

DUMAND—Deep Underwater Muon and Neutrino Detection

Steering Committee, 1980

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Hawaii DUMAND Center
University of Hawaii
2505 Correa Rd.
Honolulu, HI 96822
808 - 948-7391

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~~Hawaii DUMAND Memo 81-8~~

FROM: John Learned

SUBJECT: Rock to Water Range Conversion for Cosmic Ray Muons

Taking the energy loss rate as

$$-\frac{dE}{dx} = a + bE, \quad (1)$$

and without worrying about Landau fluctuations in range and detailed energy dependence of the parameters ($\ln E$ terms), we can write the range for a muon as:

$$R = \int_0^E \frac{dE}{a + bE} = \frac{1}{b} \ln \left[1 + \frac{b}{a} E \right], \quad (2)$$

or inverting:

$$E = \frac{a}{b} [e^{bR} - 1]. \quad (3)$$

Now if R is the range in "standard rock" (see Table I) and R' the range in sea water, we can write the range conversion (following Wright, Ref. 2) as:

$$F \equiv \frac{R}{R'} = \frac{1}{bR'} \ln \frac{a'b'}{a'b} [e^{b'R'} - 1] + 1 \quad (4)$$

using the facts that:

$$a \propto \frac{Z}{A} \quad (5)$$

$$\text{and } b \propto \frac{Z^2}{A}, \quad (6)$$

we can use the data in Table I to construct a range conversion if we supply a value for b. This coefficient (which is due to Bremsstrahlung, pair production and nuclear interactions) is not very well known ($\sim \pm 10\%$) and is in actuality a somewhat complicated function of the logarithmic terms (because of screening). Thus I've taken arbitrary values which give a good first-order correction to get from water depth to standard rock via the form of equation 4. Useful values turn out to be:

$$\frac{a'}{a} = 1.16 \quad (7)$$

$$b = \frac{1}{3} \text{ km}$$

$$b' = \frac{1}{5} \text{ km.}$$

Thus $R = 3 \text{ km} \ln\{1.933[e^{R/5 \text{ km}} - 1] + 1\}.$ (8)

This function reproduces Wright's detailed calculations (Ref. 2) within $< 1\%$ over the range from 100 to 5000 m, as shown in Figure 1 & Table II. This function has been programmed into a subroutine as shown in attached listing. It may be copied from [JGL]DEEPCON.FOR or it exists in a package in MURATE.FOR. One may excise the driver and have a function FMU(D,TH) that returns muon flux ($1/\text{m}^2 \text{ sec}$) when called with water depth (in meters) and angle (in radians). Be sure to copy subroutines STAR (which converts zenith angle to zenith angle at production) and DEPROC (which is the aforementioned depth conversion). Furthermore file TESTMUANG.FOR contains a program that uses the CERN Library routine FUNRAN (V151) to randomly generate angles according to this distribution function (FUM). TESTMUANG.COM contains the command file with appropriate linking. Sample output is attached in Appendices I and II, and a comparison with data in Figure 3.

REFERENCES

1. S. Higashi, T. Kitamura, S. Miyamoto, Y. Mishima, T. Takahashi, and Y. Watase, NC 43A, 334 (1966).
2. A. G. Wright, 13 ICRC, Denver 1973 3, 1709.

TABLE I

Effective Values of Parameters
for Energy Loss Rate Scaling

	<u>Z</u>	<u>A</u>	<u>"Z/A"</u>	<u>"Z²/A"</u>	<u>p</u>
"Standard Rock"	11.0	22.0	.50	5.5	2.65
Sea Water	7.433	14.787	.553	3.26	1.027

TABLE II

Rock to Water Range Conversion (see text)

<u>Water (km)</u>	<u>Range</u>	<u>Ratio</u>
	<u>Rock (hg/cm²)</u>	<u>F</u>
.01	11.6	1.159
.10	115	1.149
.20	228	1.139
.50	555	1.111
1.0	1069	1.069
2.0	2005	1.002
3.0	2854	.951
4.0	3644	.911
5.0	4391	.878
6.0	5107	.851

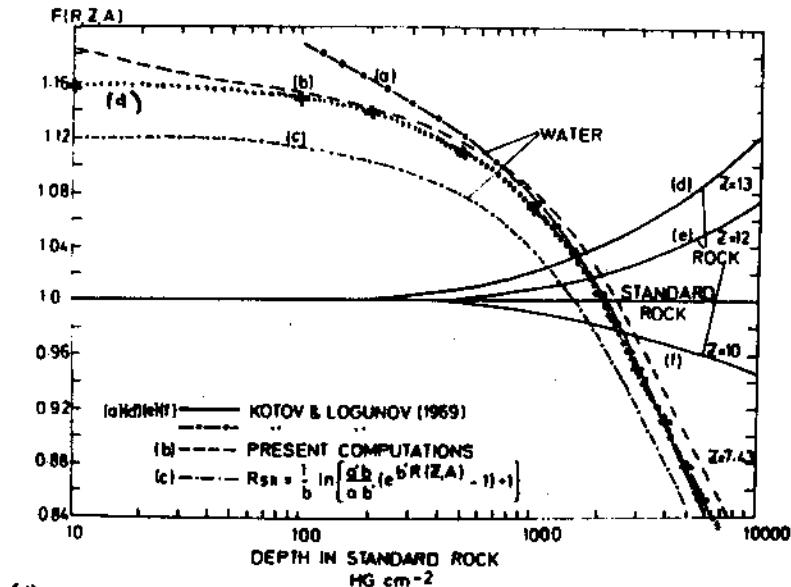
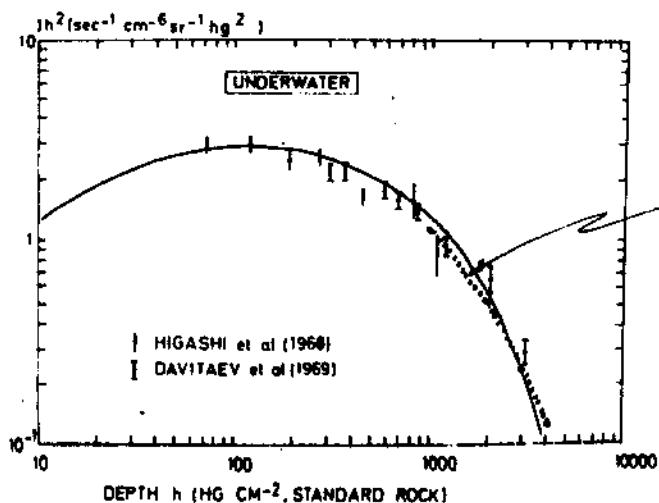
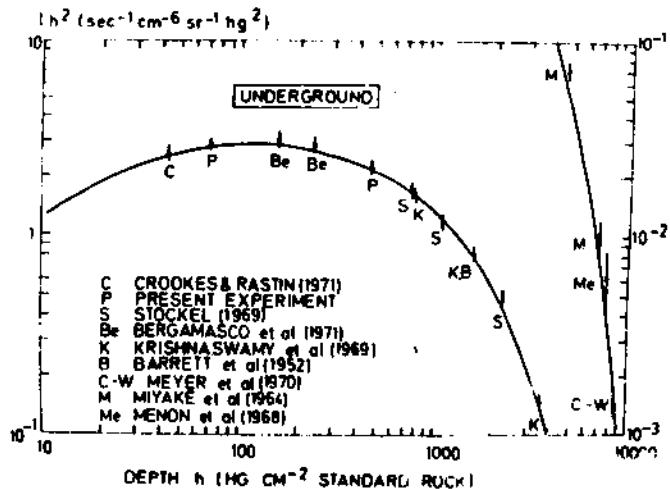


Figure 4 The conversion factor $F(R, Z, A)$ for water and rock to standard rock. (REF. 2)

(d) SAME AS (c)
BUT $a'/a = 1.16$
 $\pm b' = 1/5 \text{ km}, b = 1/3 \text{ km}$

Figure 3 Intensity-depth curve for rock. All depths measured from the atmosphere. (REF. 2)



MURATE VALUES
WITH MIYAKE PARAMETERS
& DEPTH CONVERSION AS
DISCUSSED IN TEXT

Figure 3 Intensity-depth curve for water. All depths measured from the top of the atmosphere. (REF. 2)

APPENDIX I

FILE [JGL] MURATE.FOR

DRIVER ROUTINE

```

00100      0001      PROGRAM MURATE
00200      C CALCULATES THE UNDERWATER MUON INTESITY AS A FUNCTION OF
00300      C ANGLE FOR VARIOUS ANGLES AND DEPTHS. USES MIYAKES FORM.
00400      C INTEGRATES OVER ANGLES IN COARSE ZENITH ANGLE BINS.
00500      C FD IS MU FLUX AT DEPTH D/SEC CROSSING A 1M2 SPHERE
00600      C RD IS THE TOTAL RATE THROUH A DETECTOR OF GIVEN AREA AT D
00700      C FDT IS THE FLUX OF MUONS/M2 IN GIVEN DELTA THETA BIN AT D
00800      C RDT IS THE SAME FOR DETECTOR OF GIVEN AREA
00900      C DI ARE THE DEPTHS AT WHICH FLUX WAS CALCULATED
01000      C TD ARE THE BEGINNING VALUES OF THE THETA BINS IN DEGREES.
01100      0002      DIMENSION FD(20), RD(20), FDT(20,20), RDT(20,20), DI(20), TD(20)
01150      0003      DIMENSION DRI(20)
01200      0004      DATA PI/3, 14159/
01300      0005      TWOPI=2.*PI
01400      C THETA MIN IN RADIANs FOR CALCULATION
01500      C THETA IS LOCAL ZENITH ANGLE
01600      0006      THMIN=0.0
01700      C THETA MAXIMUM ALSO IN RADIANs
01800      0007      THMAX=PI/2.
01900      C MINIMUM DEPTH FOR CALCULATION IN METERS WATER
02000      0008      DMIN=1000.
02100      C MAXIMUM DEPTH
02200      0009      DMAX=5000.
02300      C NUMBER OF THETA STEPS IN EACH BIN FOR INTEGRAL
02400      0010      NSTH=100
02500      C NUMBER OF THETA BINS
02600      0011      NTHBIN=6
02700      C NUMBER OF DEPTHS
02800      0012      ND=9
02900      C AREA IN M2 OF DETECTOR. COULD PUT FUNCTION OF THETA INSTEAD.
03000      0013      AREA=700.
03100      C DEPTH STEP SIZE
03200      0014      DELD=(DMAX-DMIN)/(ND-1)
03300      C THETA BIN STEP SIZE
03400      0015      DELTH=(THMAX-THMIN)/NTHBIN
03500      C THETA INCREMENT FOR INTEGRAL
03600      0016      DELTHS=DELTH/NSTH
03700      C
03800      C "NOW STEP THROUGH THE DEPTHS"
03900      C
04000      0017      DO 2000 ID=1, ND
04100      C DEPTH D IN METERS
04200      0018      D=DMIN+DELD*(ID-1)
04300      C SAVE VALUE FOR PRINTOUT
04400      0019      D1(ID)=D
04500      C CALCULATE EQUIVALENT DEPTH IN STANDARD ROCK
04550      0020      DRI(ID)=DEPROC(D)
04600      C ZERO THE TOTAL FLUX
04700      0021      FD(ID)=0.0
04800      C ZERO THE TOTAL RATE
04900      0022      RD(ID)=0.0
05000      C
05100      C NOW STEP THROUGH THETA BINS
05200      C
05300      0023      DO 1000 JT=1, NTHBIN
05400      C ZERO THE DEPTH-ANGLE DISTRIBUTION

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05500 0024      FDT(ID,JT)=0.0
05600 0025      C VALUE OF THETA AT START OF BIN
05700 0025      TH=THMIN+DELTH*(JT-1)
05800 0026      C SAVE THE THETA VALUE IN DEGREES
05900 0026      TD(JT)=TH*180./PI
06000
06100
06200
06300 0027      C INTEGRATE OVER THETA BIN
06400
06500 0028      DO 900 JTS=1,NSTH
06600 0028      C LOCAL THETA VALUE IN MIDDLE OF SMALL INCREMENT
06700 0029      THS=TH+DELTHS*(JTS-0.5)
06800 0030      C AT LAST CALL THE FLUX FUNCTION FMU AND MULTIPLY BY SOLID ANGLE
06900 0031      DELF=FMU(D,THS)*TWOPI*SIN(THS)*DELTHS
07000 0032      FDT(ID,JT)=FDT(ID,JT)+DELF
07100
07200
07300
07400 0033      RDT(ID,JT)=RDT(ID,JT)+DELF*AREA
07500 0034      900 CONTINUE
07600
07700 0035      C ADD TO TOTAL FLUX
07800
07900
08000
08100 0036      FD(ID)=FD(ID)+FDT(ID,JT)
08200
08300
08400
08500
08600
08700
08800
08900 0037      C AND ADD TO TOTAL RATE AT THAT DEPTH
09000 0038      RD(ID)=RD(ID)+RDT(ID,JT)
09100
09150
09200
09300 0039      1000 CONTINUE
09400
09500 0040      C
09600 0041      C
09700 0042      C
09800 0043      WRITE(6,4500) (TD(JT),JT=1,NTHBIN)
09900 0044      4500 FORMAT(1H1,'RESULTS OF MUON DEPTH INTENSITY CALCULATION'///
09900 0044      +'5X,'DEPTH, M',10X,'RATE/M2-SEC',20X,'ANGLES IN DEGREES',
09900 0044      +'5X,'ROCK DEPTH IN MG/CM2'///
09900 0044      +'13X,'TOTAL',10F10.1///)
10000 0045      DO 5000 ID=1,ND
10100 0046      WRITE(6,4900) DI(ID),FD(ID),(FDT(ID,JI),JT=1,NTHBIN),DRD(ID)
10200 0047      4900 FORMAT(1X,F10.0,1E10.3,F10.1//)
10300 0048      5000 CONTINUE
10400 0049      5500 WRITE(6,5500) (TD(JT),JT=1,NTHBIN)
10400 0050      5500 FORMAT(///40X,'MOONS/SEC'///13X,'TOTAL',10F10.1//)
10400 0050      DO 6000 ID=1,ND
10400 0050      WRITE(6,5900) DI(ID),RD(ID),(RDT(ID,JI),JT=1,NTHBIN)
10400 0050      5900 FORMAT(1X,F10.0,1E10.3//)
10400 0050      6000 CONTINUE
10400 0050      STOP
10400 0050      END

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```

10500      0001    C.....FUNCTION FMU(D,TH).....
10600      0001    C FUNCTION CALCULATES THE UNDERWATER MUON FLUX USING MIYAKE
10700      0001    C FORMULA. CALL WITH DEPTH OF WATER D IN METERS AND ANGLE
10800      0001    C THETA TH IN RADIANS. RETURNS FLUX OF MUONS/M2-SEC-SR.
10900      0002    C COMMON /FPARAM/F0,DAIR,D0,D1,D2,ALPHA,BETA
11000      0002    C THESE CONSTANTS ARE FOR MIYAKE'S ORIGINAL VERSION WHICH
11100      0002    C MAY BE FOUND IN 13ICRC, DENVER, 1973, VOLS, 3638, AND
11200      0002    C EARLIER IN J. PHYS. SOC. JAPAN VOL. 18, 1093(1963). A
11300      0002    C QUICK SUMMARY OF FORMULAE IS IN LEARNED PAPER XII, '75.
11400      0003    C DATA F0/1.74E6/,DAIR/10./,D0/400./,D1/50./,
11500      0003    C +D2/25./,ALPHA/1.53/,BETA/8.0E-4/
11600      0004    C DATA PIBY2/1.5708/
11700      0004    C THIS FORM OF THE CONSTANTS IS FROM THE MONT BLANC DATA
11800      0004    C AS FOUND IN SHELDON ET AL, PRD17, 114(1978). NOTE THAT
11900      0004    C THEIR FIT APPLIES TO VERTICAL DATA FROM 500 TO 3400 M
12000      0004    C AND AT 80DEG TO 5000 M, AND THEY DID NOT INCLUDE ANY
12100      0004    C OTHER DATA. THEIR FIT EXTRAPOLATES POORLY OUTSIDE THIS
12200      0004    C RANGE.
12300      0004    C DATA F0/2.06E6/,DAIR/10./,D0/348./,D1/50./,
12400      0004    C +D2/25./,ALPHA/1.60/,BETA/6.15E-4/
12500      0004    C THESE ARE TONY WRIGHT'S SET OF CONSTANTS AS GIVEN IN THE
12600      0004    C 13ICRC, DENVER, 1973, VOL3, 1709
12700      0004    C DATA F0/7.5E4/,DAIR/10./,D0/35.0/,D1/50./,
12800      0004    C +D2/25./,ALPHA/1.13/,BETA/8.25E-4/
12900      0004    C AND THESE ARE DAVIS AND LEARNED, PRD8, 7(1973)
13000      0004    C DATA F0/1.64E6/,DAIR/10./,D0/400./,D1/50./,
13100      0004    C +D2/25./,ALPHA/1.53/,BETA/6.5E-4/
13200      0004    C AND FINALLY THESE ARE DON GROOM'S UNPUBLISHED VALUES
13300      0004    C GIVEN TO ME IN A PRIVATE MEMO IN MAY 1977
13400      0004    C DATA F0/1.142E6/,DAIR/10./,D0/844/,D1/50./,
13500      0004    C +D2/25./,ALPHA/1.44/,BETA/7.935E-4/
13600      0005    C FMU=0.
13700      0006    IF(TH.GE.PIBY2) RETURN
13800      0007    IF(TH.LT.0.0) RETURN
13900      0007    CUST=COS(TH)
14000      0008    CALL STAR(D,CUST,CTST,STST)
14100      0009    SECT=1./COST
14200      0010    DR=DEPROC(D)
14300      0011    DR=DR*SECT
14400      0012    HP=(DR+DAIR)*SECT
14500      0013    FF=F0/(HP*CIST+D0)
14600      0014    FF=FF*(HP**-ALPHA)
14700      0015    FF=FF*(H+D1+D2)/(H+D1+D2*SECT)
14800      0016    FF=FF*EXP(-BETA*HP)
14900      0017    FMU=FF
15000      0018    RETURN
15100      0019    END

15300      0001    C SUBROUTINE TO CONVERT COS ZENITH ANGLE THETA (CT) AT DEPTH (D) ...
15400      0001    C TO COS AND SIN ZENITH ANGLE AT PRODUCTION, THETA STAR (CTST,SIST)
15500      0001    C SUBROUTINE STAR(D,CT,CTST,STST)
15600      0002    C RE IS RADIUS OF EARTH IN METERS
15700      0002    C AT IS MEAN PRODUCTION ALTITUDE ALSO IN METERS
15800      0002    C THE DEPTH BELOW THE SURFACE IS IN METERS AS WELL.
15900      0003    CTST=1.0
16000      0003    STST=0.0
16100      0004    IF(CT.GE.1.0.OR.CT.LE.-1.0) RETURN
16200      0005    ST=SQRT(1.0-CT**2)
16300      0006    STSI=ST*(RE-D)/(RE+AT)
16400      0007    CTSI=SQRT(1.0-STSI)
16500      0008    RETURN
16600      0009    END

16900      0001    FUNCTION DEPROC(DEPWAT)
17000      0001    C Converts seawater depth in meters to 'standard rock' depth
17100      0001    C in kg/cm2. Approximation used is described in NUC Memo 81-12.
17200      0001    C Precision is within 1% from 100 to 6000 meters as compared
17300      0001    C to Wright's calculations (13ICRC, Denver 1973, 3, 1709)
17400      0002    DATA APOA/1.16/,B/0.000333/,BP/0.0002/
17500      0003    CAPOA*B/BP
17600      0004    RP=DEPWAT
17700      0005    F=ALOG(C*(EXP(RP*BP)-1.0)+1.0)/B
17800      0006    DEPROC=R
17900      0007    RETURN
18000      0008    END

```

RESULTS OF MUNICIPAL DEBT INVESTIGATION

PATE

$$\text{AREA} = 750 \text{ m}^2 \text{ (isotropic)}$$

APPENDIX I
FILE TEST MUANG.FOR

AII-1

DRIVER ROUTINE

```
00100 0001      COMMON //HMEMOR(2000)
00200 0002      DIMENSION FSPACE(100)
00300 0003      EXTERNAL RFMU
00400 0004      COMMON /DEPTH/DM
00500 0005      DATA THRLO/0.0/,THRHI/1.571/,ND/9/,DMIN/1000./,DELD/500./,
00600               +NEVT/1000/
00700 0006      CALL HOUTPU(3)
00800 0007      DO 500 ID=1,ND
00850 0008      CALL HBOOK1(ID,'ANGULAR DISTSS',90,0.,90.,1000)
00900 0009      DM=DMIN+(ID-1)*DELD
01000 0010      → CALL FUNPRE(RFMU,FSPACE,THRLO,THRHI)
01100 0011      DO 200 IEVT=1,NEVT
01200 0012      → CALL FUNRAN(FSPACE,THR)
01300 0013      THD=THR*180./3.14159
01400 0014      CALL HFILL(ID,THD,1.,1.)
01500 0015      200 CONTINUE
01600 0016      500 CONTINUE
01700 0017      CALL HISTDO
01800 0018      STOP
01900 0019      END
```

CALLER FUNCTION PASSES DEPTH & MULTIPLIES
IN SOLID ANGLE FACTOR SIN(θ)

```
10000 0001      FUNCTION RFMU(TH)
10100 0002      COMMON /DEPTH/DM
10200 0003      T=TH
10300 0004      D=DM
10400 0005      RFMU=SIN(T)*FMU(D,T)
10500 0006      RETURN
10600 0007      END
```

[ADD SUBROUTINES FMU, STAR & DEPROC & LINK]
[TO CERNLIB]

A II. 7

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HANDBOOK 10 =

DATE 24 MARCH

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RELATIVE NUMBER OF MOONS

Depth (m)	(LOGO MUONS)
0	10.0
200	4.0
400	2.5
600	1.8
800	1.2
1000	0.8

30

2

@ ZENITH ANGLE (DEGREES)

81-8

M STRING DISTRIBUTION LIST (PHONE #'S)

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