

To: DUMAND Gang

From: John Learned

Subject: Attenuation of Cerenkov photons in the ocean:

- a) a new function
- b) incorporation of glass transparency and relative photo-cathode quantum efficiency
- c) analysis of existing data
- d) presentation of function coefficients for six different conditions

Abstract

A function has been found which will approximate the survival probability for Cerenkov photons versus distance in water. The attenuation of photons is not a simple exponential because of wavelength dependence of source, transmission and detection. Taking Cerenkov spectrum, glass transmissitivity, relative quantum efficiency, and various approximations of water attenuation into account a function is calculated which predicts effective numbers of photons per unit source track length versus observation distance. This function multiplied by the appropriate detector geometrical factors (detector projected area, $1/r$ or $1/r^2$, and absolute quantum times collection efficiency) gives expected photoelectron yields. The DUMAND data from Keahole is reviewed in contrast to new data from Scripps⁽¹⁾ on the "clearest ocean waters" and it is found that a) Bradner⁽²⁾ and Zaneveld's⁽³⁾ data are consistent at 1.5 km depth, b) Zaneveld's⁽³⁾ deep ocean data (4.1 km) is not as good as data reported in the new results, but c) conservatively, the new data⁽¹⁾ combined with Zaneveld's results give confidence that the water transparency is in fact better than we have heretofore assumed. More observations are obviously in

where b_b is the backscattering part of b . The distinction is not entirely trivial because our counters will receive some scattered light, molecular and particle, particularly in the far blue. For DUMAND, K is probably close to the appropriate quantity. Also plotted in figure 1 are data from Zaneveld⁽²⁾ and one point from Bradner⁽³⁾. The Zaneveld points are taken from the graphs in his DUMAND '80 paper.⁽²⁾ It is noteworthy that Zaneveld's data at 1.5 km depth is consistent with Bradner's point, even though they were taken at "slightly" different locations and at different times. Also Bradner's observation should be close to K while Zaneveld's should be nearer to c . One sees that Zaneveld's data from all three depths plotted shows an attenuation coefficient rising rapidly in the blue, much more rapidly than the clearest ocean values of Smith and Baker. Assuming the data correct it appears that scattering is significant in the Keahole basin making the attenuation coefficient 2 to 3 times worse than optimal in the 400-450 nm range. We will see the importance of this difference later.

II An "effective photon" survival function

The diffuse attenuation coefficient (K) shown in figure 1 (ref. 1) is seen to exhibit several features. First the bumps on the red side apparently coincide with harmonics of the O-H stretch in water (6th at ~514 nm, 5th at ~604 nm, and 4th at 743 nm, ref. 4). The wavelength dependence of the coefficient can be looked at as a sequence of increases $\propto \lambda^m$ ($m \sim 10$), approaching the resonance from the blue side, and $\propto \lambda^2$ thereafter. On the ultraviolet side it appears to be $\propto 1/\lambda^4$ (suspiciously like Rayleigh scattering). The red side behaviour makes sense as a high order resonance phenomena, but I don't understand the blue side. An analytical approach from first principles looks difficult, and probably not useful. We can get a clue to finding a function that will be

$$F_1 = F(x_1)$$

$$F_2 = F(x_1) \equiv F(x_1 + D) \quad (10)$$

$$F_3 = F(x_3) \equiv F(x_1 + 2D).$$

The solutions may be written

$$\gamma = \frac{\ln(F_2^2/F_1 F_3)}{\ln(x_1 x_3/x_2^2)}$$

$$\beta = \frac{\ln(F_2/F_3) - \gamma \ln(x_3/x_2)}{D} \quad (11)$$

$$A = F_1 e^{\beta x_1 x_1^\gamma}$$

$$x_o = (A/F_o)^{1/\gamma}.$$

III Calculating the Appropriate Functions

A useful function can be calculated numerically as the product of the number of Cerenkov photons per unit track length (dn/dl) times the attenuation ($\exp -a(\lambda)x$) to some distance x , times the relative quantum efficiency ($Q(\lambda_i)/Q_{\max}$), times the phototube envelope transparency (raised to a power equal to the actual thickness divided by nominal thickness, (t/t_o)), summed over wavelength. I call this function the "effective number of photons per unit track length", as a function of distance:

$$\frac{dN_{\text{eff}}}{dl} \equiv 2\pi\alpha(1 - \frac{1}{n^2}) 10^7 \sum_i \left\{ \frac{\Delta\lambda}{\lambda_i^2 - \frac{\Delta\lambda^2}{2}} [G(\lambda_i)]^{t/t_o} \frac{Q(\lambda_i)}{Q_{\max}} e^{-a(\lambda_i)x} \right\} \text{photons/cm} \quad (12)$$

$\alpha = 1/137$, $n = 1.35$ for sea water at depth, and the 10^7

converts wavelength λ_i in nm to cm.

To apply this source function in calculations we must convert it in either one of two cases, radiation from a line or from a point. For consistency with Stenger and Roberts (ref. 5) I take

above ~650 nm. The maximum number of photons visible near a radiating track converges then to a value of 237/cm. (This is surprisingly very close to the nominal value given in the particle handbook⁸, which yields 225 photons/cm).

In figure 2 one sees plotted the various components used in calculating the effective Cerenkov source distribution. The data is also tabulated in Appendix I. One should observe that the effective distribution is peaked at about 340 nm while the attenuation is least in the range of 450-500 nm. This emphasizes the importance of the Keahole observations which imply several times the ideal attenuation coefficient in the 350-450 nm range. Six functions of attenuation have been parameterized as indicated in figure 2 by $m = 1$ through 6. The first five employ the Smith and Baker data for the range of 500-800 nm, where almost all observations agree, and employ Smith and Baker data multiplied by a constant factor between 200-400 nm. The range of 400-500 nm is spanned by sketched in functions. The first corresponds to the old Stenger and Roberts approximation. The third is consistent with Zaneveld's 4.1 km data and the fifth is the Smith and Baker data. The sixth corresponds to Smith and Baker's "pure water" absorption coefficient. Given present data $m = 3$ seems the most reasonable and conservative choice.

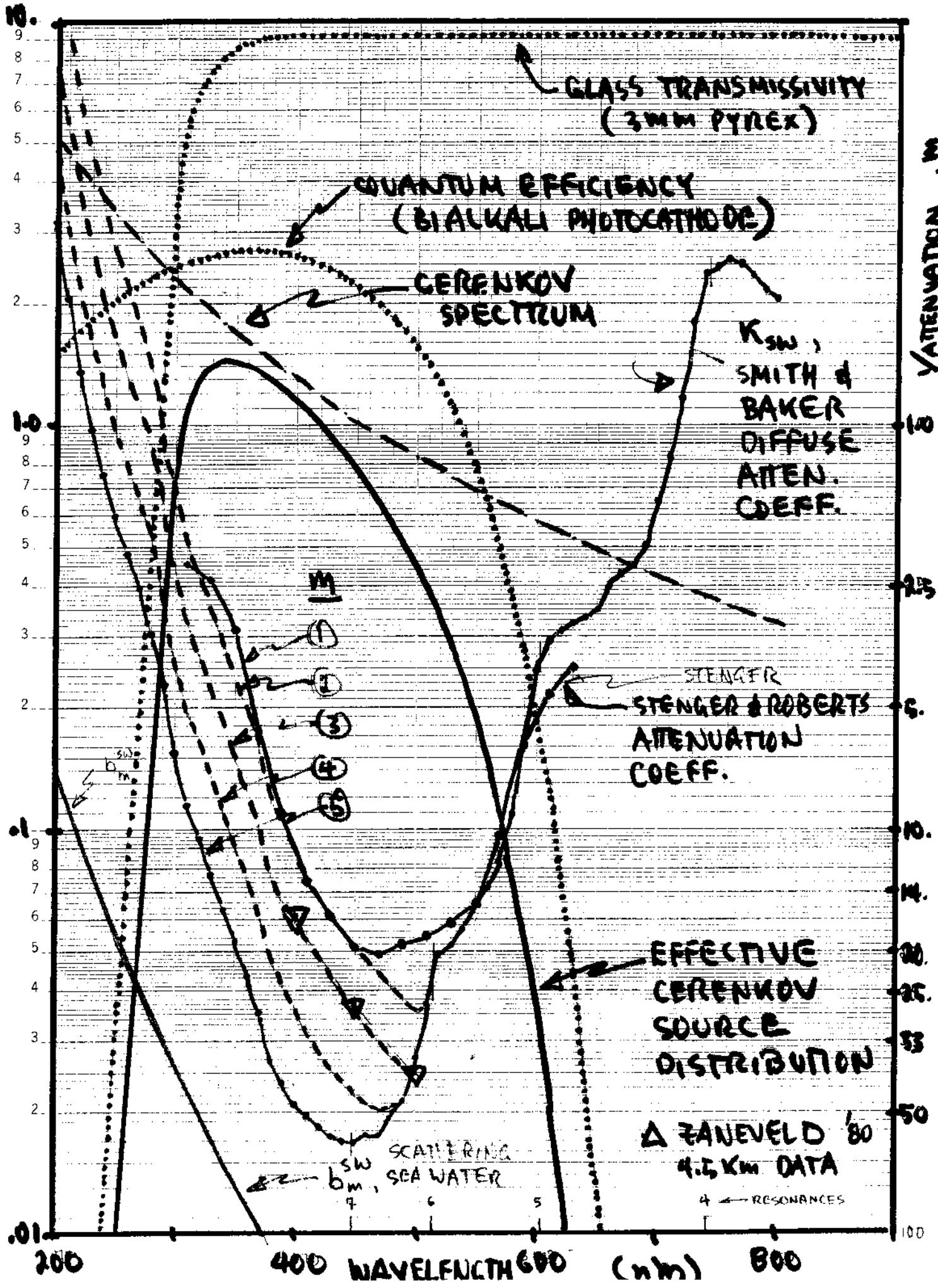
Figure 3 and 4 show results of various calculations of the photon density from cascades and from maximum ionizing tracks and the data is tabulated in Appendix II. The new calculations and fitted function agree satisfactorily and they agree with the Stenger and Roberts function (reported in 5) up to about 25 m. That function, which utilized a power series approximation, diverges at large and small distances. Also shown in figure 3 is the enhance-

References

- 1) Raymond C. Smith and Karen S. Baker, "Optical Properties of the Clearest Natural Waters (200-900)", *Applied Optics*, 20 177 (1981). This paper also contains an excellent review of previous data except that in refs. 2 and 3, below.
- 2) J.R.V. Zaneveld, "Optical Properties of the Keahole DUMAND Site", *Proceedings of the 1980 International DUMAND Symposium*, July 24-August 2, V.J. Stenger ed., I, 1 (1981). Hereafter referred to as D'80.
- 3) H. Bradner and G. Blackinton, "Long Baseline Measurements of Light Attenuation", D'80 I, 9 (1981).
- 4) C.K.N. Patel and A.C. Tam, *Nature* 280, 302 (1979).
- 5) V.J. Stenger and A. Roberts, "The Generation and Propagation of Cerenkov Light in the DUMAND Monte Carlo Program", D'80 I, 161 (1981).
- 6) See EMI Industrial Electronics Ltd. 1979 Catalogue.
- 7) AIP Handbook 3rd Edition, D.W. Gray, Ed., AIP, McGraw-Hill, NY (1972), 6 92.
- 8) Particle Properties Data Booklet, N. Barash-Schmidt et al, *RMP* 52 77 (1980).
- 9) The functions are available on the HDC VAX as [JGL]ATTEN.FOR, [JGL]ATTEN2.FOR and [JGL]WHAT.FOR.

is overly pessimistic, while, $m = 3$, fits the new Zaneveld data (ref. 2) best, $m = 5$ corresponds to best ocean conditions (ref. 1), and $m = 6$ is for pure water (ref. 1). On the right hand scale sensitivity is indicated in stengers (photoelectrons/100 photons/ m^2), with plausible values ranging up to $\sim s = 6$. Note "spectacular" gain in seeing distance between $m = 1, 3$ and $m = 5$, particularly at large s values.

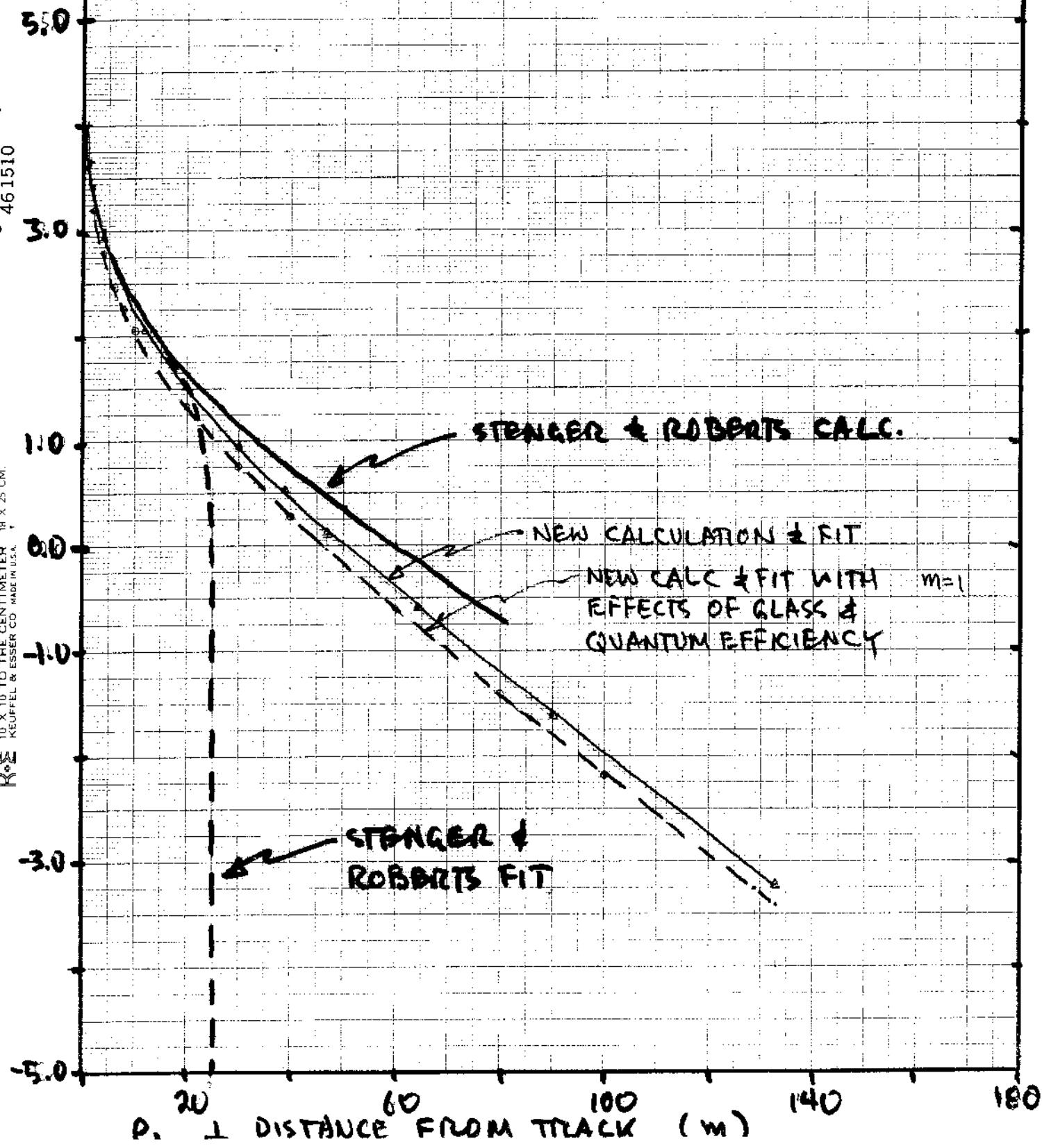
ATTENUATION COEFFICIENT (m^{-1})
K+E SEMILOGARITHMIC • 3 CYCLES X 70 DIVISIONS
REIFFEL & ESSER CO. MADE IN U.S.A.



LOG₁₀(γ PHOTON DENSITY, PHOTONS/ m^2)

461510

K&E 10 X 10 TO THE CENTIMETER 18 X 25 CM.
KEUFFEL & ESSER CO. MADE IN U.S.A.



FILE [JGL]ATTEN.FOR

PROGRAM ATTEN
 C PROGRAM FOR CALCULATING ATTENUATION OF CERENKOV
 C LIGHT IN SEA WATER USING DATA ON THE DIFFUSE
 C ATTENUATION COEFFICIENT FROM THE PAPER OF SMITH
 C AND BAKER, APPL. OPTICS 20, 177, 1981.

CCC LAM IS WAVELENGTH IN NM
 CCC DNNDT IS NUMBER OF PHOTONS/CM TRACK AT SLANT DIST X
 CCC X IS THE SLANT DIST TO TRACK IN METERS
 CCC DNDE IS THE NUMBER OF PHOTONS/M²*GEV FROM ISOTROPIC BURST
 C DNDA IS THE NUMBER OF PHOTONS/M² AT SLANT DIST X
 C RHO IS THE RADIAL DIST FROM A TRACK (AT SLANT DIST X)
 COMMON /STUFF/ LAM(61), DAC(61), DNNDT(100), X(100), DNDE(100),
 +DNDA(100), RHO(100), DN(61), NL, DL, LMIN, COEF, FDNDT(100),
 +OE(61), GL(61), DNE(61), PWAC(61)
 COMMON /PARAM/ AA, GAM, BETA, XX0
 DIMENSION FACTOR(3), PATCH(10, 3)
 DATA NX/100/, DX/2.0/, XMIN/2.0/
 DATA NL/61/, DL/10., LMIN/200., SUM/0./, SUMPE/0./
 DATA THICK/0.3/, THNUM/0.3/, OEM/0.27/
 DATA ALPHA/7.299E-3/, RINDEX/1.35/, PI/3.14159/, B/2.18E-3/
 C B IS ENERGY LOSS RATE IN GEV/CM
 C RINDEX IS SEA WATER INDEX OF REFRACTION AT DEPTH
 C DAC IS THE DIFFUSE ATTENUATION COEFFICIENT IN 1/M
 DATA DAC/3.14, 2.05, 1.36, 0.968, 0.754, 0.588, 0.481, 0.394,
 +0.306, 0.230, 0.154, 0.116, 0.0944, 0.0765, 0.0637, 0.053, 0.0439,
 +0.0353, 0.0267, 0.0233, 0.0209, 0.0196, 0.0184, 0.0172, 0.017,
 +0.0168, 0.0176, 0.0175, 0.0194, 0.0212, 0.0271, 0.037, 0.0489,
 +0.0519, 0.0568, 0.0648, 0.0717, 0.0807, 0.109, 0.158, 0.245,
 +0.29, 0.31, 0.32, 0.33, 0.35, 0.4, 0.43, 0.45, 0.5, 0.65, 0.834,
 +1.17, 1.18, 1.38, 2.47, 2.55, 2.51, 2.36, 2.16, 2.07/
 C FACTOR SCALES THE ATTENUATION IN THE FAR BLUE UP TO 400NM.
 C MM DETERMINES THE TRANSPARENCY TO BE USED. ROUGHLY MM=1, 2, 3, 4
 C CORRESPONDS TO 25, 30, 40, 50 METER WATER. M=4 GIVES THE SMITH
 C AND BAKER. M=5 GIVES PURE WATER ABSORPTION COEF (ALSO S&B). M = MM-1
 DATA FACTOR/4.29, 2.86, 1.71/, MM/5/
 C PATCH SMOOTHES THE ATTEN COEF IN THE 410 TO 500 NM REGION.
 DATA PATCH/.075, .068, .061, .051, .050, .048, .044, .041, .038, .035,
 +.054, .05, .045, .04, .036, .032, .03, .028, .026, .021,
 +.032, .028, .026, .024, .022, .021, .02, .02, .021, .021/
 C PWAC IS THE ABSORPTION COEF FOR PURE WATER
 DATA PWAC/
 +3.0700, 1.9900, 1.3100, 0.9270, 0.7200, 0.5590, 0.4570, 0.3730, 0.2880,
 +0.2150, 0.1410, 0.1050, 0.0844, 0.0678, 0.0561, 0.0463, 0.0379, 0.0300,
 +0.0220, 0.0191, 0.0171, 0.0162, 0.0153, 0.0144, 0.0145, 0.0145, 0.0156,
 +0.0156, 0.0176, 0.0196, 0.0257, 0.0357, 0.0477, 0.0507, 0.0558, 0.0638,
 +0.0708, 0.0799, 0.1080, 0.1570, 0.2440, 0.2890, 0.3090, 0.3190, 0.3290,
 +0.3490, 0.4000, 0.4300, 0.4500, 0.5000, 0.6500, 0.8390, 1.1690, 1.7990,
 +2.3800, 2.4700, 2.5500, 2.5100, 2.3600, 2.1600, 2.0700/
 C QE IS THE QUANTUM EFFICIENCY VERSUS WAVELENGTH FOR A BIALKALI
 C PHOTOCATHODE. DATA FROM EMI SPECIFICATIONS
 DATA QE/0.15, 0.16, 0.172, 0.184, 0.196, 0.207, 0.216, 0.225,
 +0.233, 0.241, 0.248, 0.253, 0.257, 0.261, 0.265, 0.268, 0.269,
 +0.27, 0.269, 0.268, 0.263, 0.258, 0.25, 0.24, 0.23, 0.22, 0.21,
 +0.198, 0.185, 0.172, 0.158, 0.142, 0.127, 0.111, 0.095, 0.08,
 +0.063, 0.05, 0.0375, 0.028, 0.02, 0.013, 0.0084, 0.005, 0.0028,
 +0.0014, 0.0015/

[JGL]ATTEN2.FOR IS FOR m=1 (OLD
 STENGER & ROBERTS APPROXIMATION)

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550 CONTINUE
  WRITE(6,600)
600 FORMAT('1 WAVELENGTH DEPENDANCE',
+//5X,' LAMBDA NM',5X,'ATTEN COEF,1/M',5X,
+' QUANTUM EFFIC ',5X,' GLASS TRANS ',5X,
+' NUMBER PHOTONS ',5X,' EFFECTIVE PHOTONS',//)
  WRITE(6,650)(LAM(JL),DAC(JL),QE(JL),GL(JL),DN(EJL),DNE(JL),JL=1,NL)
650 FORMAT(100(1X,I10,3(10X,F10.4),10X,F10.2,10X,F10.2//))
  WRITE(6,700)
700 FORMAT('1 NUMBER OF PHOTONS/CM TRACK VERSUS DISTANCE'///
+' DIST,M',8X,'PHOTONS/CM',7X,
+' PHOT/M2*GEV ',1X,' PHOT/M2 ',3X,' RADIAL DIST,M',
+5X,' FITTED FUNC '//)
  WRITE(6,800)(X(IX),DN(DT(IX)),DN(DE(IX)),RHU(IX),
+FDN(DT(IX)),IX=1,NX)
800 FORMAT(100(1X,F10.0,5X,3(E10.3,5X),F10.1,5X,E10.3//))
  LAMTOT=LAM(NL)-LAM(1)
  BINV=1./BETA
  WRITE(6,900) SUM,LAMTOT,SUMPE,XX1T,XX2T,XX3T,AA,GAM,XX0,BETA,BINV,
+MM,KK
900 FORMAT(//,'1 SUM= ',F10.1,'PHOTONS/CM IN ',15,'NM'/
+' EFFECTIVE TOTAL PHOTONS/CM= ',F10.2//)
+' XX1= ',F10.2/
+' XX2= ',F10.2/
+' XX3= ',F10.2//
+' AA= ',F10.3/
+' GAM= ',E10.3/
+' XX0= ',F10.3/
+' BETA= ',F10.5/
+' 1/BETA= ',F10.2//
+' MM= ',I5/
+' KK= ',I5//)
  STOP
END

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FUNCTION FOT(XX)
C THIS PHENOMENOLOGICAL FUNCTION PRODUCES THE SURVIVING
C NUMBERS OF PHOTONS/CM TRACK THAT REACH A DISTANCE X
C IT MUST HAVE GEOMETRIC FACTORS ADDED TO REPRESENT
C RADIATION FROM A LINE OR A POINT
COMMON /PARAM/AA,GAM,BETA,XX0
FOT=0.0
IF(XX.LT.0.0)RETURN
FOT=AA*(XX+XX0)**(-GAM)*EXP(-BETA*XX)
RETURN
END

```

```

FUNCTION PHOTONS(XX)
C DOES THE INTEGRAL OVER WAVELENGTH FOR CERENKOV LIGHT
C SEEN FROM DISTANCE X. UNITS ARE PHOTONS/CM,
COMMON /STUFF/ LAM(61),DAC(61),DN(DT(100),X(100),DN(DE(100),
+DN(DA(100),RHO(100),DN(61),NL,DL,LMIN,COEF,FDN(DT(100),
+QE(61),GL(61),DNE(61)
PHOTONS=0.0
IF(XX.LT.0.0) RETURN
DP=0.0
DO 200 JL=1,NL
DP=DP+DNE(JL)*EXP(-XX*DAC(JL))
200 CONTINUE
PHOTONS=DP
RETURN
END

```

LENGTH DEPENDANCE

LAMBDA, NN ATTEN CUEF, 1/M

QUANTUM EFFECT

GLASS TRANS

NUMBER PHOTONS

EFFECTIVE PHOTON

PROGRAM TO TEST PHOTON DENSITY FUNCTIONS
(PHOCAS & PHOMIZ)

FILE [JGL] WHERE.FOR

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PROGRAM WHEREAS
C TEST ROUTINE FOR FILE WHAT.FOR, THE CERENKOV PHOTON
C DENSITY FUNCTIONS PHOMIZ AND PHOCAS.
DO 500 IM=1,6
RMIN=2.0
DELR=2.0
NR=100
E=1.0
FN=1.35
THC=ACOS(1/FN)
TH=THC
M=IM
WRITE(6,50)
50 FORMAT(1H1,5X,'DISTANCE,M',5X,'DNDA',10X'DNDE'///)
DO 200 IR=1, NR
R=RMIN+(IR-1)*DELR
DNDA=PHOMIZ(R,M)
DNDE=PHOCAS(R,TH,THC,M,E)
WRITE(6,100) R, DNDA, DNDE
100 FORMAT(5X,F10.1,2E15.3)
200 CONTINUE
WRITE(6,300) TH,THC, FN, M, E
300 FORMAT(///3F10.3,15,F10.3)
500 CONTINUE
STOP
END
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[JGL] WHAT.FOR CONT.

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FUNCTION PHOMIZ(RHO,M)
C CALL WITH RHO IN METERS AND INDEX M FOR WATER CLARITY
C ROUGHLY M=1,2,3,4,5,6 CORRESPONDS TO 20,25,30,40,50,60M
C WATER. FIRST IS POORER THAN OBSERVED WHICH CORRESPONDS
C TO THIRD. FIFTH REPRESENTS CLEAREST OBSERVED ACTUAL
C OCEAN WATERS. SIXTH IS FOR PURE WATER AS IN PDK DETECTORS.
C THIS FUNCTION CALCULATES THE EFFECTIVE NUMBER OF PHOTONS
C PER M**2 AT A PERPENDICULAR DISTANCE RHO FROM A SINGLE
C CERENKOV RADIATING TRACK. THE INDEX OF REFRACTION IN
C THE MEDIUM IS RINDEX. REQUIRES THE FUNCTION DNDT TO
C CALCULATE THE NUMBER OF PHOTONS THAT SURVIVE TO REACH
C SLANT DISTANCE XX.
C MULTIPLY BY DETECTOR PROJECTED AREA IN DIRECTION OF
C CERENKOV ANGLE FROM TRACK(NOT PERPENDICULAR). ALSO
C MULTIPLY BY MAXIMUM QUANTUM TIMES COLLECTION EFFICIENCY.
C FUNCTION INCORPORATES RELATIVE RESPONSE OF BIALKALI
C PHOTOCATHODE AND TRANSMISSION OF 3MM PYREX.
C RHOMIN IS THE MINIMUM DISTANCE AT WHICH THE PHOTON
C DENSITY MAKES SENSE, THAT IS ABOUT ONE DETECTOR
C RADIUS.
C
DATA RINDEX/1.35/, RHOMIN/0.5/, PI/3.14159/, KALL/0/
C INITIALIZE CONSTANT FACTORS
    IF(KALL.GT.0) GO TO 100
C CONSTANT CONVERTS DNDT UNITS OF PHOTON/CM OF SOURCE
C TRACK TO PHOTONS/M**2
    CONST=100./(2.*PI*(1-1/(RINDEX**2)))
C SINE OF THE CERENKOV ANGLE
    STC=SQRT(1-1/(RINDEX**2))
C MINIMUM SLANT DISTANCE
    XXMIN=RHOMIN/STC
    100 CONTINUE
    KALL=KALL+1
C CONVERT TO SLANT DISTANCE
    XX=RHO/STC
C NO DIST TOO CLOSE
    IF(RHO.LT.RHOMIN) XX=XXMIN
C NOW CALCULATE PHOTONS/M**2
    PHOMIZ=DNDT(XX,M)*CONST/XX
    RETURN
END

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```

FUNCTION DNDT(X,M)
C FUNCTION CALCULATES THE EFFECTIVE NUMBER OF PHOTONS
C PER CM OF CERENKOV RADIATING TRACK THAT SURVIVE TO
C A DISTANCE X IN METERS. SEE HDC MEMO NUMBER 81-10
C 4/81 BY J. LEARNED FOR DETAILS.
C THIS FUNCTION IS CALLED BY PHOMIZ TO PRODUCE PHOTON
C DENSITY FROM A SINGLE RELATIVISTIC TRACK, AND FROM
C PHOCAS TO PRODUCE PHOTON DENSITIES FROM A CASCADE.
C DIMENSION A(4,6)
DATA A/
+290.9,0.375,1.869,0.04952,
+713.6,0.660,5.303,0.03757,
+935.6,0.694,7.250,0.02722,
+698.4,0.530,7.674,0.02041,
+547.9,0.397,8.231,0.01679,
+530.2,0.377,8.432,0.01434/
DNDT=A(1,M)*(X+A(3,M))**(-A(2,M))*EXP(-X*A(4,M))
RETURN
END

```

DISTANCE, M

DNDA PHOTONS / m⁻²

