segments between E_0 and E (10 gives same results) and the range for each segment calculated assuming a constant value of a and b calculated at the energy E_i at the beginning of the segment. That is, for segment i ,

$$R_\mu^i = \frac{1}{b} \ln \frac{a + b E_i}{a + b E_{i+1}}$$

where $a = a(E_i)$, $b = b(E_i)$, and $R_\mu = \sum R_\mu^i$.

This method has been used to calculate the muon flux expected at DUMAND depth from the neutrino flux from Active Galactic Nuclei by Stecker et al.³ It is compared in Fig. 3 with the Monte Carlo calculation of Stanev for the same flux.⁴ The results are in good agreement, except for almost straight up-going tracks. The difference in that region could be the result of range straggling, which can only be accommodated in a Monte Carlo calculation. Although straggling can result in some muons reaching far greater than their mean range, the mean range itself is smaller than when straggling is neglected. This would be expected to give a lower flux, as observed.

The method described here offers an alternative to Monte Carlo means for calculating the undersea muon flux from an assumed neutrino spectrum. Since it can be done quickly, it may be convenient when one is exploring sensitivity to a large number of parameters.