MULTIPLICATION OF EFFECTIVE DETECTOR VOLUME FOR FAST MUONS

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Consider a spherical detector of radius r_o. Let the range of a muon of energy E_μ be R_μ. What is the effective gain in detector volume as a function of energy?

Let the source density of muons be ρ. The contribution from an element of area dA of the thin shell of thickness dR is

$$d^2N = \rho dA dR \Omega,$$

where Ω is the solid angle within which muons must be emitted in order to be registered. It is given by

$$\Omega = \alpha \pi r_o^2/4\pi R^2,$$

where α represents the fraction of the projected area A_o of the detector πr_o^2, within which the muon track is long enough to be counted.

The element dA thus contributes

$$d^2N = \rho dA dR \; \alpha \pi r_o^2/4\pi r^2$$

We now integrate, first over the area of the shell of radius R, and then over R. The first integration gives

$$\int_A \rho dA dR . \alpha \pi r_o^2/4\pi R^2 = \rho dR \alpha \pi r_o^2$$

The second gives

$$\int_{r_o}^{R_\mu} \rho \alpha \pi r_o^2 dR$$

$$= \rho \alpha \pi r_o^2 (R_\mu - r_o)$$

This is the external contribution.

The volume gain is the ratio of the effective external volume to the internal volume, which is the ratio of the above volume to the detector volume:

$$\text{Gain} = 0.75 \; \alpha (R_\mu - r_o)/r_o$$

Table 1 shows the effective volume for muons of a given range, for detectors of several different radii, with the approximation $\alpha = 1$.

Table 1. Effective volume for muon detection, km^3

r_o(km)	A_o(km^2)	Muon range, km = 0.2	0.5	1.0	2.0	5.0
.05	.0079	.021	.053	.111	.230	.583
.1	.0314		.094	.212	.448	1.155
.2	.1257		.141	.377	.848	2.263
.5	.785				1.767	5.30

Since the effectiveness of detection thus is proportional to radius rather than area, five units, each of radius 0.1 km, will have the same muon detection efficiency as a single unit of radius 0.5 km. This saves a factor of 5 in total volume, assuming we are considering only detectors of equal thickness. Thus no detector need be thicker than the range required to measure the direction of the muon (or its energy, whichever is desired.)

Following is a table of mean muon range vs. energy.

$E\mu$(TeV)	Mean Range, km
0.10	0.325
.16	.486
.25	.745
.38	1.06
.62	1.60
1.00	2.29
1.55	3.10
2.50	4.15