

A Simple Functional Form for the Reno-Quigg (Anti)Neutrino Total Cross sections

The differential cross section for neutrino nucleon scattering, given by Halprin and Oakes (DUMAND '78 Workshop, pg. 43), includes the effect of the W propagator;

$$\frac{d\sigma}{dx dy} = \left[\frac{\beta}{\beta + xy} \right]^2 \frac{G^2}{2\pi} (s = 2M_N E_\nu) [q(x) + \bar{q}(x)(1 - y)^2] \quad (\text{eq. 1})$$

where x and y are the Bjorken scaling variables, and $\beta = (M_W)^2 / (s = 2M_N E_\nu)$.

1. Total Cross Sections

a) Neutrinos

I choose an approximate form of $q(x) = 1-x$, $\bar{q}(x) = 0$, and find a simple functional form for the total cross section;

$$\sigma = [(GM_W)^2 / 2\pi = 5,468 \times 10^{-38} \text{cm}^2][(1 + \beta) \ln(\beta^{-1} + 1) - 1] \quad (\text{eq. 2})$$

It is easy to verify that at low energy (large β),

$$[(1 + \beta) \ln(\beta^{-1} + 1) - 1] \approx \beta^{-1} = (s = 2M_N E_\nu) / (M_W)^2$$

and at high energy (small β),

$$[(1 + \beta) \ln(\beta^{-1} + 1) - 1] \approx \ln(\beta^{-1} = 2M_N E_\nu / (M_W)^2)$$

I use this as a comparison function for the Reno and Quigg Neutrino total cross section that I scaled from their Fig.1 (PR D37, 657(1988)). My Fig.1 is the ratio of their cross section to mine. The scatter of the points is caused by my inaccurate reading of their figure. The agreement up to 3×10^4 GeV is made better if I use a slightly higher value of 90 GeV for M_W in the evaluation of β . The constant in front of (eq. 2) is for $M_W = 80$ GeV. Note that the departure from unity above 3×10^4 GeV is a straight line on this log-log plot and suggests that I multiple my simple function (eq.2) by the factor $(E_\nu/E_b)^{0.23}$ above $E_b = 3 \times 10^4$ GeV.

Fig.2 is the result and shows exceptional agreement. Fig.3 is my resulting neutrino total cross section vs energy which I label Sig "0.23".

Quigg, Reno, and Walker in their earlier paper (PRL 57, 774 (1986), Fig. 1) show that above E_b , the contribution of the ocean quarks dominates over the valence quarks, and the neutrino and anti neutrino cross sections become equal.

b) Antineutrinos

For antineutrinos it is more complicated. Fig. 4 displays the Reno Quigg anti neutrino cross section divided by my corrected function [factor $(E_\nu/E_b)^{0.23}$, above $E_b = 3 \times 10^4$ GeV]. There are two regions of disagreement. The first, from 10 to 10^3 GeV, showing that my number is a factor 2 too high. Thus, in this region, I multiply my corrected function by 0.5. The second region, from 10^3 GeV to 10^5 GeV, is again a straight line on a log-log plot and a correction factor, $(E_\nu/E_a)^{0.1505}$ is applied between $E_a = 10^3$ GeV and $E_b = 3 \times 10^4$ GeV. Fig. 5, again

the ratio, shows this function gives very good agreement. Fig. 6 is my anti neutrino total cross section vs energy.

2. Differential Cross Section, $d\sigma/dy$.

a) Neutrinos

On integrating (eq.1) over x , with my crude approximation, $q(x) = 1-x$, $\overline{q}(x) = 0$, I get;

$$\frac{d\sigma}{dy} = \frac{G^2(s\beta - M_W^2)}{2\pi} \left[\frac{1}{y} - \frac{\beta}{y^2} \ln\left(1 + \frac{y}{\beta}\right) \right] \quad (\text{eq. 3})$$

The integral of this over y , of course, yields (eq. 2).

Fig. 7a displays $(1/E_\nu)d\sigma/dy$, again modified by the factor $(E_\nu/E_b)^{0.23}$, above $E_b = 3 \times 10^4$ GeV. It agrees well with Fig. 3 of Quigg, Reno, and Walker for energies up to 10^6 GeV. Figures 7b,c,d are the ratio of their values (Reno) to mine (MLS) versus y for neutrino energies, 10^4 , 10^6 , and 10^8 GeV, resp.

Fig. 7e is a log-log plot of my $d\sigma/dy$ that emphasizes the behavior at low y .

b) Antineutrinos

I multiply (eq. 3) by $(1 - y)^2$ to get a similar expression for the anti neutrino differential cross section. However, when I integrate it, I get a term,

$$\int \frac{1}{y} \ln\left(1 + \frac{y}{\beta}\right) dy$$

which, Gradshstein/Ryzhik say, "... cannot be expressed as a finite combination of elementary functions." Therefore, I give up on antineutrinos.

3. Total (CC+NC) Cross Sections

a) Neutrinos

Fig. 8 is the ratio of Reno-Quigg's (CC+NC)/CC total cross section on a log-log plot. The fitted line through their calculations is shown. Thus to obtain the total (CC+NC) Neutrino Cross Section using my simple modified (eq.2), one multiplies it by;

$$1.293 [E_\nu / (E_0=10 \text{ GeV})]^{0.01145}.$$

b) Antineutrinos

Fig. 9 is the same as Fig. 8, but for anti neutrinos. Thus to obtain the total (CC+NC) Antineutrino Cross Section, one multiplies my correction factors of section 1 (b) by;

$$1.335 [E_\nu / (E_0=10 \text{ GeV})]^{0.00612}.$$

4. Summary of (Anti)Neutrino Cross Section Parameterization Factors*

Total Neutrino CC

$$\begin{aligned} [(1+\beta)\ln(\beta^{-1}+1) - 1] & \quad 10 < E_\nu < E_b \equiv 3 \times 10^4 \text{ GeV} \\ [(1+\beta)\ln(\beta^{-1}+1) - 1] (E_\nu/E_b)^{0.23} & \quad E_\nu > E_b \\ \text{For Total (CC+NC), multiply all above by } 1.293 [E_\nu/(E_0=10 \text{ GeV})]^{0.01145}. \end{aligned}$$

Total Antineutrino CC

$$\begin{aligned} [(1+\beta)\ln(\beta^{-1}+1) - 1] 0.5 & \quad 10 < E_\nu < E_a \equiv 1 \times 10^3 \text{ GeV} \\ [(1+\beta)\ln(\beta^{-1}+1) - 1] 0.5 (E_\nu/E_a)^{0.1505} & \quad E_a < E_\nu < E_b \\ [(1+\beta)\ln(\beta^{-1}+1) - 1] 0.5 (E_b/E_a)^{0.1505} (E_\nu/E_b)^{0.23} & \quad E_\nu > E_b \quad \text{CORRECTED} \\ \text{For Total (CC+NC), multiply all above by } 1.335 [E_\nu/(E_0=10 \text{ GeV})]^{0.00612} \end{aligned}$$

Differential Neutrino CC, $d\sigma/dy$

$$\begin{aligned} [y^{-1} - \beta y^{-2} \ln(y\beta^{-1}+1)] & \quad 10 < E_\nu < E_b \equiv 3 \times 10^4 \text{ GeV} \\ [y^{-1} - \beta y^{-2} \ln(y\beta^{-1}+1)] (E_\nu/E_b)^{0.23} & \quad E_\nu > E_b \end{aligned}$$

* These factors must be multiplied by $[(GM_W)^2/2\pi = 5,468 \times 10^{-38} \text{ cm}^2]$ to obtain cross sections, where $M_W = 80 \text{ GeV}$.
 Recall that, $\beta = (M'_W = 90 \text{ GeV})^2/(s = 2M_N E_\nu)$.

5. Acknowledgements

I wish to thank Chris Quigg and Hallsie Reno for supplying their numbers to me, and John Learned for encouraging me to write this note.

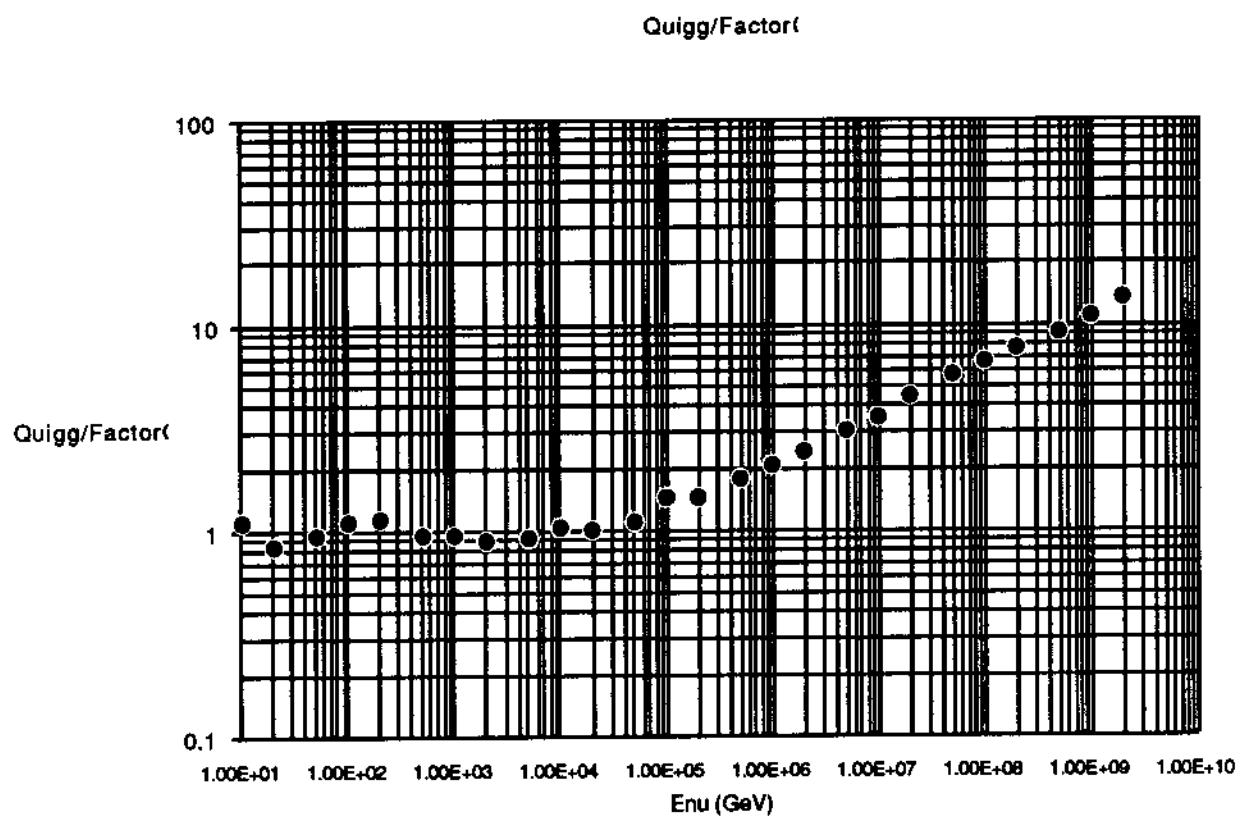


Fig. 1

Quigg/Corr"(E/Eb)^{0.23} above Eb = 3E4 GeV

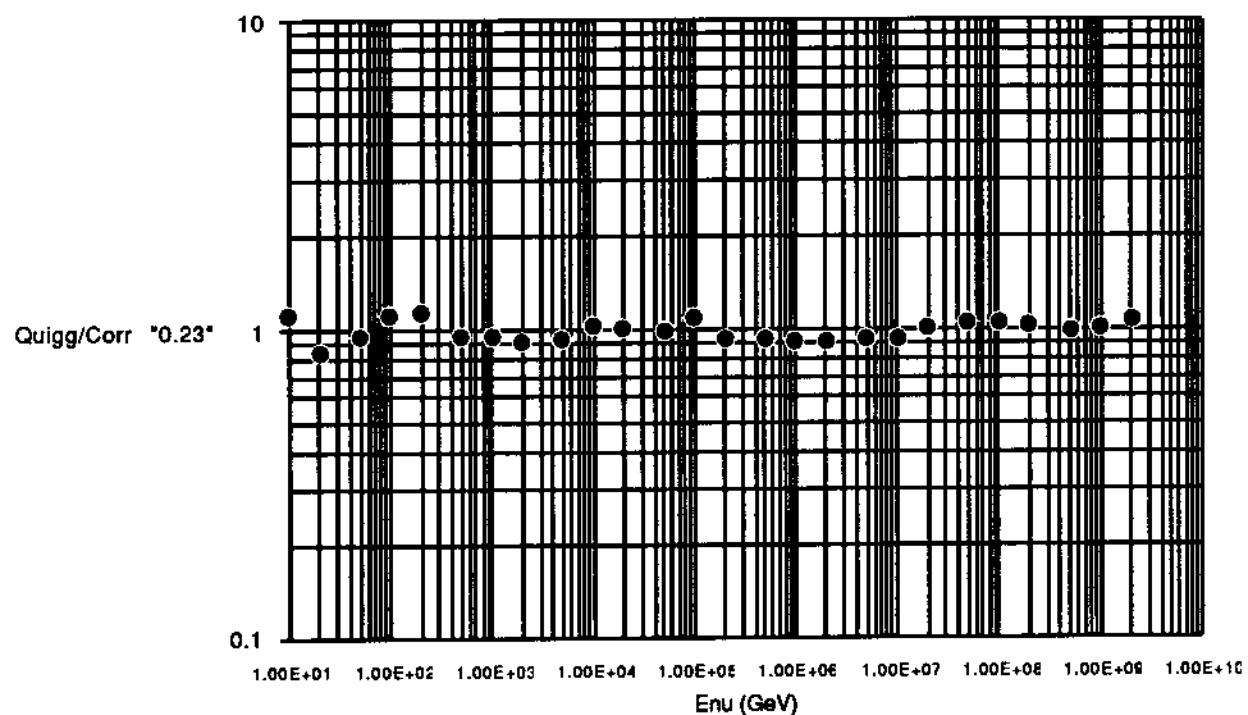


Fig. 2

Corr"0.23**5468

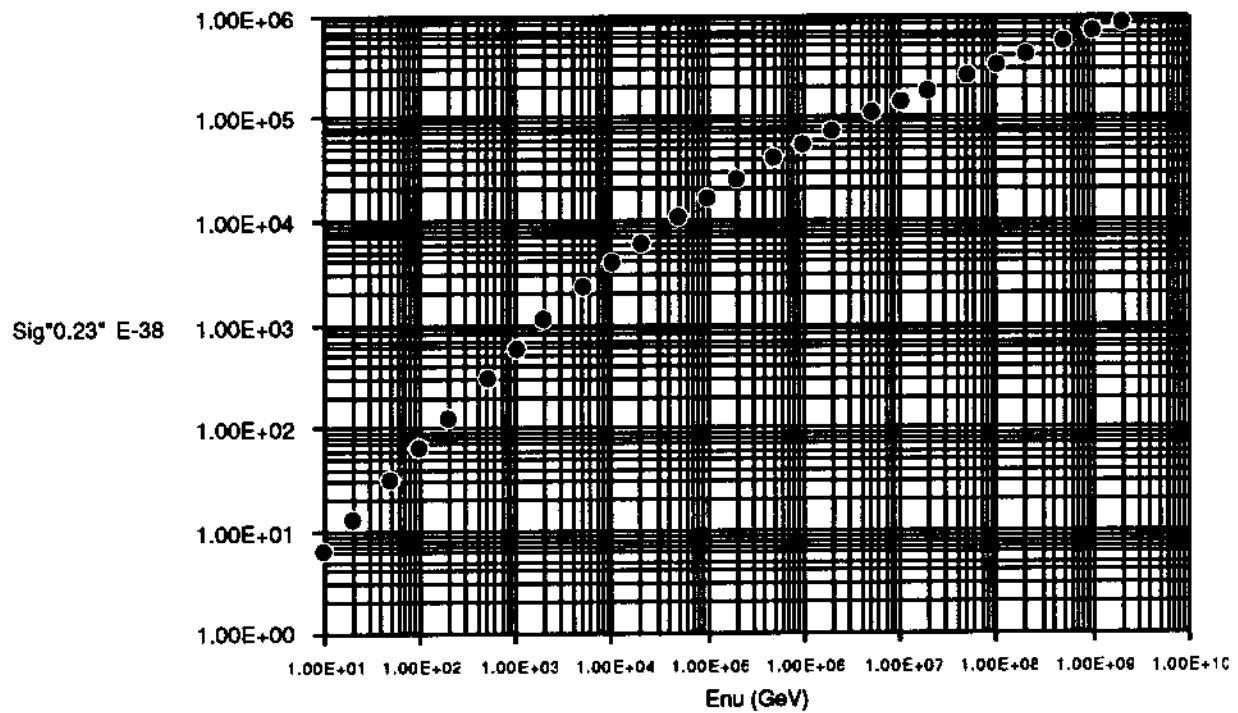


Fig. 3

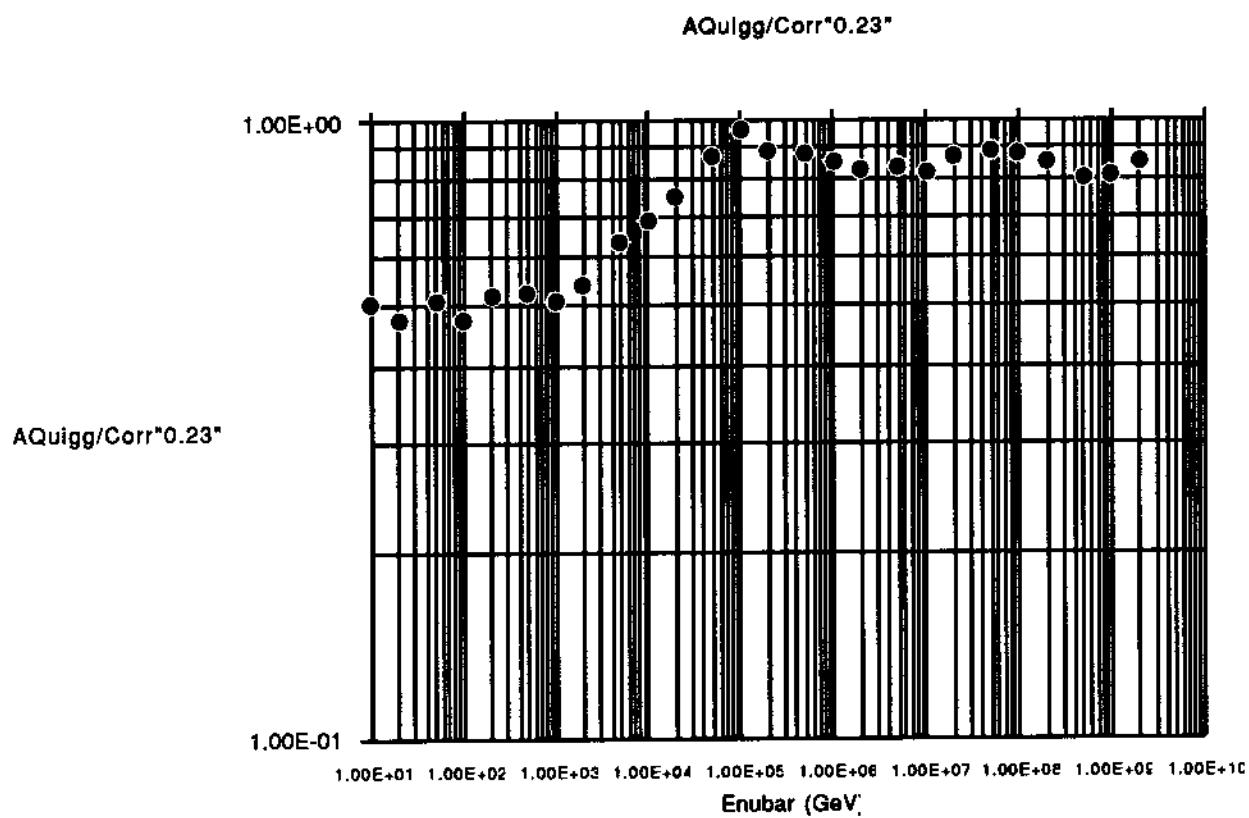


Fig. 4

AQuigg/FactorC:Parameters= 0.5, "0.1505, 0.23 powers"

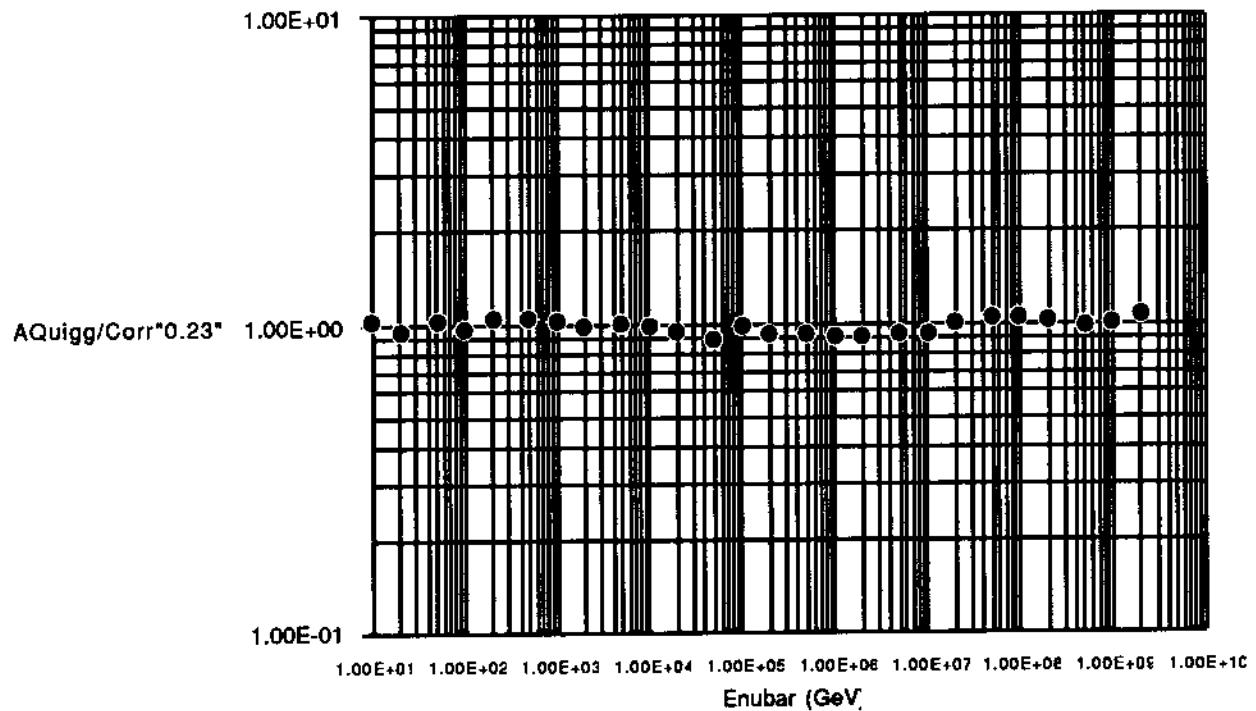


Fig. 5

ASig:Correction parameters = 0.5,"0.1505, 0.23"

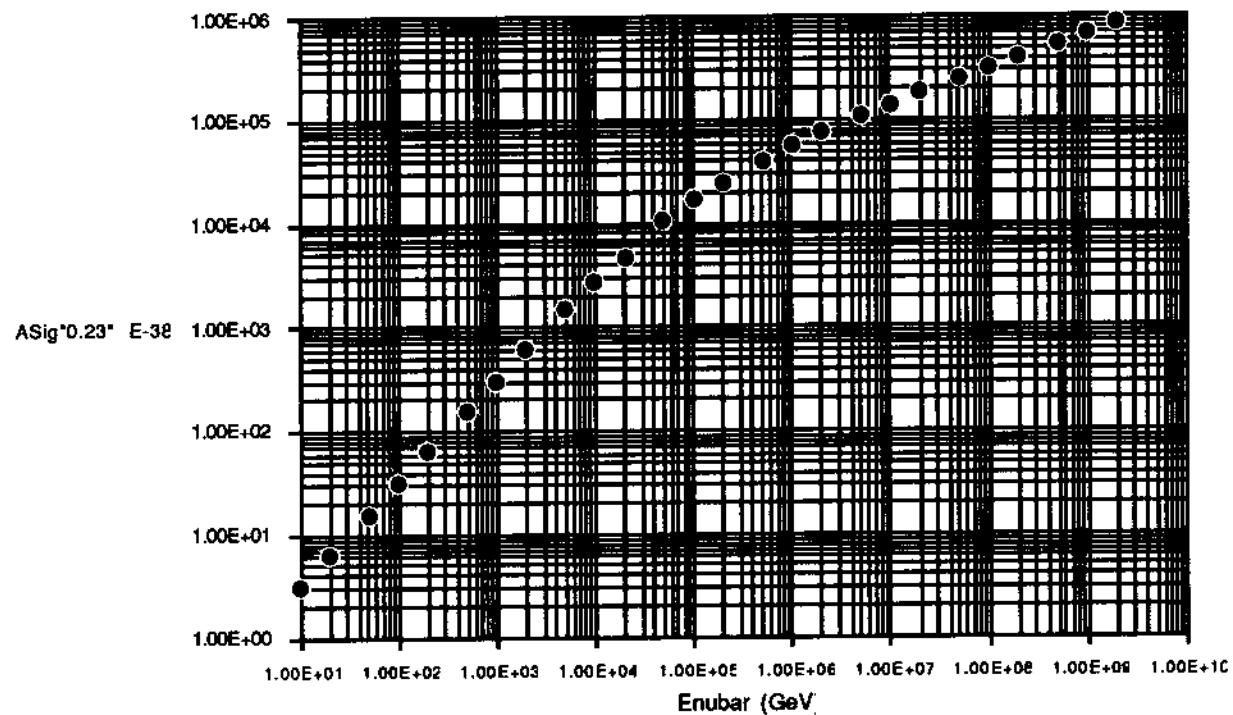


Fig. 6

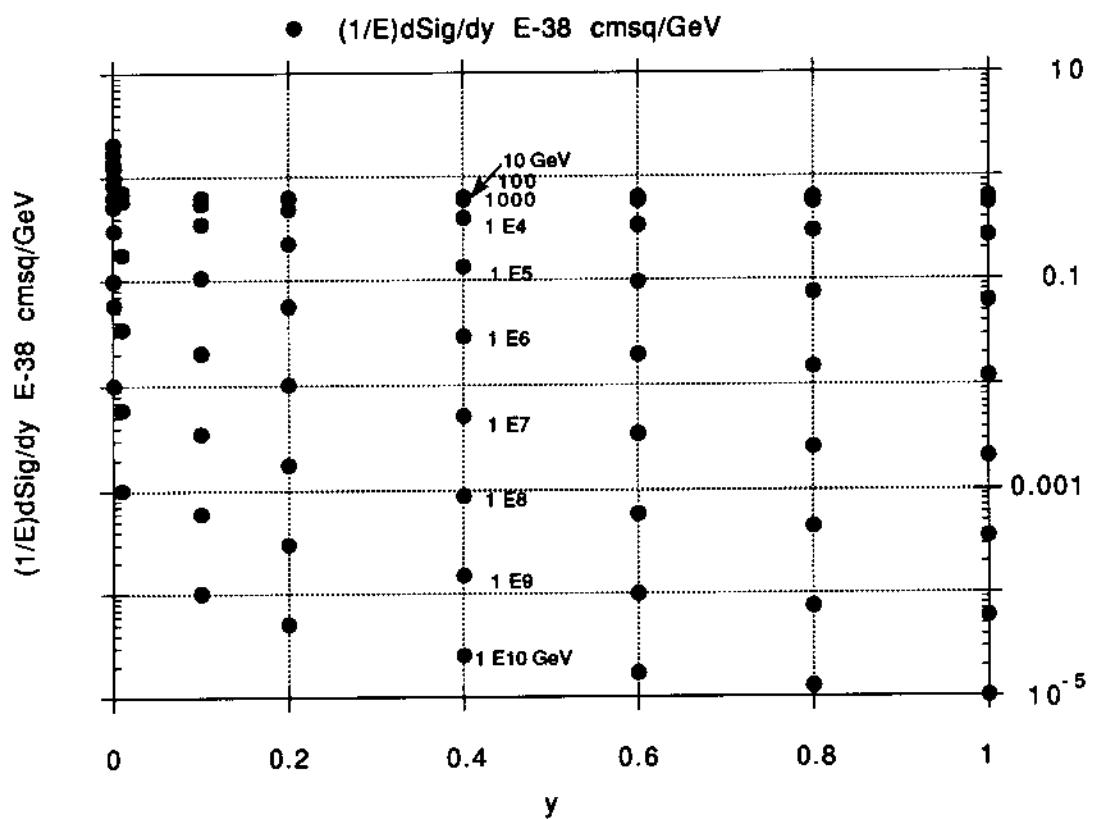


Fig. 7a

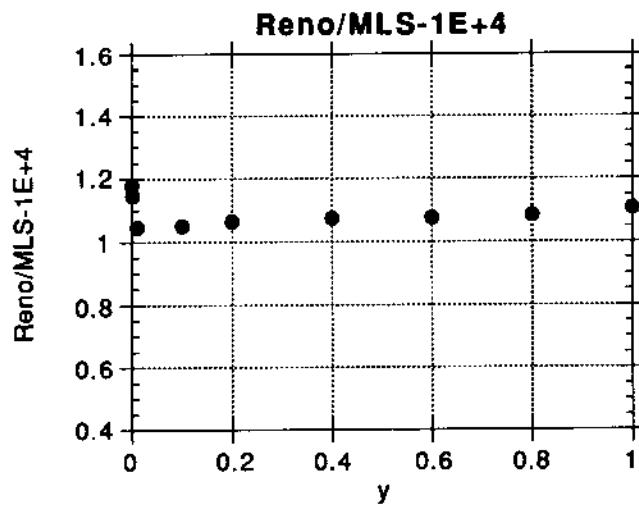


Fig. 7b

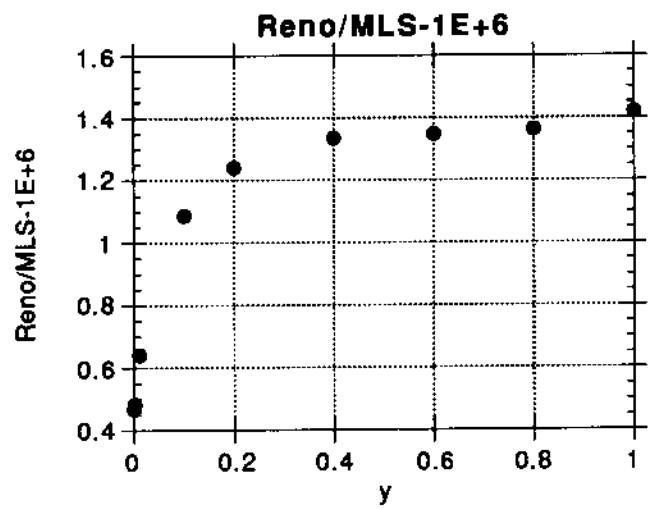


Fig. 7c

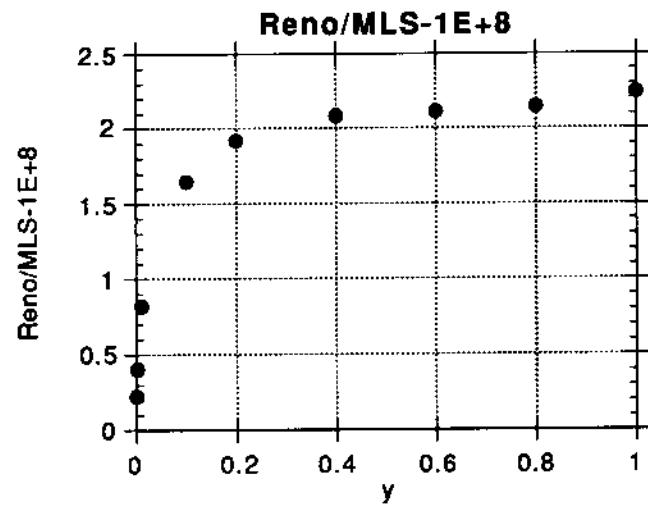


Fig. 7d

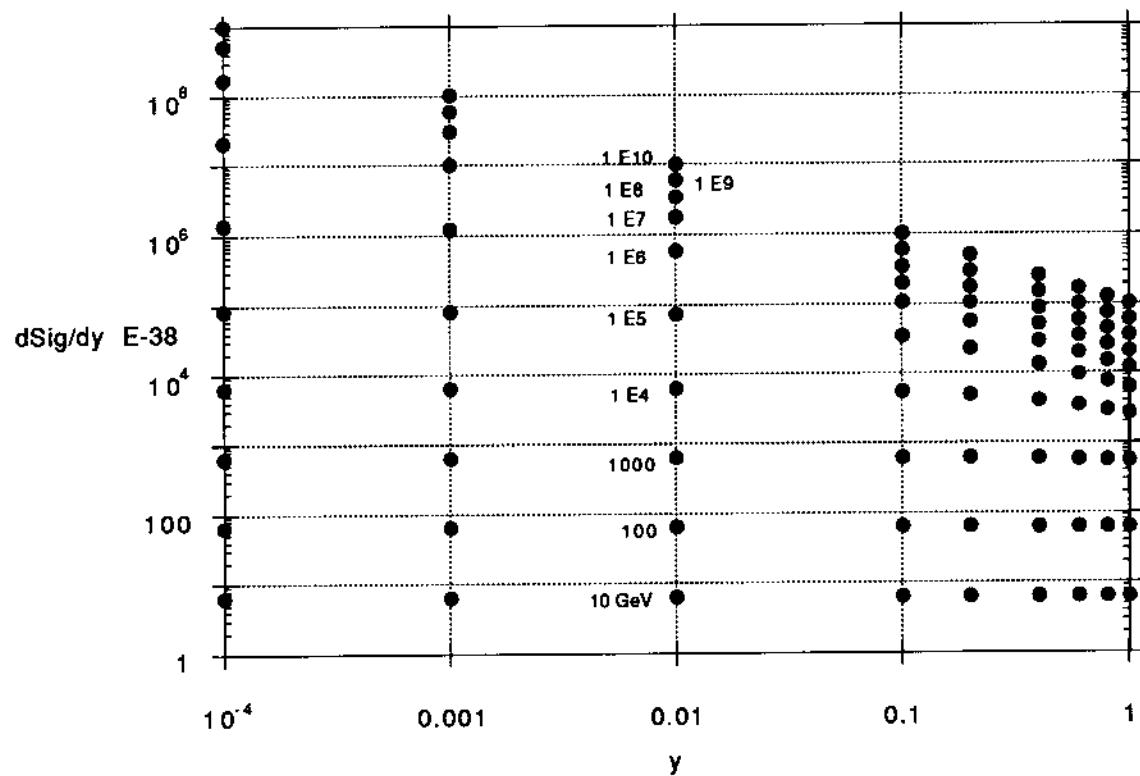


Fig. 7e

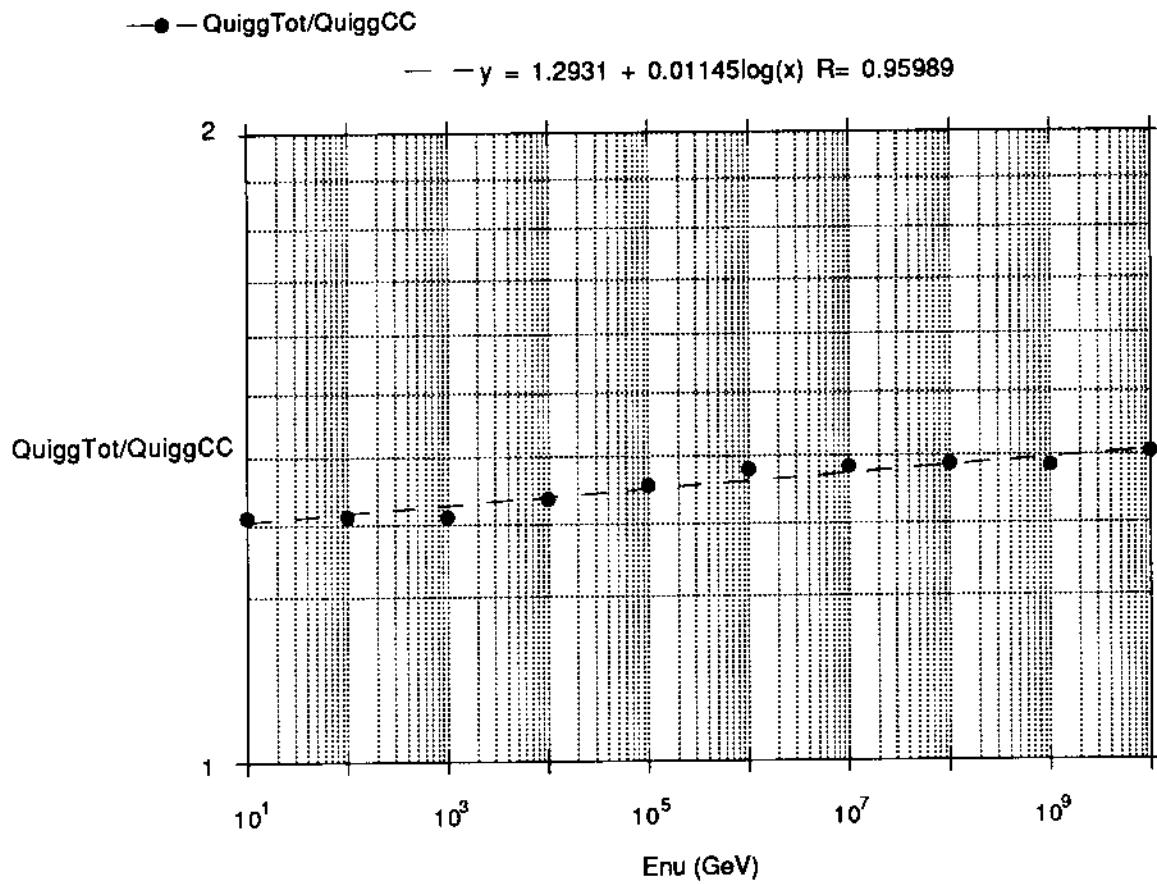


Fig. 8

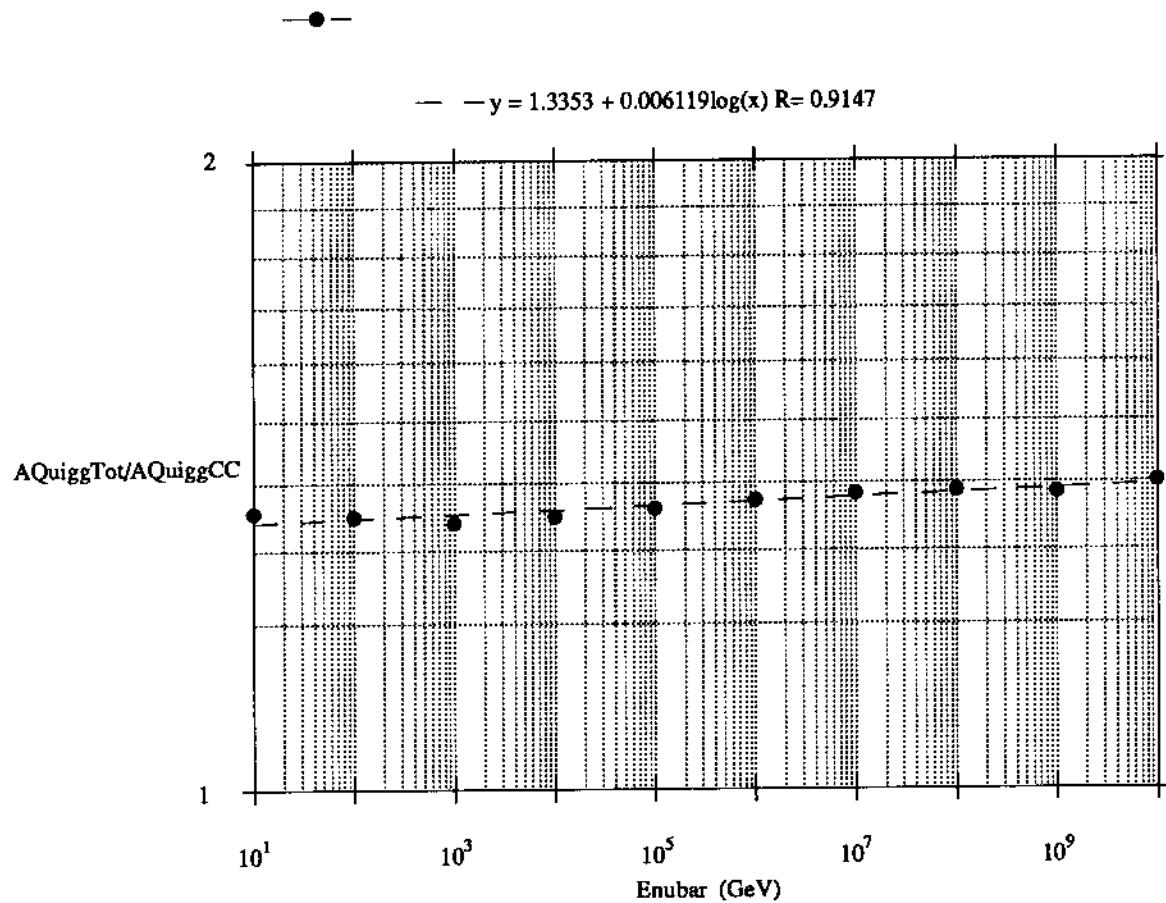


Fig. 9