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Acceleration Mechanisms:

The "Dynamo" Model (Lovelace)
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It has been proposed (1) that a supermassive black hole ($\sim 10^8 M_\odot$) acting as an electric dynamo can produce two oppositely-directed beams of ultra-relativistic protons. The generator action is inherent in the inward rotation of a highly-conducting, magnetized accretion disk circling the black hole. The collimated beams propagate in the direction of the disk angular momentum vector and have an output power $\sim 10^{44}$ erg/s, corresponding to maximum proton energies $\sim 10^{19}$ eV, for a 1 Kgauss poloidal disk magnetic field. The proton beams eventually interact with the ambient plasma and, via the two-stream interaction, can produce collimated beams of relativistic electrons. The electron beams are denser than the tenuous proton beams and release energy in the form of synchrotron radiation, which appears as the familiar "radio jets."

(1)

Assume that a massive black hole lies in the center of a galaxy and is surrounded by a large accretion disk. Material escaping from local stars and ionized interstellar plasma is drawn into the disk. The disk angular momentum is correlated with that of the accreting matter and with the general motion of the host galaxy. Weak magnetic field lines trapped in the plasma become concentrated during the inward spiral as the surface density increases. The radial dependence of the surface density of a flat disk can be approximated as follows:

Let the orbital velocity $u_\phi(r) = \left(\frac{GM_H}{r}\right)^{1/2}$, where M_H is the black hole mass and the equality derives from standard Keplerian motion. Also let the surface density $\sigma(r) = \frac{\text{mass}}{\text{area}}$ at radius "r".

Now make the approximation that the inward radial motion of the matter is proportional to the orbital velocity, but much smaller --

(2)

$$U_r(r) = \epsilon U_\phi(r) = \epsilon \left(\frac{GM_H}{r} \right)^{1/2}, \text{ where } \epsilon \ll 1.$$

Consider an annular section of the disk containing a mass element "dm" of the total disk mass "m",

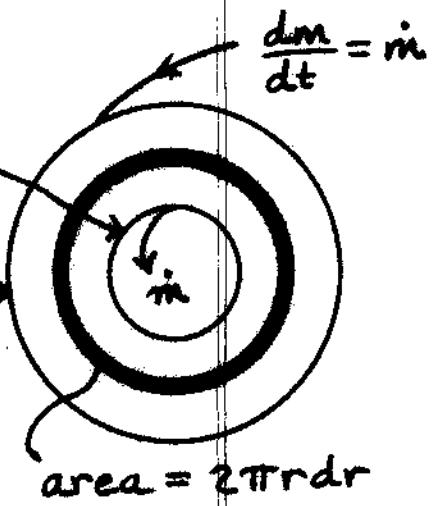
$$\sigma(r) = \frac{dm}{2\pi r dr}$$

$$= \frac{\frac{dm}{dt}}{2\pi r \frac{dr}{dt}}$$

$$= \frac{\dot{m}}{2\pi r U_r(r)} = \frac{\dot{m}}{2\pi r \epsilon \left(\frac{GM_H}{r} \right)^{1/2}} = \frac{\dot{m}}{2\pi r^{3/2} \epsilon \sqrt{GM_H}}$$

r_1 (inner radius)

r_2 (outer radius)



area = $2\pi r dr$

Thus $\sigma(r)$ is proportional to the rate at which mass enters and leaves the disk (herein assumed equal and constant) and inversely proportional to the square root of the black hole mass and the disk radius, so for constant M_H ,

$$\sigma(r) \propto r^{-1/2} \propto B_z(r),$$

where $B_z(r)$ corresponds to the lines of flux trapped in the disk with the accumulated plasma.

(3)

To an observer in a non-rotating reference frame there is a radial electric field,

$$E_r(r) = -\frac{u_\phi(r)}{c} B_z(r)$$

The potential across the disk is then

$$V_{12} = \int_{r_1}^{r_2} E_r(r) dr = -\frac{1}{c} \int_{r_1}^{r_2} u_\phi(r) B_z(r) dr$$

where the inner radius r_1 is determined by assuming a Schwarzschild (non-rotating) black hole (2) and taking the smallest stable circular orbit,

$$r_1 \sim \frac{6GM_H}{c^2} \approx \frac{\left(6 \times 6.672 \times 10^{-11} \frac{m^3}{Kg \cdot s^2}\right) \left(2 \times 10^{38} Kg\right)}{9 \times 10^{16} m^2/s^2} \left(\frac{M_H}{10^8 M_\odot}\right)$$

$$\approx 10^{12} m \left(\frac{M_H}{10^8 M_\odot}\right) = 10^{14} cm \left(\frac{M_H}{10^8 M_\odot}\right)$$

where $M_\odot = 1$ solar mass $\approx 2 \times 10^{30}$ Kg, and the outer radius $r_2 \gg r_1$ is taken to be the point where the rotating disk can be distinguished from the ambient plasma. To determine the magnitude of the disk potential, let

$$B_z(r) = A r^{-1/2}; \text{ then } A = B_z(r) r^{1/2},$$

(4)

$$\begin{aligned}
 \text{and } V_{12} &= -\frac{1}{c} \int_{r_1}^{r_2} \left(\frac{GM_H}{r} \right)^{1/2} \frac{A}{r^{1/2}} dr \\
 &= -A \left(\frac{GM_H}{c^2} \right)^{1/2} \int_{r_1}^{r_2} \frac{dr}{r} \\
 &= -A \left(\frac{GM_H}{c^2} \right)^{1/2} \ln r \Big|_{r_1}^{r_2} = -A \left(\frac{GM_H}{c^2} \right)^{1/2} \ln \left(\frac{r_2}{r_1} \right)
 \end{aligned}$$

Now assume that A corresponds to the maximum field strength which occurs at $r = r_1$, so

$$A = B_z(r_1) r_1^{1/2} = B_* \left(\frac{6GM_H}{c^2} \right)^{1/2}$$

where $B_* = B_z(r_1)$ (maximum field strength).

Thus $V_{12} \approx -\sqrt{6} B_* \left(\frac{GM_H}{c^2} \right) \ln \left(\frac{r_2}{r_1} \right)$. Shifting to more practical MKS units (volts),

$$\begin{aligned}
 V_{12} &\approx -\frac{\sqrt{6}}{c} B_* GM_H \ln \left(\frac{r_2}{r_1} \right) \\
 &\approx \frac{-\sqrt{6} B_*}{3 \times 10^8 \text{ m/s}} \left(\frac{10^3 \text{ gauss}}{\text{K gauss}} \right) \left(6.672 \times 10^{-11} \frac{\text{m}^3}{\text{Kg} \cdot \text{s}^2} \right) \\
 &\quad \times \left(2 \times 10^{38} \text{ Kg} \right) \left(\frac{M_H}{10^8 M_\odot} \right) \ln \left(\frac{r_2}{r_1} \right) \\
 &\approx -10^{19} \left(\frac{B_*}{\text{K gauss}} \right) \left(\frac{M_H}{10^8 M_\odot} \right) \ln \left(\frac{r_2}{r_1} \right) \text{ Volts}
 \end{aligned}$$

(5)

For the assumed values $B_z(r_1) = B_* \approx 10^3$ gauss,
 $M_H \approx 10^8 M_\odot$, and $r_2 \sim 10 r_1$,

$$eV_{12} \approx -2 \times 10^{19} \text{ eV}$$

To find the magnetic flux through the disk,

$$\Phi = \int_{\text{disk}} \vec{B} \cdot d\vec{S} = \int_{r_1}^{r_2} B_z(r) 2\pi r dr$$

$$= 2\pi A \int_{r_1}^{r_2} r^{1/2} dr = \frac{2\pi A}{3/2} (r_2^{3/2} - r_1^{3/2})$$

Using $r_1 \approx 10^{14} \text{ cm} \left(\frac{M_H}{10^8 M_\odot} \right)$ and multiplying and

dividing by $r_1^{3/2}$ gives

$$\Phi \sim \frac{4\pi}{3} B_* \left(\frac{10^{14} \text{ cm } M_H}{10^8 M_\odot} \right)^{1/2} \left(\frac{10^{14} \text{ cm } M_H}{10^8 M_\odot} \right)^{3/2} \left(\frac{r_2^{3/2}}{r_1^{3/2}} - 1 \right)^{\text{small}}$$

$$\sim 4 \times 10^{31} \left(\frac{B_*}{\text{Kgauss}} \right) \left(\frac{M_H}{10^8 M_\odot} \right)^2 \left(\frac{r_2}{r_1} \right)^{3/2} \text{ gauss-cm}^2$$

To set the time scale for disk fluctuations,
use the orbital period at the inner radius,

$$\omega \sim \frac{U_\phi(r_1)}{r_1} = \left(\frac{GM_H}{r_1^3} \right)^{1/2}$$

(6)

$$\text{Since } r_1 \approx 6 \frac{GM_H}{c^2} \approx 10^{14} \text{ cm} \left(\frac{M_H}{10^8 M_\odot} \right),$$

$$GM_H \approx 6 \times 10^{14} \text{ cm} \left(\frac{M_H}{10^8 M_\odot} \right) c^2,$$

$$\text{and } \omega(r_1) \approx \frac{\left[6 \times 10^{14} \text{ cm} \left(\frac{M_H}{10^8 M_\odot} \right) c^2 \right]^{1/2}}{\left[10^{14} \text{ cm} \left(\frac{M_H}{10^8 M_\odot} \right) \right]^{3/2}}$$

$$\approx (2.45 \times 10^{-14}) c \left(\frac{M_H}{10^8 M_\odot} \right)$$

$$\tau (\text{period}) \approx \frac{2\pi}{\omega} \approx 10^4 \left(\frac{M_H}{10^8 M_\odot} \right) \text{ seconds}$$

For large-scale changes in the established magnetic field, the time scale can be estimated by $\tau_B = L/R$ (inductance over resistance).

Since the disk is very large and highly conducting, the lifetime of the magnetic field is expected to be the same as the lifetime of the disk itself. The axisymmetric field may, however, be occasionally upset by turbulent fluid motions within the disk.

(7)

The disk potential provides the mechanism for particle acceleration. Assume that the regions just above and below the disk contain no material, while the area outside the cylinder $r = r_2$ contains electrically neutral plasma. The disk potential causes a vacuum field above and below the disk, shown here in cross-section,

$$\nabla^2 V = 0, \text{ where } V(r, 0) \text{ is given,}$$

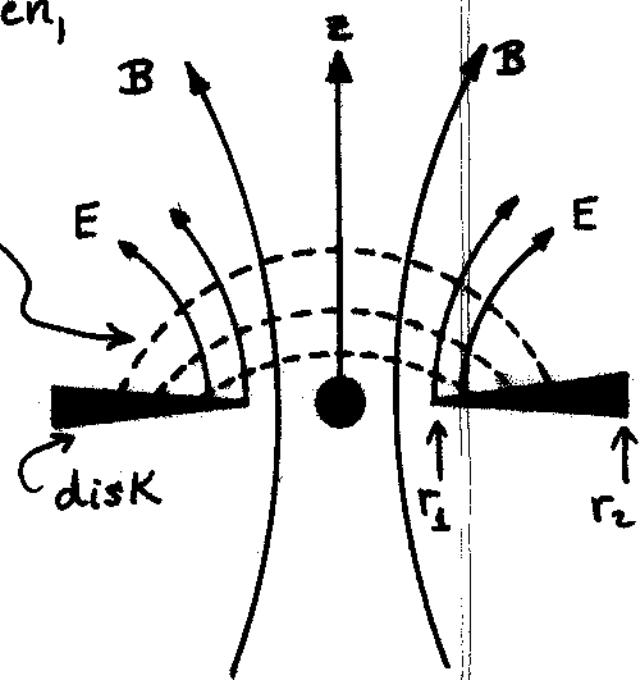
$$V(r_2, 0) = V(r, \infty) = 0$$

The equipotential surfaces are intersected at right angles by the lines of electric force. The

order of magnitude of the electric field near the inner edge of the

disk can be estimated as $V_{12}/r_1 \sim 10^7 \text{ volts/m.}$

The lines of magnetic flux embedded in the disk are expected to reconnect in the region $r < r_1$, so that a flux buildup does not occur through the $z=0$ surface. The trapped lines are assumed arbitrarily to point in the negative z direction.



(8)

The reconnection lines thus point in the positive z direction, as drawn.

The force on a proton by the electric field is

$$eE \sim (1.6 \times 10^{-19} \text{ coul})(10^7 \text{ V/m}) \sim 10^{12} \text{ nt},$$

while the force due to the gravitational pull of the black hole is

$$\frac{GM_H m_p}{r_1^2} \sim \left(\frac{6.7 \times 10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2}{10^{24} \text{ m}^2} \right) (2 \times 10^{38} \text{ kg})(1.7 \times 10^{-27} \text{ kg})$$

$$\sim 2 \times 10^{-23} \text{ nt}$$

The electric force thus exceeds the gravitational force by a factor of about 10^{11} . The electric field is smaller than the poloidal magnetic field, since

$$\frac{E^2}{B_z^2} \sim \frac{4\phi}{c^2} = \frac{GM_H}{c^2 r_1} \sim \frac{1}{6}$$

Protons are accelerated off the inner disk surface with energies $\sim 10^{19} \text{ eV}$ to form two oppositely-directed highly-relativistic beams. Charge buildup is prevented by a slower electron current in the region $r > r_2$ and $|z| \gg r_2$.

(9)

Each proton beam is space-charge limited, with current $I = \kappa C V_{12}$, where κ depends on the geometry and is on the order of unity.

Then (in the cgs system),

$$I = \kappa \left(3 \times 10^{10} \text{ cm/s} \right) \left(\frac{1}{300} \right) \left(10^{19} \text{ statvolts} \right) \left(\frac{B_*}{\text{Kgauss}} \right) \left(\frac{M_H}{10^8 M_\odot} \right) \ln \left(\frac{r_2}{r_1} \right)$$

$$\cong 10^{27} \kappa \left(\frac{B_*}{\text{Kgauss}} \right) \left(\frac{M_H}{10^8 M_\odot} \right) \ln \left(\frac{r_2}{r_1} \right) \text{ statamps}$$

and using the conversion $1 \text{ amp} = 3 \times 10^9 \text{ statamps}$,

$$I \cong 3 \times 10^{17} \kappa \left(\frac{B_*}{\text{Kgauss}} \right) \left(\frac{M_H}{10^8 M_\odot} \right) \ln \left(\frac{r_2}{r_1} \right) \text{ amps}$$

The power in each beam is thus

$$P_{\text{beam}} \cong I V_{12} \cong 3 \times 10^{43} \kappa \left(\frac{B_*}{\text{Kgauss}} \right)^2 \left(\frac{M_H}{10^8 M_\odot} \right)^2 \left(\ln \frac{r_2}{r_1} \right)^2 \text{ ergs/s}$$

The driving force is supplied by the gravitational force of the black hole which powers the accretion disk. Some disk angular momentum is lost in supporting the self-electric field of the beams, and some is carried out by the poloidal magnetic field.

(10)

The limiting Lorentz factor is obtained from the maximum accelerating potential,

$$\gamma_0 = \frac{e V_{12}}{m_p c^2} \sim 10^{10} \left(\frac{B_*}{\text{Kgauss}} \right) \left(\frac{M_H}{10^8 M_\odot} \right) \ln \left(\frac{r_2}{r_1} \right)$$

The density of protons in the beams can be estimated from the current by assuming the protons are rising from the total disk area,

$$n_{\text{beam}} \sim \frac{I}{eCA} \sim \frac{(3 \times 10^{17} \text{ amp}) K \left(\frac{B_*}{\text{Kgauss}} \right) \left(\frac{M_H}{10^8 M_\odot} \right) \ln \left(\frac{r_2}{r_1} \right)}{e\pi (r_2^2 - r_1^2) c}$$

Using the conversion factor from amperes to the number of protons, ie; $1 \text{ coul/s} = 6.24 \times 10^{18} \text{ e/s}$,

$$n_{\text{beam}} \sim \frac{(3 \times 10^{17} \text{ coul/s}) (6.24 \times 10^{18} \text{ e/coul}) K \left(\frac{B_*}{\text{Kgauss}} \right) \left(\frac{M_H}{10^8 M_\odot} \right) \ln \left(\frac{r_2}{r_1} \right)}{e\pi \left(\frac{r_2^2}{r_1^2} - 1 \right) r_1^2 (3 \times 10^{10} \text{ cm/s})}$$

and using $r_1 \sim 10^{14} \text{ cm} \left(\frac{M_H}{10^8 M_\odot} \right)$,

$$n_{\text{beam}} \sim 2 \times 10^{-3} K \left(\frac{r_1}{r_2} \right)^2 \ln \left(\frac{r_2}{r_1} \right) \left(\frac{B_*}{\text{Kgauss}} \right) \left(\frac{10^8 M_\odot}{M_H} \right) \text{ cm}^{-3}$$

The ratio of the energy density of the proton beam to the energy density of the magnetic field can now be found using the above equations,

(11)

where the ratio is denoted by β_0 near the disk,

$$\beta_0 = \frac{8\pi n_{beam} T_0 m_p c^2}{B_z^2}$$

$$\sim \frac{8\pi (2 \times 10^7) \times}{B_0^2} \left(\frac{B_0}{\text{Kgauss}} \right)^2 \left(\frac{r_1}{r_2} \right)^2 \left(\ln \frac{r_2}{r_1} \right)^2 m_p c^2$$

$$\sim 0.7 \times \left(\frac{r_1}{r_2} \right)^2 \left(\ln \frac{r_2}{r_1} \right)^2$$

For $r_2 \gg r_1$, $\beta_0 \ll 1$ and the magnetic field energy density is much greater than the beam energy density. The accelerated protons tend to follow the magnetic field lines in a force-free configuration. A small value of β_0 lends magneto hydrodynamic stability to beams which are neutralized by plasma (3).

A small toroidal self-field surrounds each beam, but this is small if $\beta_0 \ll 1$. The field opposes the effect of charge repulsion and tends to collimate the beam (pinch effect).

(12)

Away from the disk, the ratio of the beam to field energy densities (β) may be very high. Beam divergence then occurs as a result of ballistic motion, possibly resulting in radiation due to the track curvature. The resultant photons would have an energy about

$$\frac{3}{2} \gamma^3 \frac{\pi c}{p} \sim 3 \times 10^{10} \left(\frac{\gamma}{10^{10}} \right)^3 \left(\frac{10^{15} \text{ cm}}{p} \right) \text{ eV}$$

where p is the radius of curvature of the track. At some distance $|z| > r_2$ the proton beam begins to interact with the ambient plasma via the two-stream interaction and produces a relativistic electron beam. The two beams thus formed are collimated in the direction of the original beam axis, but they rapidly lose energy in the form of incoherent synchrotron emission. The final result is two large areas of radio emission extended axially along the symmetry axis of the supermassive black hole system.

(13)

Some protons may accelerate to great distances without collision, ie, $|z| \gg r_2$ ($1\text{ pc} \rightarrow 1\text{ Mpc}$). These protons will escape the system without large synchrotron losses provided the ambient magnetic field strengths are small, $\sim 10^{-3}$ gauss. The production of electron-positron pairs off low energy photons would become significant if the photon luminosity is high, $> 10^{46}\text{ erg/s}$. Dynamo sources near the Earth may thus be responsible for an upper portion of the cosmic ray spectrum ($10^{16} \rightarrow 10^{20}\text{ eV}$).

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