

A COMPARISON OF UNDERGROUND NEUTRINO EVENT RATE CALCULATIONS

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There are now several estimates of the sensitivity of underground and undersea experiments to point sources of extraterrestrial neutrinos. During my recent trip to Italy I held discussions with G. Auriemma, Jim Musser and others of the MACRO/Gran Sasso collaboration. While we basically agreed that the average neutrino energy will be high for the type of power law spectra expected - several TeV even when the muon threshold energy is a few GeV - there still appeared to be about a factor of ten disagreement in overall event rates. In this note I show that the various calculations considered actually agree within a factor of two, with differences mainly resulting from assumptions about the νN cross section at very high energy.

Although the authors were not present at the discussion, the calculation of Gaisser and Stanev [BA-85-9] has been used in estimates made by MACRO and it was said that these agreed with the Monte Carlo results of Auriemma. Let me use their method for reference. They define $P(E) = N_\mu/N_\nu$ as the probability that a neutrino of energy E produces a muon in an underground detector with a minimum energy E_0 . I have calculated $P(E)$ using the following formula, which is equivalent to the one used by Gaisser and Stanev, except that I integrate over muon range R_μ rather than muon energy at the point of interaction:

$$P(E) = \frac{n_A}{E} \int_{E_0}^E dE_\mu \int_0^{R_\mu^{\max}} dR_\mu \int_0^1 dx \frac{d\sigma_{\nu N}}{dx dy} \quad (1)$$

where the notation is conventional. Note that this has the very nice feature of not depending on the neutrino flux. Although I did not compute $P(E)$ directly, this is the method I used in my recent paper HDC-4-85.¹

A more approximate way to compute $P(E)$, equivalent to the calculation of Jim Musser (private communication), is:

$$P(E) = n_A \sigma(E) R_\mu(E/2) \quad (2)$$

That is, assume that the muon always has half the neutrino energy.

Fig. 1 shows $P(E)$ calculated from (1) and (2) above, compared with Fig. 1 of Gaissner and Stanev, for $E_0 = 1$ GeV. In calculating (2) the same total cross section formula used by Musser was used. In calculating (1) the same differential cross section subroutine was used as in HDC-4-85. The relative ν and $\bar{\nu}$ contribution was taken to be 55%-45% in all calculations here, although I used 67%-33% in HDC-4-85 so my results here are slightly different than previously reported. The muon energy loss was assumed to be given by $-dE_\mu/dx = 2.2 \times 10^{-3} + 3 \times 10^{-6}$ GeV/cm, somewhat different than assumed by the other guys but not different enough to matter much.

Note from Fig. 1 that Musser and I agree pretty well, while Gaissner and Stanev get significantly higher probabilities (note that its a log scale) above 1 TeV. This probably results from different assumptions on the cross section, since that is the only real difference in the calculations.

1./ Here, for simplicity, I take $x_{\max} = 1$. Also, please note that equation (4) in HDC-4-85, as originally distributed, has a typographical error: E_ν should be in the denominator.

I must say I was surprised to see Musser's approximation come so close, especially since the average muon energy in neutrino interactions is a lot less than half the neutrino energy for both ν and $\bar{\nu}$ at the high energies involved. This fortuitous agreement is presumably an effect of the fact that the interaction volume weights events toward higher muon energy.

Let me more directly compare these results with HDC-4-85. There I defined an efficiency $\epsilon = N_{\mu}[>E_0]/N_{\nu}[>E_0]$, which we see is not the same as $P(E)$. If we write the integral neutrino flux as the power law $F_{\nu}[>E] = F_{\nu}[>1]E^{-\gamma}$, where E is in GeV and where I use γ as the integral spectral index, then ϵ can be related to $P(E)$ by

$$\epsilon = \gamma E_0^{\gamma} \int_{E_0}^{\infty} P(E) E^{-\gamma-1} dE \quad (3)$$

Let us just consider $E_0 = 1$ GeV and $\gamma = 1$. Then [3] can be written as simply the integral of $2.303 P(E)/E$ over $\log E$. This integrand is plotted in Fig. 2 for the three $P(E)$ given in Fig. 1. Note how, what appears to be a small difference in $P(E)$ in Fig. 1 translates into a big difference in the integral to get ϵ . Thus I get $\epsilon = 2.7 \times 10^{-9}$ using the $P(E)$ I compute with (1), 2.8×10^{-9} using the $P(E)$ obtained with Musser's approximation (2) and 6.8×10^{-9} using $P(E)$ from Geisser and Stanev. My value is somewhat higher than quoted in HDC-4-85 because of the different assumption on mix of ν and $\bar{\nu}$. Also note that the most probable neutrino energy is about 6 TeV in my case, 10 TeV for Musser and about 20 TeV for Geisser and Stanev. The range of a 20 TeV muon is about 10 kmwe, so we are talking about enormous detection volumes.

Table 1 of BA-85-9 lists the "rates of upward muons for power law spectra" for $E_0 = 2$ and 100 GeV. This is normalized to a flux of 1 neutrino above 2 GeV, according to the text, so it seems to be equivalent to my ϵ except for the normalization at 2 GeV rather than 1 GeV. For an integral spectral index of 1 and $E_0 = 2$ GeV, a value of 1.5×10^{-8} is quoted.

From (3), I need to multiply the previous result for $E_0 = 1 \text{ GeV}$ by 2, giving 1.4×10^{-8} , in good agreement with BA-85-9.

The efficiency ϵ can be used directly to make an estimate of the expected event rates. The event rate to be expected for an underground detector of area A and threshold energy E_0 is simply $N_{\mu}[>E_0] = \epsilon A F_{\nu}[>E_0]$. For example, if we take $F_{\nu}[>E_0] = 3 \times 10^{-8} E_0^{-1} \text{ cm}^{-2} \text{ s}^{-1}$ [i.e., equal to the γ -ray flux from Cygnus X-3], $A = 1000 \text{ m}^2$, and $E_0 = 1 \text{ GeV}$, we get .025, .027 and .064 events per year for the three calculations considered. As I show in HDC-4-85 this result is essentially independent of E_0 to 100 GeV. So there is really only about a factor of 2 disagreement, which probably results primarily from differences in the assumed high energy νN cross sections.

One factor which has not been considered in these calculations is straggling. R. Svoboda of the Hawaii group has been doing this and suggests that it will tend to increase our event rate estimates.

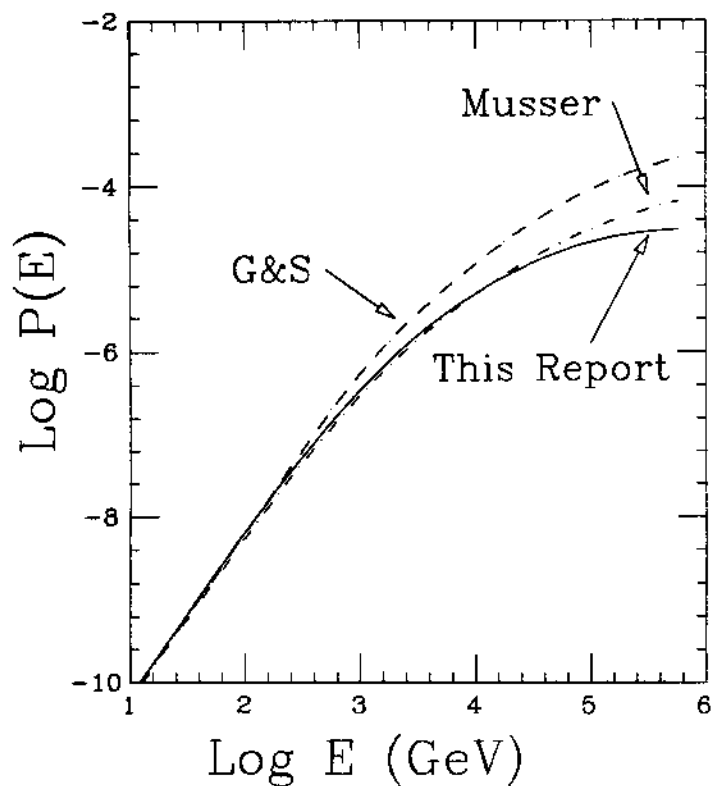


Fig. 1. The probability $P(E)$ that a neutrino of energy E will make a muon of at least 1 GeV in an underground detector. The dashed curve labelled G&S is from Gaisser and Stanev [BA-65-8]. The other curves are calculations described in the text.

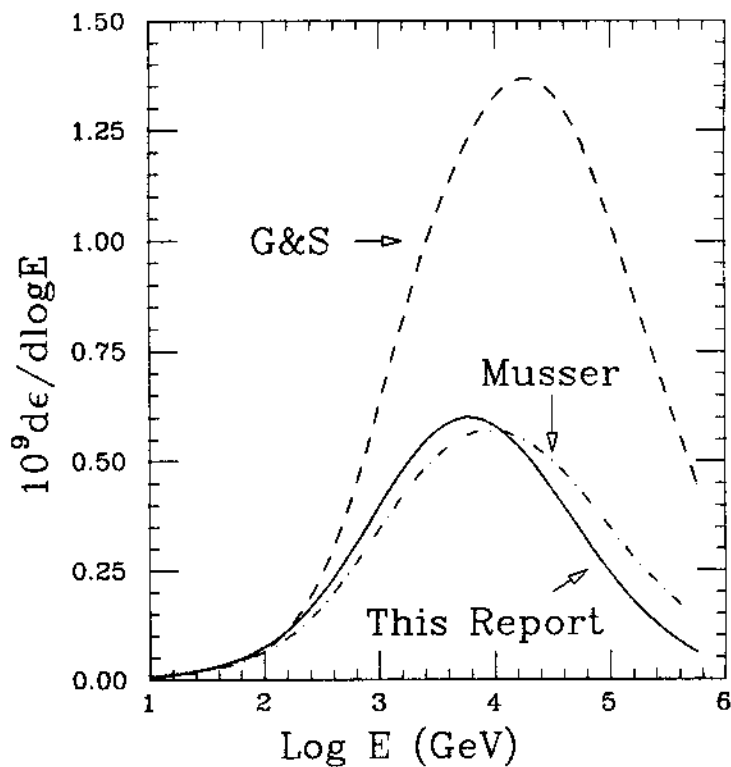


Fig. 2. $d\epsilon/d\log E$ vs. $\log E$ for the same three calculations.