Estimating the Total Energy Trigger Rate DUMAND II

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Abstract

We fit a gamma distribution function to measured pulse height distributions for the Hamamatsu photomultipliers and discuss the possible noise rates thus implied for the TE trigger. It appears that the TE trigger threshold will be between 25 and 31 photoelectrons in $1\mu sec$ to keep the rate below 1000 triggers/sec.

DIR-19-90

1 A Function that Describes the Hamamatsu Pulse Height Distribution

A pulse height distribution from a new style Hamamatsu photomultiplier was measured and the results were reported in DIR-13-90 (Okada and Uehara). Subsequent to that a simple function has been fitted to the pulse height distribution. The data and fitted function are shown in Figure 1. The function for an illumination level of 1 photoelectron (PE), is of the normalized, one free parameter form:

$$F_1 = b^2 Q e^{-bQ}.$$

The fit gives a value of $b = 2.0$, but we caution that this is only for one PMT, and other samples may well give different b values, though hopefully the same functional form. In any case, this function can be convolved with itself to generate the distribution for an arbitrary number of photoelectrons:

$$F_n = b^{2n} Q^{2n-1} e^{-bQ}/(2n-1)!$$

This is the gamma distribution (also known as the Erlangian distribution). This is a slightly peculiar result in that it is a Poisson distribution (for finding $2n - 1$ with mean bQ) with the usual variables interchanged. We do not understand why this is so, though it seems that it may be significant. Anyway, the mean and standard deviation are:

$$< Q >= 2n/b$$

and

$$\sigma_Q = \sqrt{2n}/b.$$

Note that the mean for 1 PE illumination is $2/b$. The fractional width of the distribution scales as we would expect:

$$\sigma_Q / < Q >= 1/\sqrt{2n}.$$

The most probable value for the distribution is a bit odd,

$$Q_m =< Q_1 > (n - 1/2),$$

where $Q_1 = 2/b$. Thus the maxima occur at 0.5 PE, 1.5 PE, 2.5 PE, etc.

Unfortunately this function is not partially integrable in closed form, being an incomplete gamma function, so one must do a numerical integration for finding out the probability of being greater or less than some particular value. One can, however use a trick to interchange the incomplete integral with a finite sum:

$$\int_Q^\infty F_n(Q) dQ = \sum_{k=0}^{2n-1} P(k \mid bQ),$$

where P is the Poissonian function. We show the function in differential and integral form for 1 to 4 PE, in Figure 2.

We should note that we have explored some other functions, and none really have the correct shape. The famous convolution of Polya distributions, proposed by Prescott[1], does have the right sort of shape, but is very difficult to calculate. Moreover the physics motivation for the Polya distribution is in our view rather unconvincing. When we have more data from a larger tube sample (and of the new Hamamatsu phototube) it will be interesting to explore this question further.

2 Calculating the TE False Trigger Rate

Having a simple function to describe the pulse height we can do a simple calculation of the rate versus summed pulse height from the whole array. We assume that the noise rate per PMT, R_m, is constant for all N_m PMTs. Thus because the probability per PMT of getting one or more hits in an interval τ is constant, we can describe the probability distribution of number of PMTs with hits in the array in time interval τ as Poisson distributed with mean $m = R_m N_m \tau$, and distribution function

$$P(k|m) = m^k e^{-m}/k!.$$

That $R_m \tau$ is small (0.06), and that the probability of getting more than one PE per K^{40} decay is also very small ($<< 1\%$), means that the high pulse heights from individual PMTs will mostly arise from the long tail of the single PE distribution. We assume also that multiple PEs from afterpulsing are also negligible (to be demonstrated experimentally, however).

We have already solved the problem of the total pulse height to be expected from some given number of PMTs which have hits: it is given by the same convolution as discussed in Section 1 for muliple PEs from one PMT. Hence, for the total rate in the array at some summed pulse height we need only add the probability of getting some number, say j, times the probability for getting some sum, Q given that number of hits, summed over the possible number of hits (0 to ∞). Thus for the total rate we have

$$R_Q = \sum_{j=1}^{\infty} bP(2j-1|bQ)P(j|m)/\tau,$$

[1] J.R. Prescott, Nuclear Instruments and Methods 39, 173 (1966)

where the $P(x|y)$ is the Poisson distribution, for x with mean y.

This is illustrated in Figure 3 for $m = 12.96$ and $b = 2$. Note that this rate prediction is for an array of all Hamamatsu PMTs. Presumably the better first stage gain and pulse height resolution of the Philips PMTs would make for a lower rate. The dashed curve in Figure 3 is computed for the case of infinitely good resolution, and is just the Poisson case discussed in DIR-12-90. Barring any large contribution from ions (afterpulsing) or other correlated multi-PE sources, we would then expect the TE random trigger rate to lie between the solid and dashed curves in Figure 3. If we wish the trigger rate to be less than 1000 per second, the the threshold setting should be between 26 and 31 PE from the entire array in $1\mu sec$.

3 Summary

We have found that the simple gamma distribution function roughly describes the pulse height distribution for a sample Hamamatsu PMT. This permits easy calculation of the expected rate from the entire array for the TE trigger, as a function of summed pulse height threshold as indicated in Figure 3. As shown in DIR-12-90, this level of threshold will allow efficient collection of muon, cascade, and other relativistic physics events. This calculation should be redone more carefully after we have results from a meaningful sample of the new version of the Hamamatsu DUMAND photomultipliers.

Figure 1: Hamamatsu PMT pulse ht data and fitted function

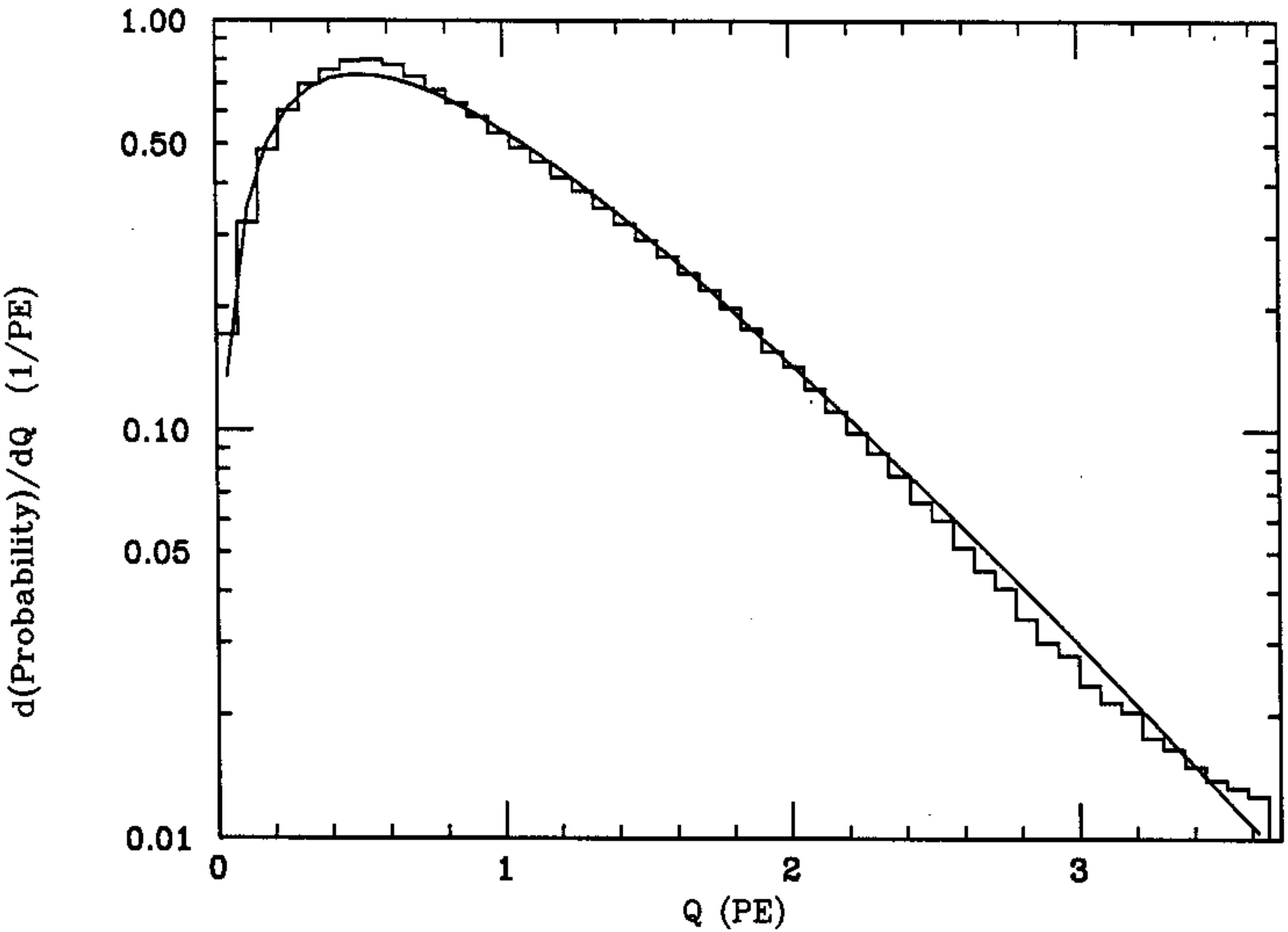

Figure 2: Pulse height distribs and integrals, 1–4 PE.

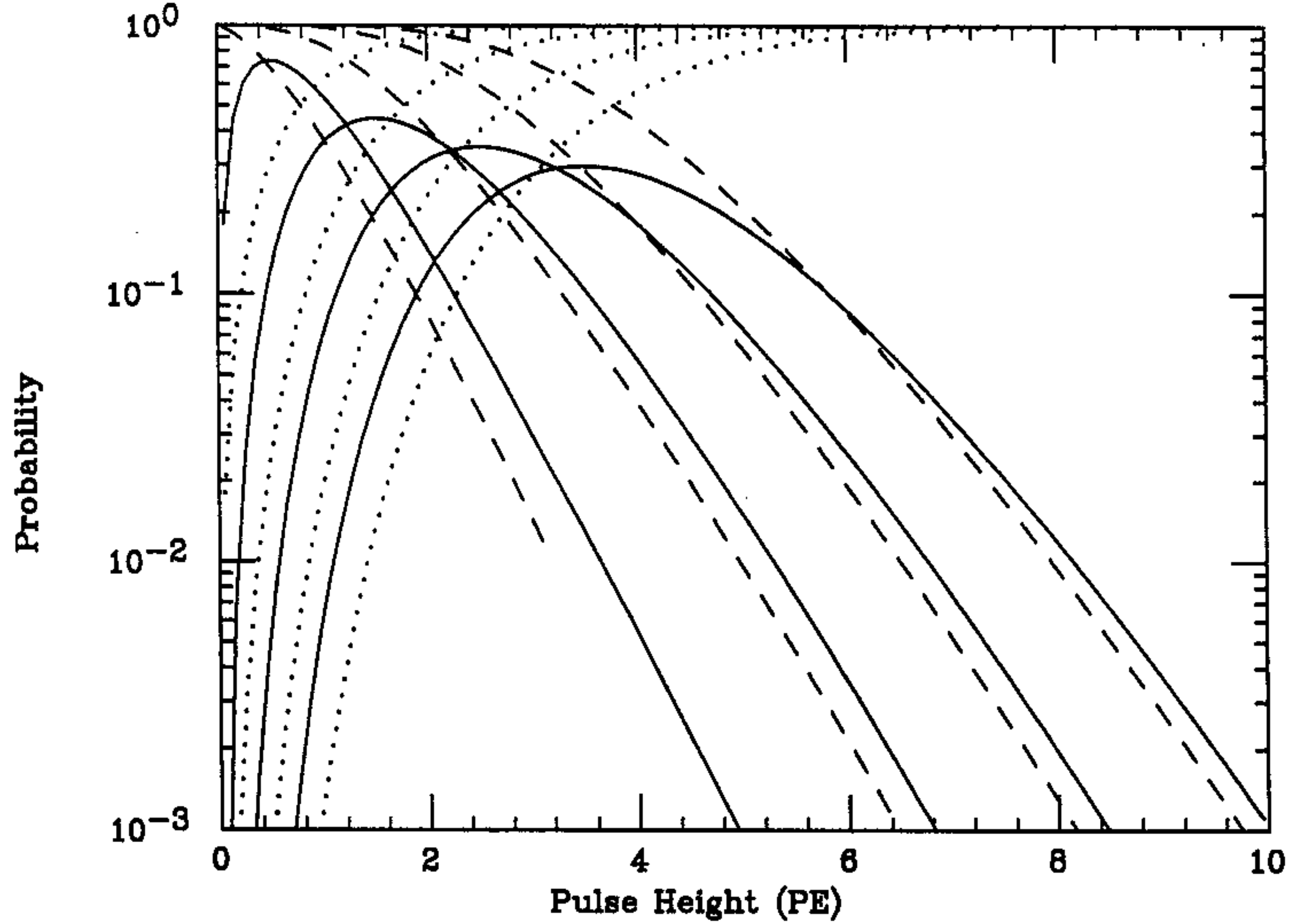

Figure 3: TE rate prediction for all Hamamatsu PMTs.