Hawaii DUMAND
Note 81-11
4/28/81

To: MDC's (not MCP's)

From: John Learned

Subject: Directly Produced μ's in the Monte Carlo Program

I've added a little function to the routine that generates cosmic ray muons at depth with the appropriate angular distribution (see function FMU in J.L.'s note 81-8, and in the Monte Carlo, [VJS.DUM]DUMC3, it is called RFMU). The function added multiplies the muon rate at depth, D, by

$$F' \approx F\left[1 + R_{DIR}\cdot\frac{h'}{6300}^{3.0}\cdot\frac{h'\cos\theta^{*} + D_O}{h' + D_O}\right]$$

where

$$h' = (D + D_{AIR})\sec\theta \qquad \text{(under ocean slant depth).}$$

The new parameters are 3.0, 6300 m, and RDIR. The latter represents the ratio of direct production to ordinary decay at 10 TeV. (Under optimistic assumptions it is 1.0, pessimistic = 0.0).

<u>Derivation</u>:

In Fig.1 (from Elbert, Gaisser, and Stanev, D'80 <u>I</u> 224) one sees the variation of direct production of C.R. μ's with energy. The optimistic model #1 crosses ordinary decay (π and k) μ's at 10 TeV. The energy scaling is seen to be quite straight on the log-log plot and close to

$$\frac{\text{Flux direct}}{\text{Flux ordinary}} = \left(\frac{E}{10\ \text{TeV}}\right)^{1.2} \tag{1}$$

Using the rather good Miyake energy formula for μ spectra at the surface (see his rapporteur talk in 13 ICRC, Denver 1973, <u>5</u> 3638)

$$I_\mu(>E) = \frac{174}{5E\cos\theta + 400}\,(5E + 10\sec\theta)^{-1.57}\,\frac{E + 15}{E + 10 + 5\sec\theta} \tag{2}$$

We can approximate the direct flux as

$$I_{\mu D}(>E) = I_\mu(>E)\cdot\left(\frac{E}{10\ \text{TeV}}\right)^{1.2}. \tag{3}$$

Miyake's underground formula is

$$I(>h) = \frac{I_o}{h'\cos\theta^* + d_o} \cdot (h')^{-\alpha} \cdot \frac{h + d_1 + d_2}{h + d_1 + d_2\sec\theta^*}\, e^{-\beta h'} \tag{4}$$

where $h' = (d + d_{AIR})\sec\theta$ = slant depth under ocean, $h = d\sec\theta$

$\sec\theta^*$ = zenith angle at top of atmosphere

d_{AIR}, d_1, d_2, α, β, I_o = constants

(This is the function used in FMU, now). Note that the angular enhancement (the famous "$\sec\theta$ effect") enters in the first term; that is we would get a $\sec\theta$ dependence under a hemispherical mountain of height d. We are under a flat ocean so the enhancement is cancelled by the increase in slant depth. If we have a direct flux component it will have then a similar form, but no $\cos\theta^*$ and no third term (which also traces back to π & K decay, and which is unimportant at our depth anyway). Thus

$$I_d(>h) = \frac{I_o}{h' + d_o} \cdot (h')^{-\alpha}\, e^{-\beta h'} \cdot R_{DIR} \cdot f(h) \tag{5}$$

and hence

$$I_{TOT}(>h) = I(>h) \cdot \left[1 + R_{DIR} \cdot f(h) \cdot \frac{h'\cos\theta^* + d_o}{h' + d_o} \cdot \frac{h + d_1 + d_2\sec\theta^*}{h + d_1 + d_2}\right] \tag{6}$$

Now $R_{DIR} \cdot f(h)$ is the function that translates the energy dependence of direct production into depth dependence. By analogy with equations (1) and (2) we can see that

$$f(h) \approx \left(\frac{h'}{6300\ \text{m}}\right)^{\alpha'} \tag{7}$$

The distance 6300 m corresponds (roughly) to the 10 TeV range but more importantly fits the graph given by Elbert at D'80, quoted from his transparencies by Learned (D'80 I, 257 Fig. 10) and reproduced here as Figure 2. Varying RDIR between 0. and 1.0 will then roughly span the various model predictions. Looking at Elbert's graph we get $\alpha' \approx 3.0$.

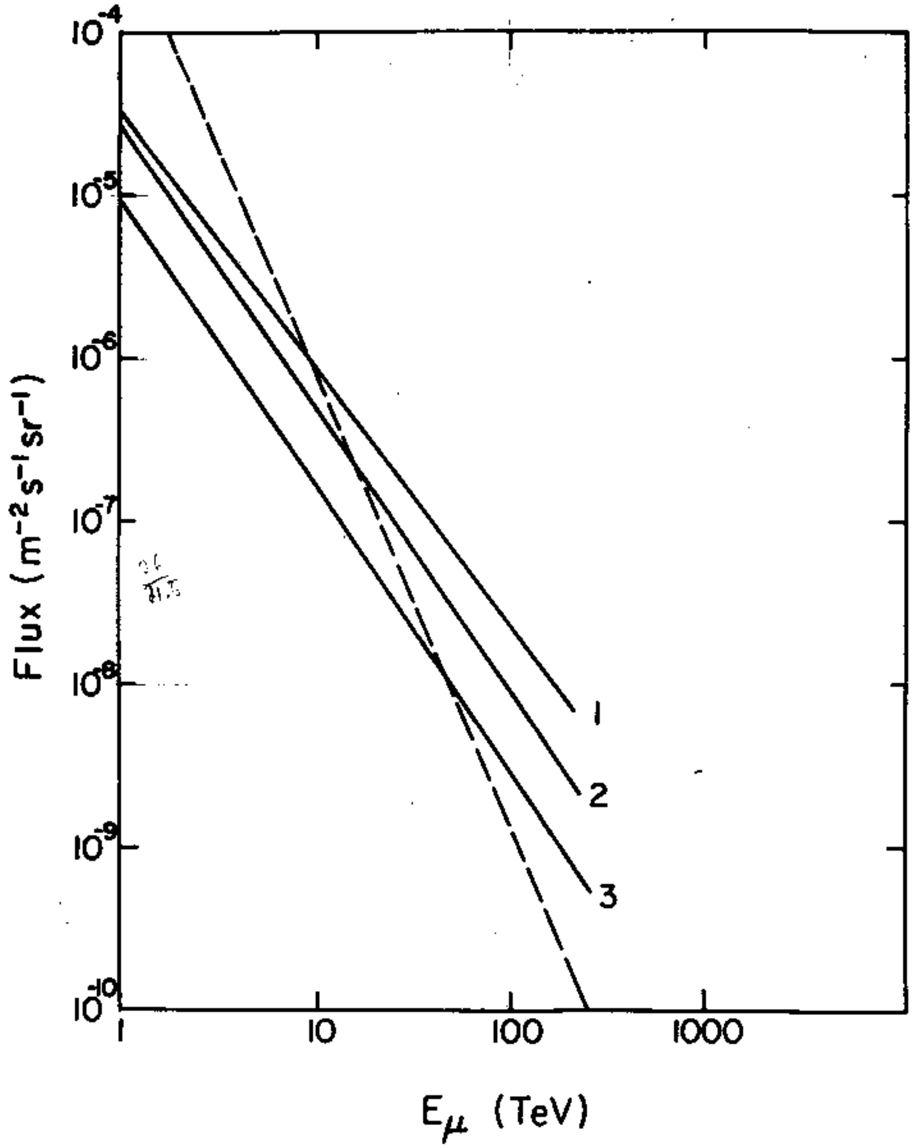

Fig. 1: Comparison of integral muon spectra from ordinary muons (dashed line) to prompt muon production by the models 1-3 (solid lines) described in the text.

(FROM ELBERT D'80)

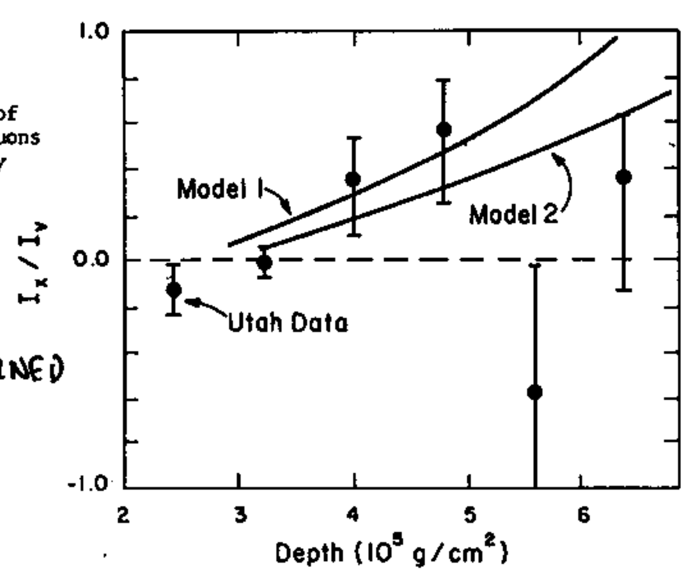

Fig. 2 Fraction of directly-produced muons relative to ordinary pion and kaon decay vertical muons as a function of depth. Several Utah data points and model calculations are shown (Ref. 19).

(FROM LEARNED D'80)