Course Updates
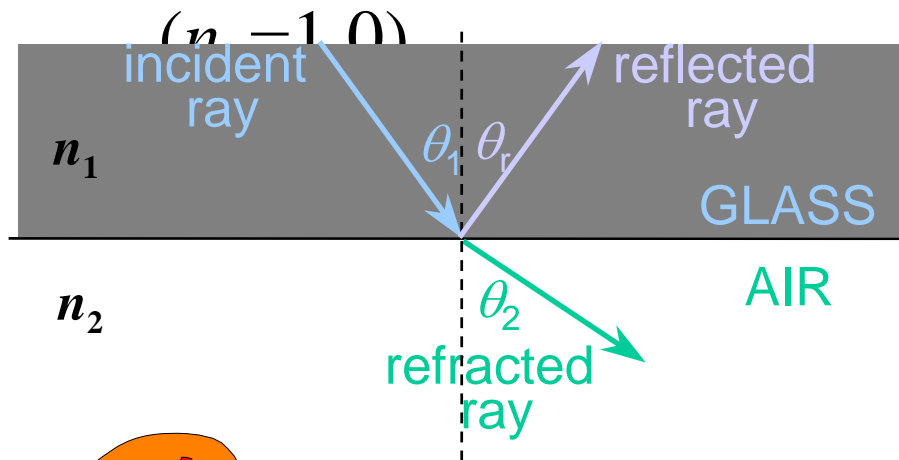
Reminders:

1) Assignment #12 → due today

2) Polarization, dispersion

3) Last HW (#13 posted) → due Monday, May 3rd

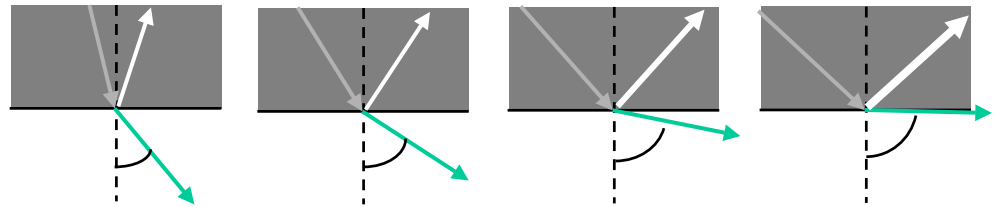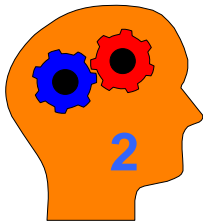# Total Internal Reflection

– Consider light moving from glass (n_1=1.5) to air $(n_2=1.0)$

incident ray

reflected ray

n_1

GLASS

θ_1 θ_r

θ_2

AIR

refracted ray

n_2

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{n_1}{n_2} > 1 \implies \theta_2 > \theta_1$$

I.e., light is bent away from the normal. as θ_1 gets bigger, θ_2 gets bigger, but θ_2 can never get bigger than $90°$!!
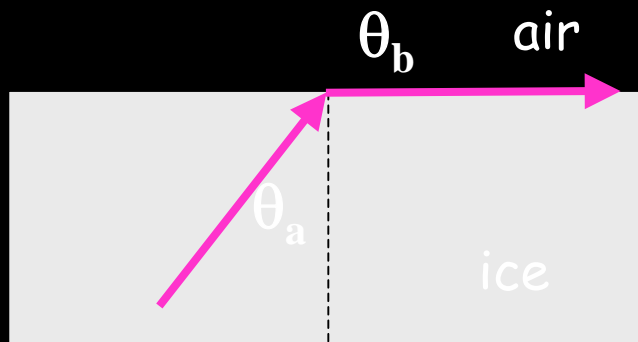
In general, if $\sin\theta_1 > (n_2 / n_1)$, we have NO refracted ray; we have TOTAL INTERNAL REFLECTION.

For example, light in water which is incident on an air surface with angle $\theta_1 > \theta_c = \sin^{-1}(1.0/1.33) = 48.8°$ will be totally reflected. This property is the basis for the optical fiber communication.

"Mini-Antarctica" at Stanford Linear Accelerator

Problem!

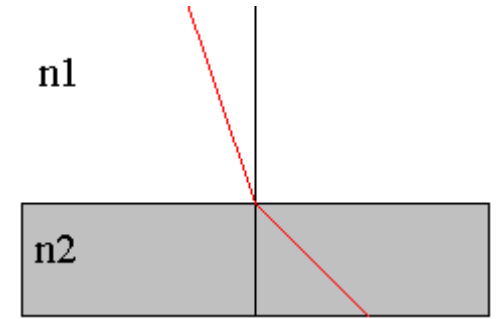θ_b air

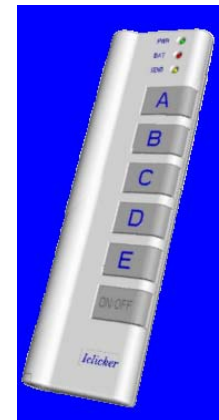θ_a

ice

There is no escape!!!

Not always a bad thing

Question 1

The path of light is bent as it passes from medium 1 to medium 2. Compare the indexes of refraction in the two mediums.
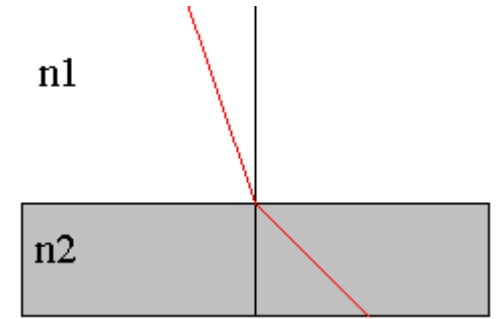
n1

n2

a) $n_1 > n_2$

b) $n_1 = n_2$

c) $n_1 < n_2$

The path of light is bent as it passes from medium 1 to medium 2. Compare the indexes of refraction in the two mediums.

n1

n2

a) $n_1 > n_2$

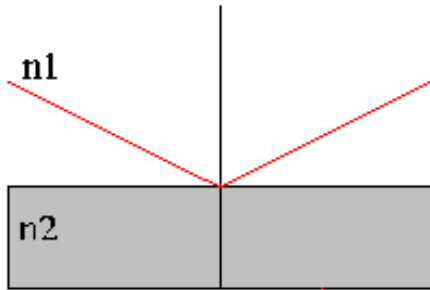b) $n_1 = n_2$

c) $n_1 < n_2$

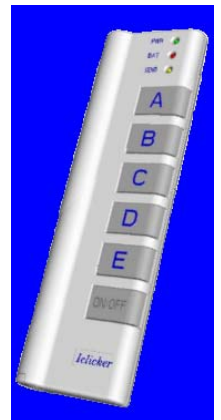Snell's Law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Here, $\theta_2 > \theta_1$ implies $n_2 < n_1$

Question 2

A light ray travels in a medium with n_1 and completely reflects from the surface of a medium with n_2. The critical angle depends on:

a) n_1 only

b) n_2 only

c) n_1 and n_2

Question 2
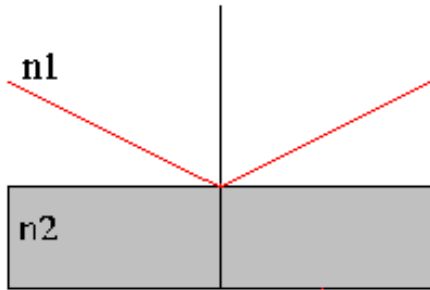

A light ray travels in a medium with n_1 and completely reflects from the surface of a medium with n_2. The critical angle depends on:

a) n_1 only

b) n_2 only

c) n_1 and n_2

Critical angle occurs when $\theta_2 = 90°$
Therefore, $\sin\theta_{critical} = n_2/n_1$

Question 3: Critical Angle…

An optical fiber is surrounded by another dielectric. In case I this is water, with an index of refraction of 1.33, while in case II this is air with an index of refraction of 1.00.
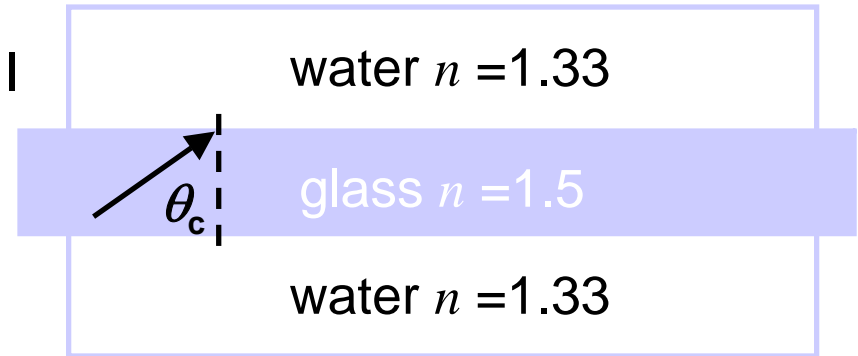
Compare the critical angles for total internal reflection in these two cases
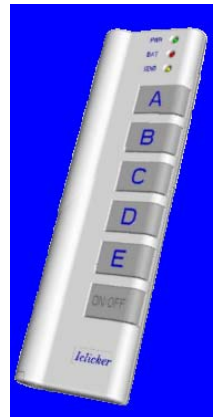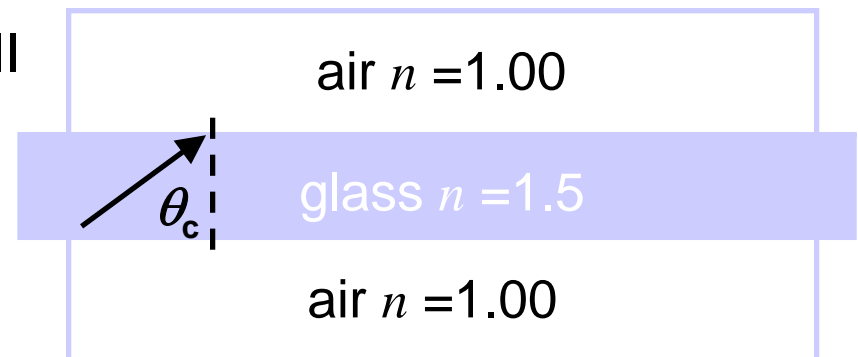
a) $\theta_{cI} > \theta_{cII}$

b) $\theta_{cI} = \theta_{cII}$

c) $\theta_{cI} < \theta_{cII}$

Case I

water $n =$1.33

θ_c | glass $n =$1.5

water $n =$1.33

Case II

air $n =$1.00

θ_c | glass $n =$1.5

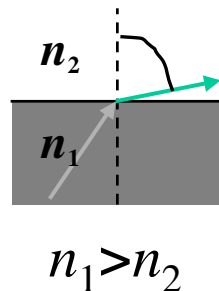air $n =$1.00

Question 3: Critical Angle…

An optical fiber is surrounded by another dielectric. In case I this is water, with an index of refraction of 1.33, while in case II this is air with an index of refraction of 1.00.

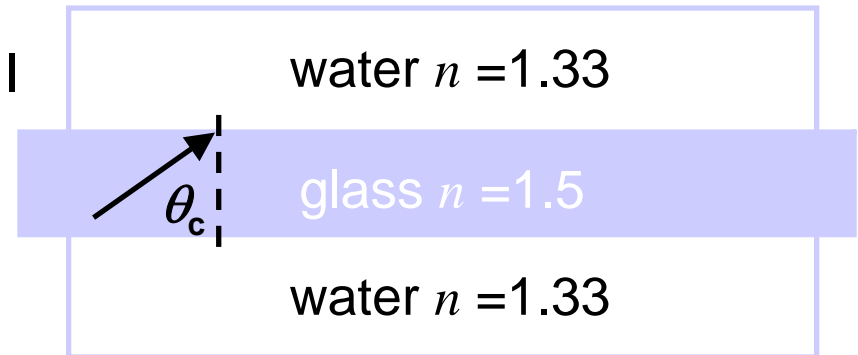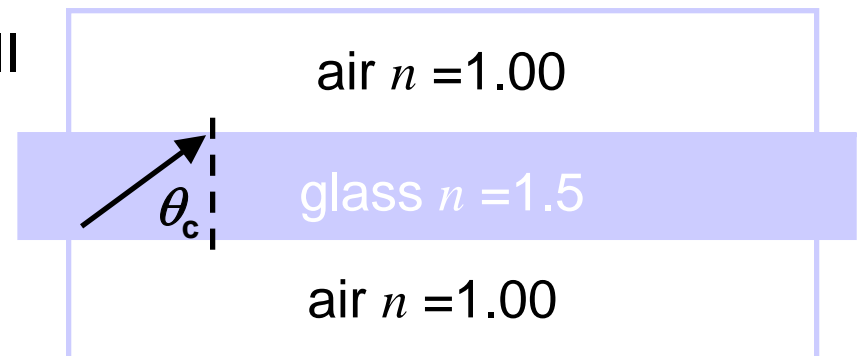**Compare the critical angles for total internal reflection in these two cases**

Case I

water $n = 1.33$

glass $n = 1.5$ θ_c

water $n = 1.33$

Case II

air $n = 1.00$

glass $n = 1.5$ θ_c

air $n = 1.00$

a) $\theta_{cI} > \theta_{cII}$

b) $\theta_{cI} = \theta_{cII}$

c) $\theta_{cI} < \theta_{cII}$

n_2

n_1

$n_1 > n_2$

Since $n_1 > n_2$ TIR will occur for $\theta >$ critical angle. Snell's law says $sin\theta_c = n_2/n_1$.

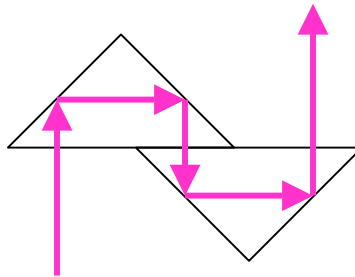If $n_2 = 1.0$, then θ_c is as small as it can be.

So $\theta_{cI} > \theta_{cII}$.
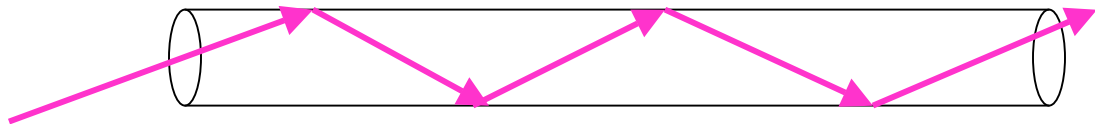
Total Internal Reflection

Total internal reflection occurs when $\theta > \theta_c$ and provides 100% reflection. This has better efficiency than silvered mirror.

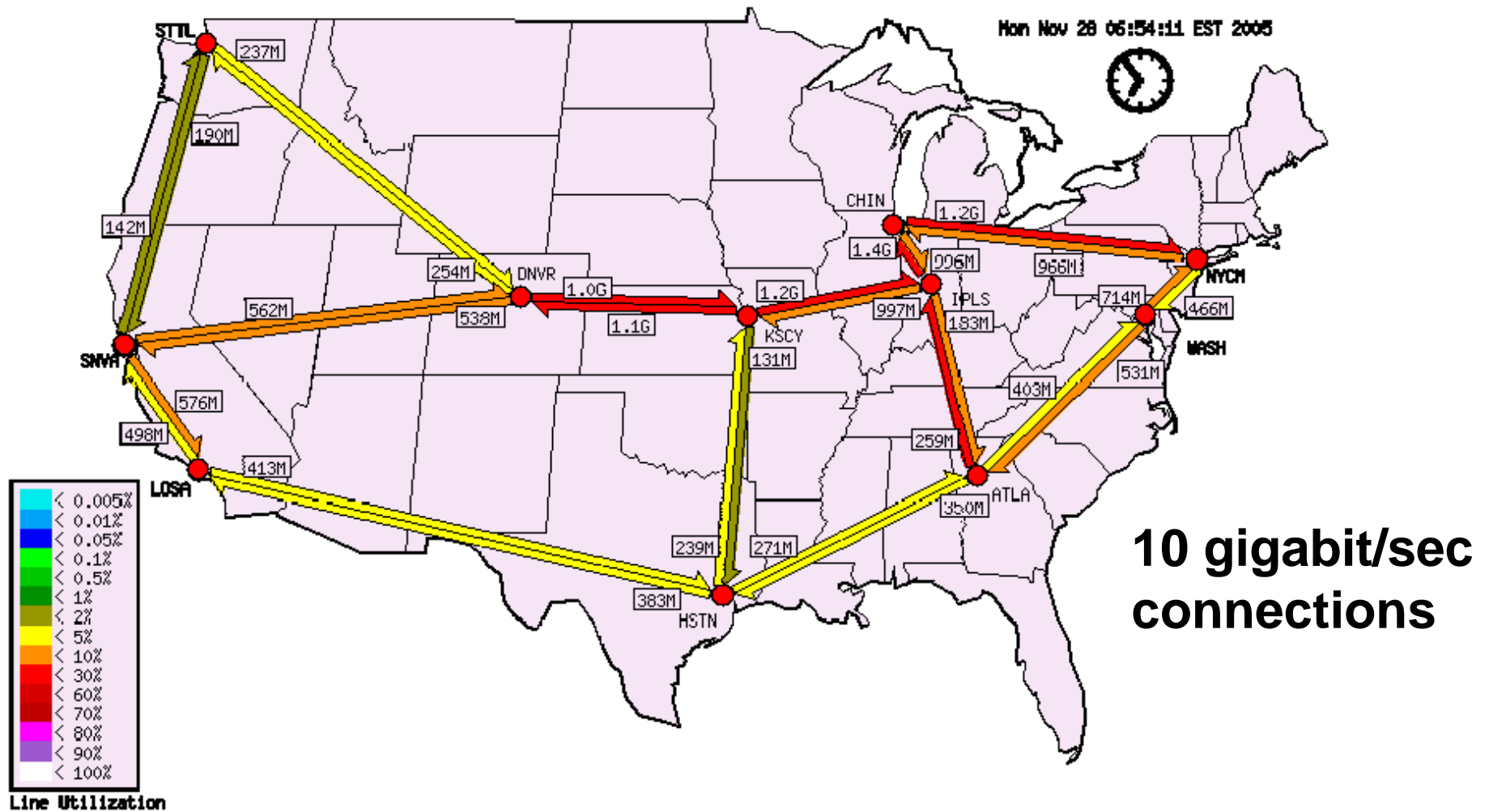### Examples of devices using Critical Angle

• Prism Binoculars

• Fiber Optics

Fiber optics is extremely important for high speed Internet and digital data transfer at long distances. Many companies (Lucent) have laid fiber over long Distances to provide internet service.
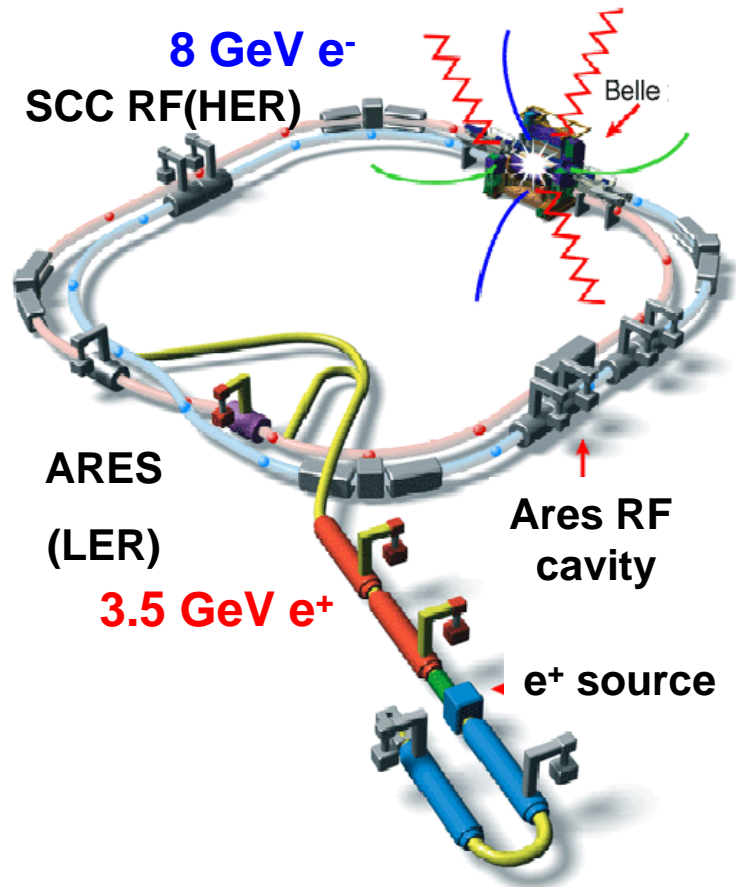
I2 (academic) network of fiber connections
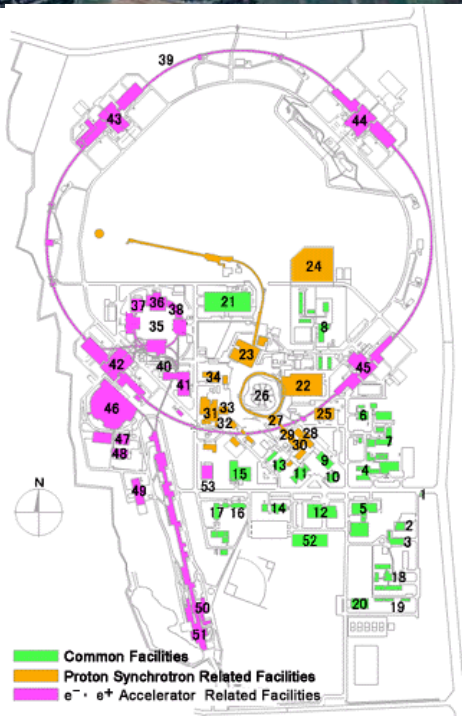

10 gigabit/sec connections

Mt. Tsukuba

KEK
Tsukuba, Japan

~2mi circ. ring

World's highest Luminosity collider

8 GeV e−
SCC RF(HER)

Belle

ARES
(LER)

3.5 GeV e+

Ares RF cavity

e+ source

Common Facilities
Proton Synchrotron Related Facilities
e− · e+ Accelerator Related Facilities

Belle Detector

SC solenoid
1.5T

CsI(Tl) $16X_0$

TOF counter

8GeV e^-

Aerogel Cherenkov cnt.
n=1.015~1.030

3.5GeV e^+

Tracking + dE/dx
small cell + He/C_2H_5

Si vtx. det.
3 lyr. DSSD

μ / K_L detection
14/15 lyr. RPC
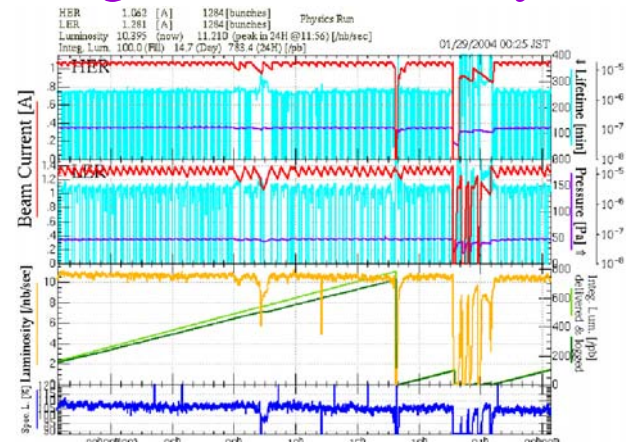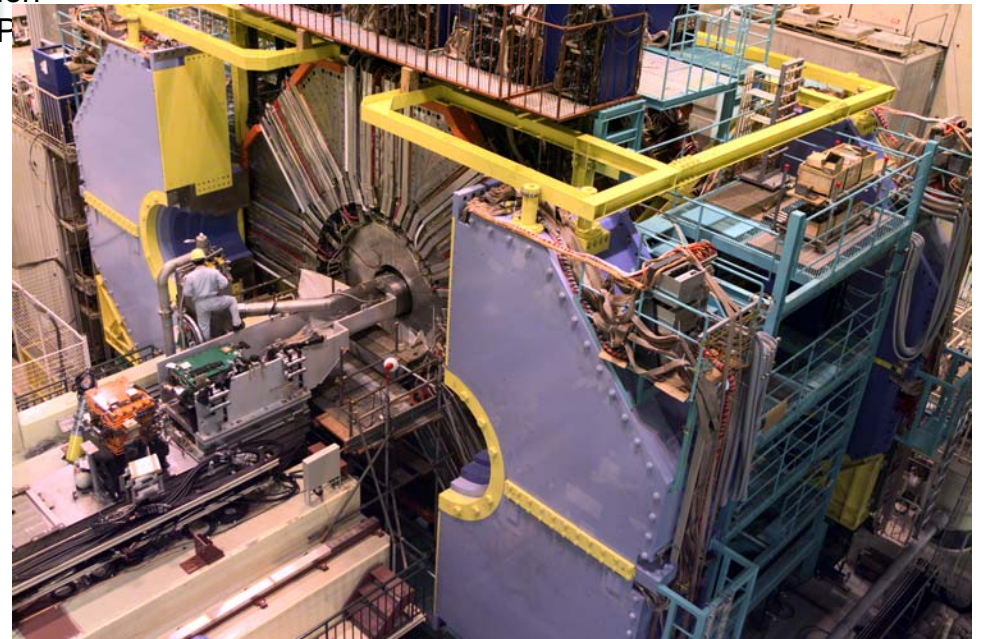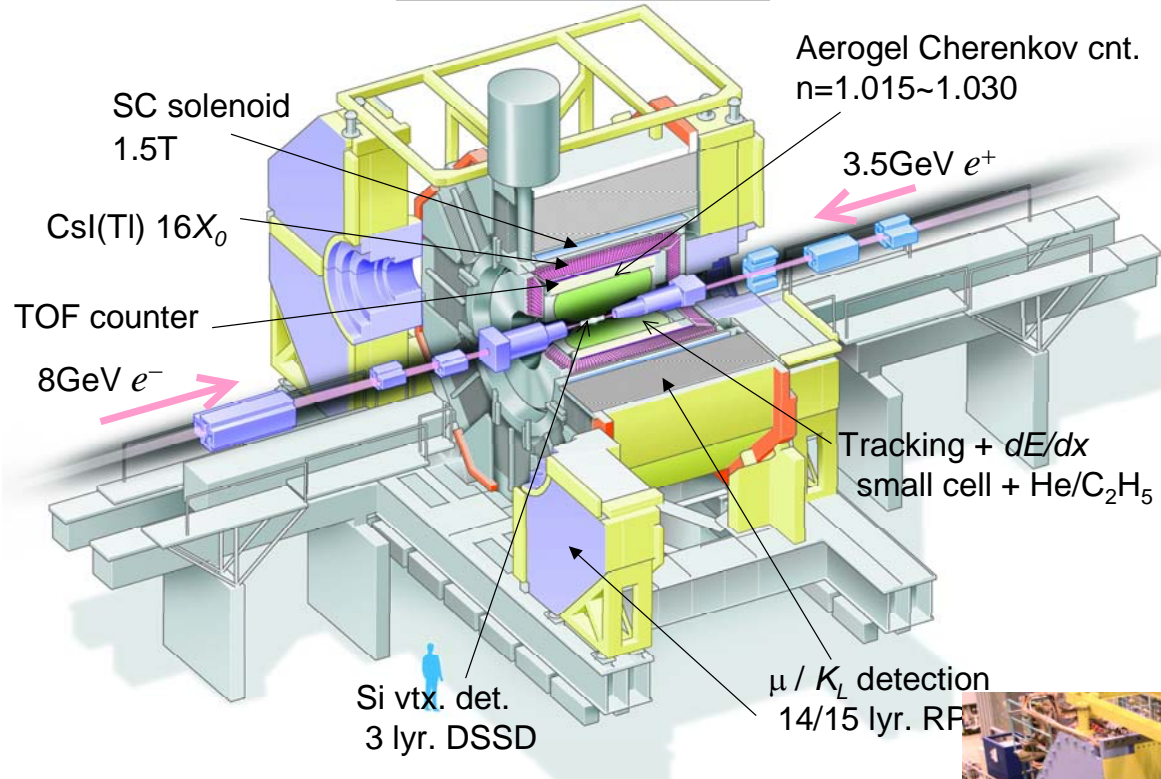
$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

$$V_{ij} = \begin{bmatrix} 0.97383 \\ 0.2271 \\ 0.00814 \end{bmatrix}$$

小林益川理論が正解だった！
Bファクトリーが放った決定打

$(1-\lambda^2/2)$

$\boxed{\pi - \beta - \gamma}$
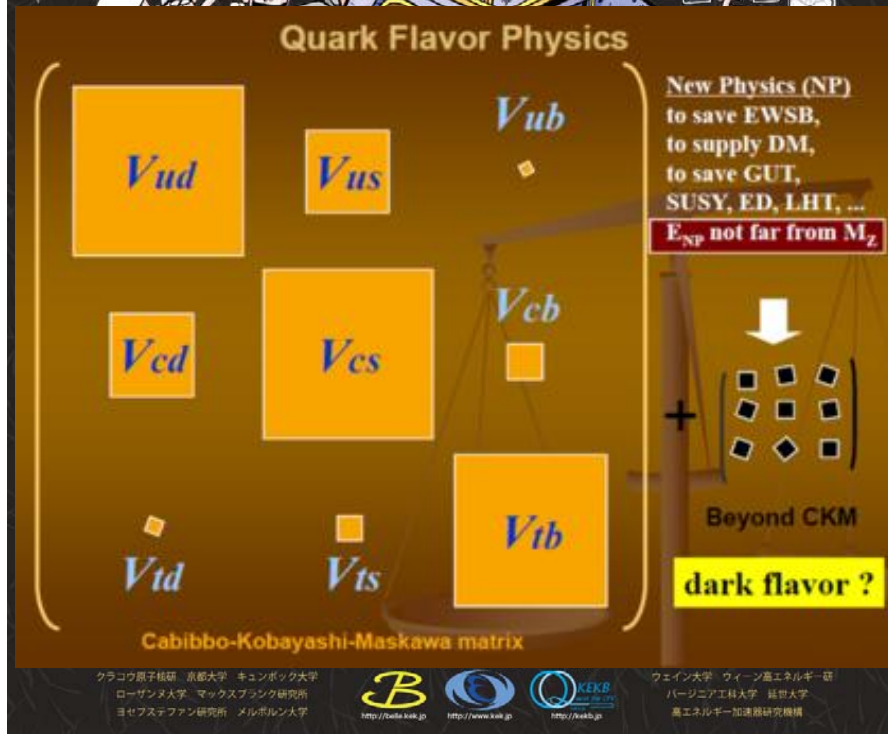
$\sqrt{(1-\overline{\rho})^2 + \overline{\eta}^2}$

$\left[\dfrac{V_{cb}^*}{V_{tb}^*} \right] = \boxed{\arg\left[V_{td}^* \right]}$

Belle Upgrade (Run stops June 2010)

CsI(Tl) $16X_0$
→ pure CsI (endcap)

SC solenoid
1.5T

Aerogel Cherenkov counter
+ TOF counter
→ "TOP"/F-DIRC

μ / K_L detection
14/15 lyr. RPC+Fe
→ tile scintillator

Tracking + dE/dx
small cell + He/C_2H_5
→remove inner lyrs.
Use fast gas

New readout
and
computing
systems

Si vtx. det.
4 lyr. DSSD
→ 2 pixel/striplet lyrs.
+ 4 lyr. DSSD

imaging TOP (iTOP)
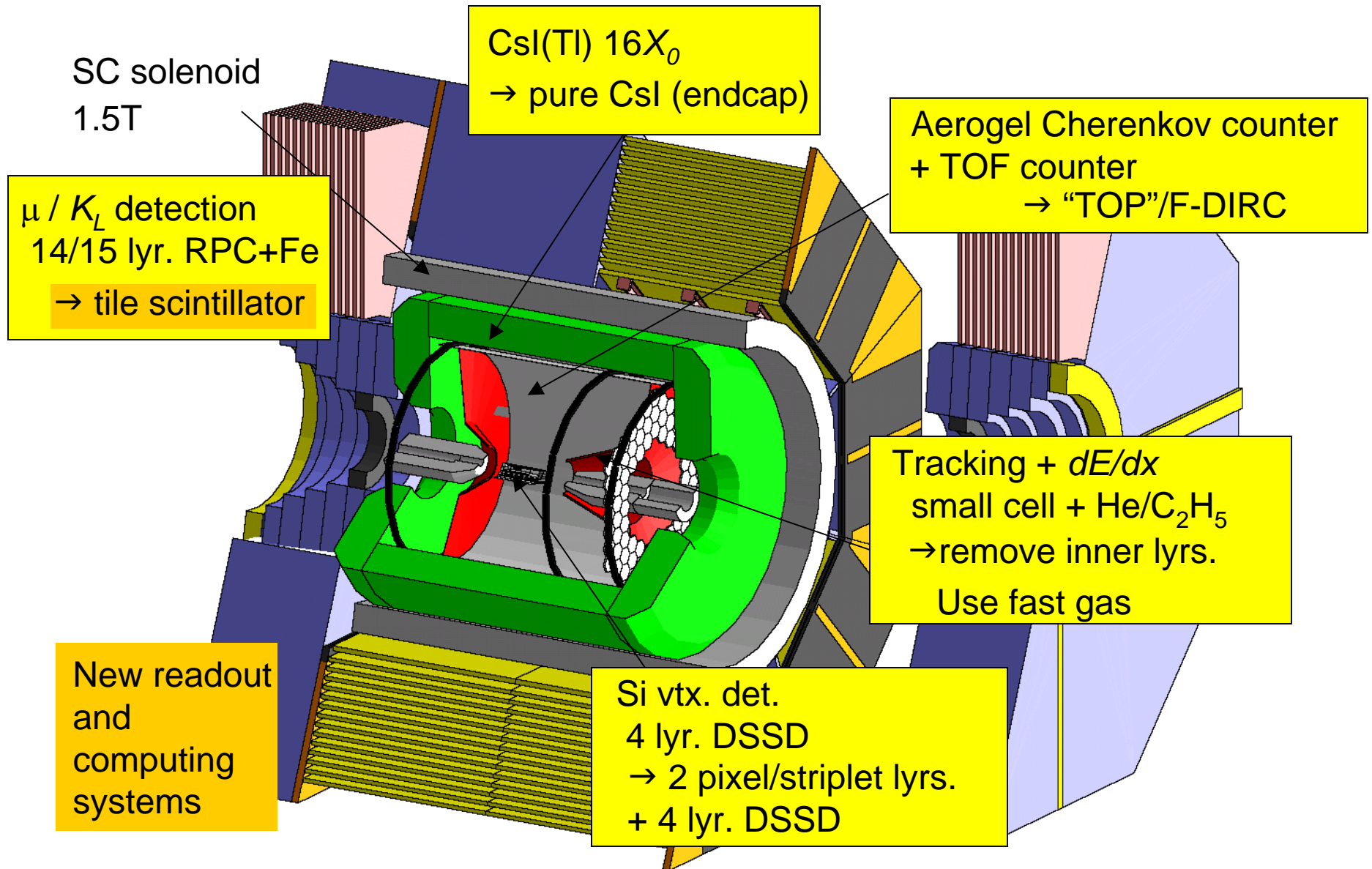
Total internally reflected Photons used for "Particle ID"
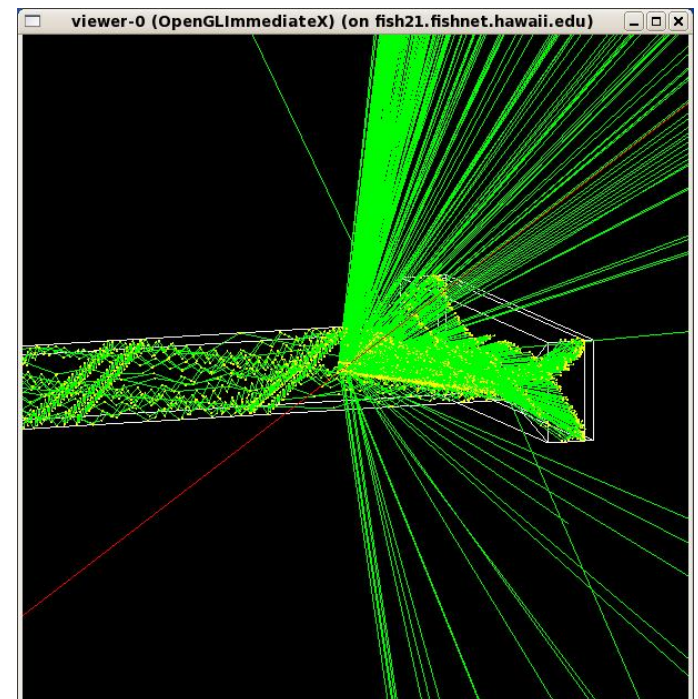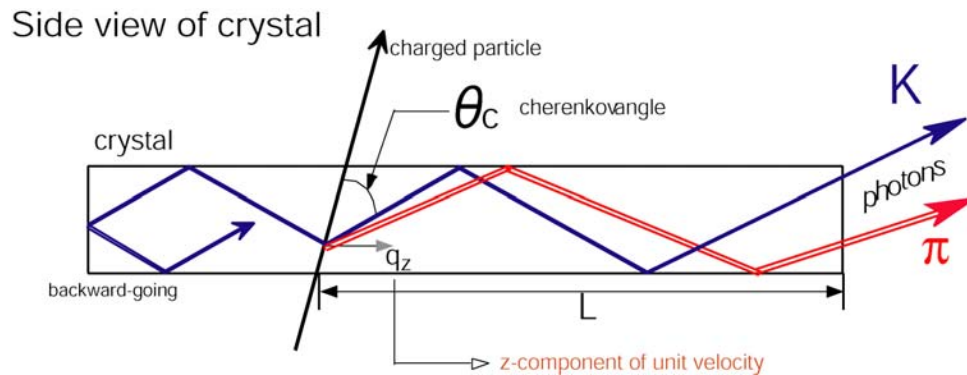
Side view of crystal

charged particle

θ_C cherenkov angle

crystal

backward-going

q_z

K

photons

π

L

z-component of unit velocity

Linear-array type photon detector

Quartz radiator

~400mm

20mm

L

X

y

z

x

viewer-0 (OpenGLImmediateX) (on fish21.fishnet.hawaii.edu)

TOP counter principle

- Measure Position+Time

Linear-array type
photon detector Compact detector

Quartz radiator

~400mm

20mm

L ~2m

TDC (ps)

Side view of crystal

charged particle

θ_C cherenkov angle

crystal

q_z

backward-going

photons

K

π

L

z-component of unit velocity

~200ps
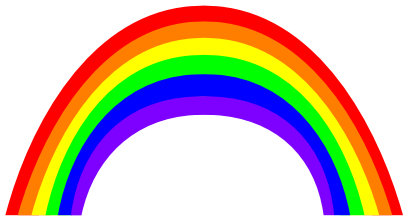
K

π
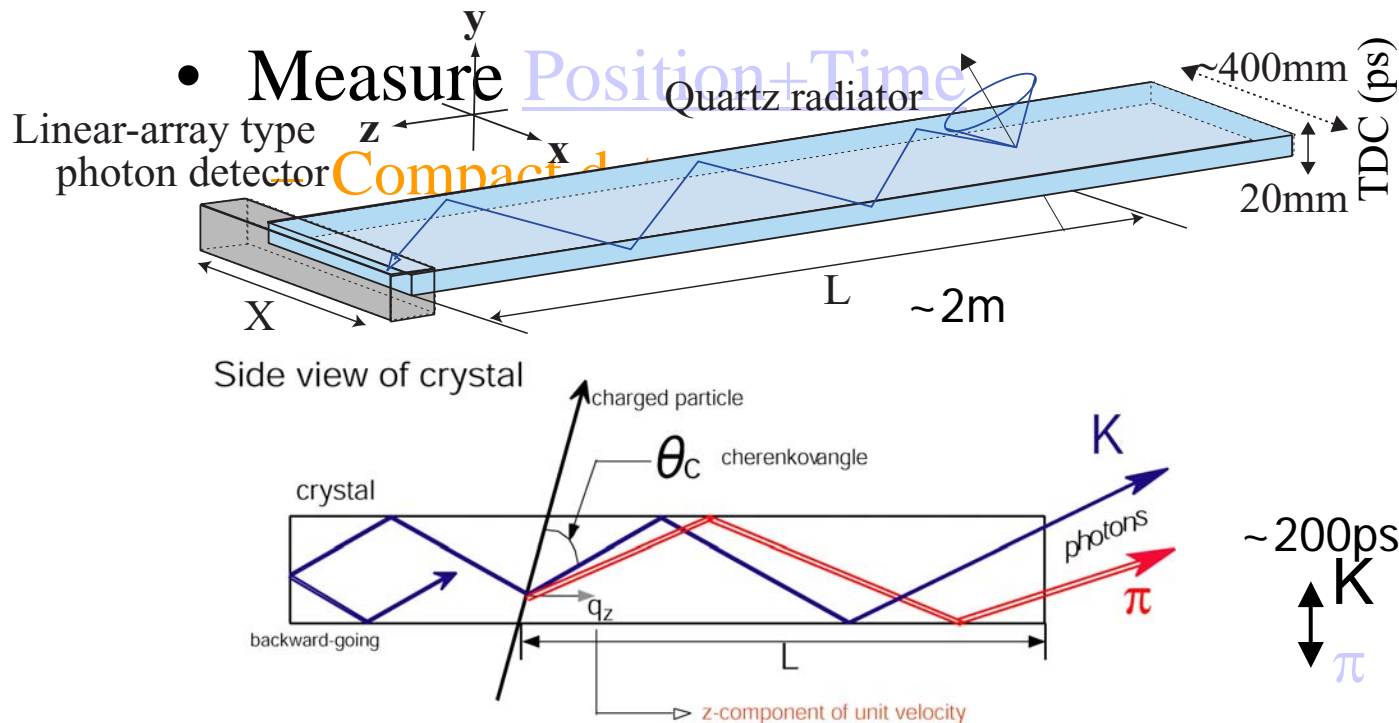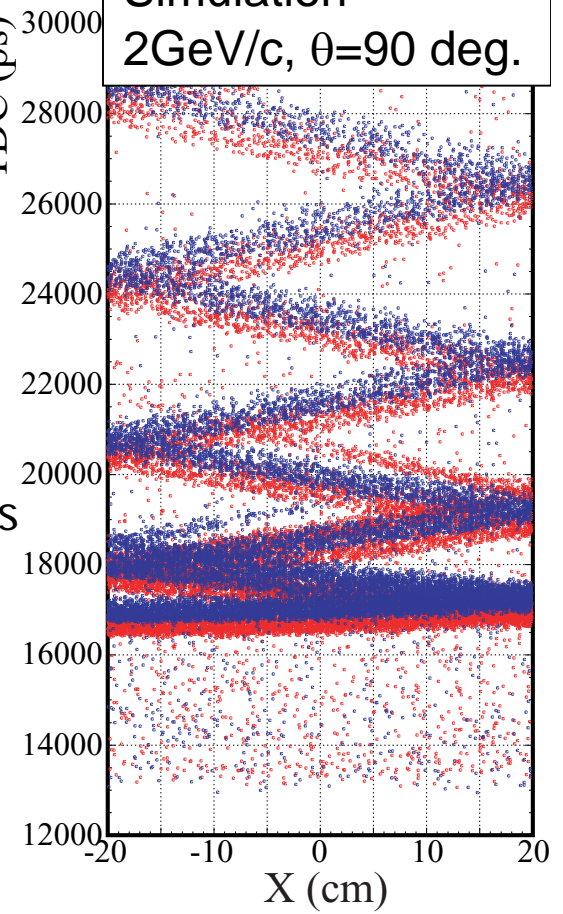
Different opening angle for the same momentum
→ Different propagation length(= propagation time)
  + TOF from IP works additively.

Simulation
2GeV/c, θ=90 deg.

30000
28000
26000
24000
22000
20000
18000
16000
14000
12000
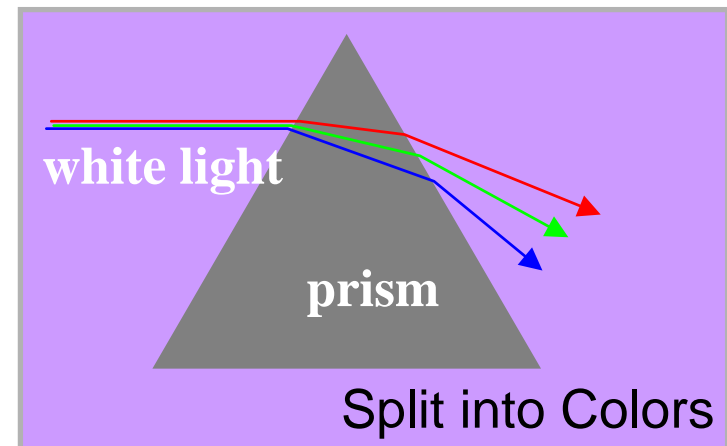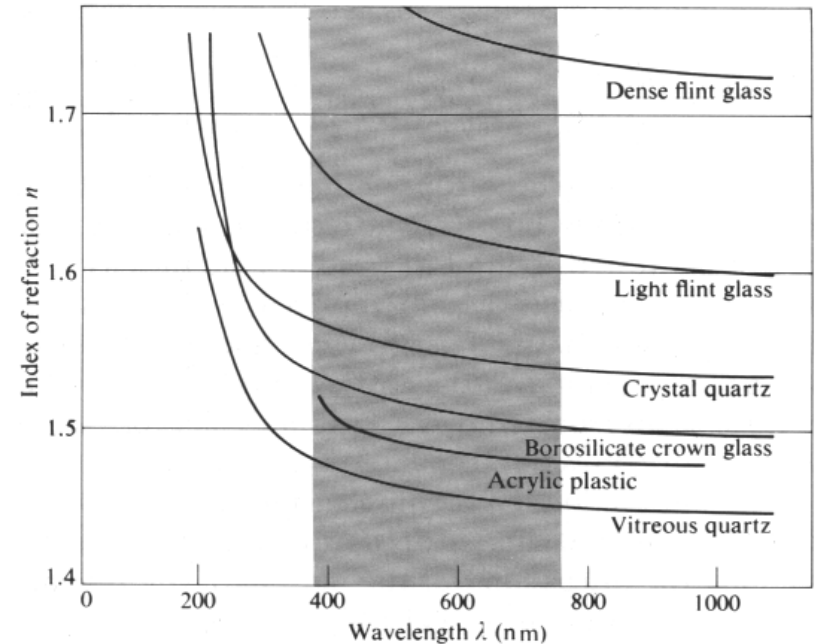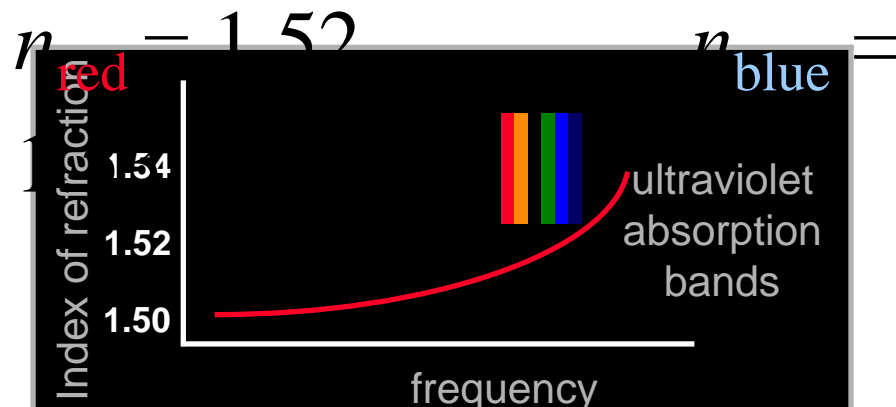
-20   -10   0   10   20
X (cm)

Dispersion: $n = n(\omega)$

The index of refraction depends on frequency, due to the presence of resonant transition lines.

For example, ultraviolet absorption bands in glass cause a rising index of refraction in the visible, i.e.,
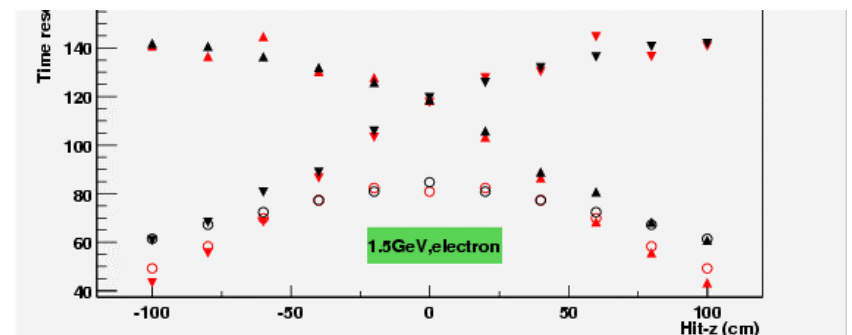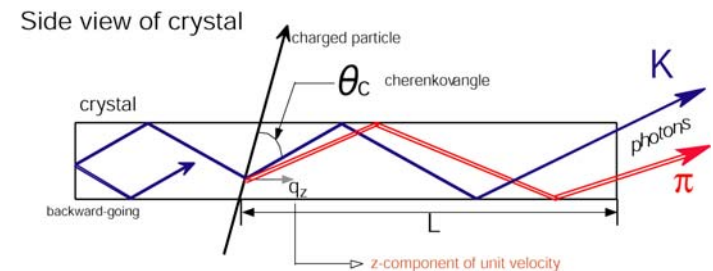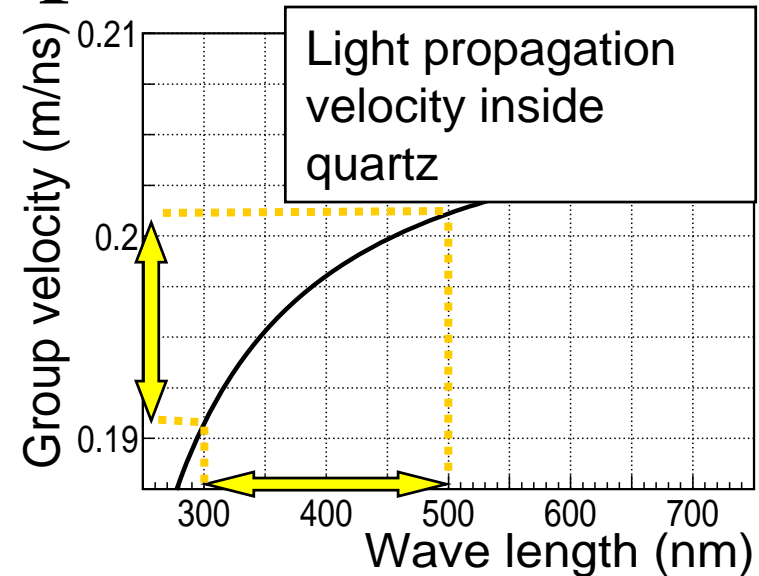
$n(\text{higher } \omega) > n(\text{lower } \omega)$:

$n_{red} = 1.52 \qquad n_{blue} = $

Dense flint glass
Light flint glass
Crystal quartz
Borosilicate crown glass
Acrylic plastic
Vitreous quartz

Index of refraction n

Wavelength λ (nm)

red
blue
Index of refraction
1.54
1.52
1.50
ultraviolet absorption bands
frequency

white light
prism
Split into Colors

Chromatic dispersion

Variation of propagation velocity depending on the wavelength of Cherenkov photons

- Due to wavelength spread of detected photons
- → propagation time dispersion

- Longer propagation length
→ Improves ring image difference
But, decreases time resolution.

Light propagation velocity inside quartz

Group velocity (m/ns) vs Wave length (nm)

Side view of crystal

θ_C cherenkov angle

charged particle

crystal

K

photons

π

backward-going

q_z

z-component of unit velocity
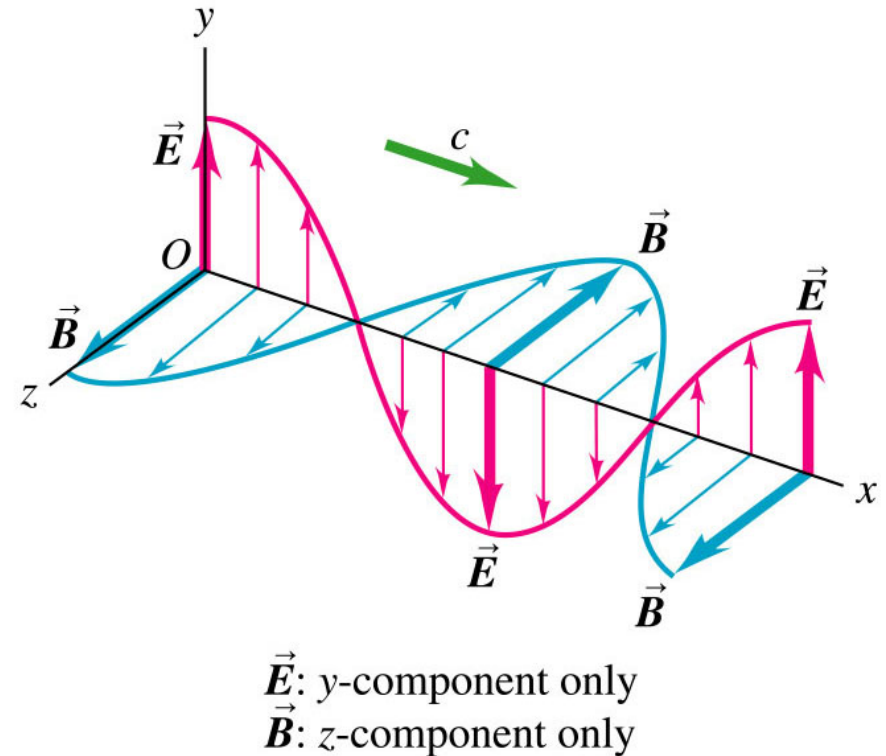
L

Time res

1.5GeV,electron

Hit-z (cm)

Polarization

Consider our EM plane wave. The E field is polarized in the Y-direction. We say this is "linearly Polarized light".

$$E_y = E_0 \sin(kx - \omega t)$$

$$B_z = \frac{E_0}{c} \sin(kx - \omega t)$$

$\vec{E}$: y-component only
$\vec{B}$: z-component only

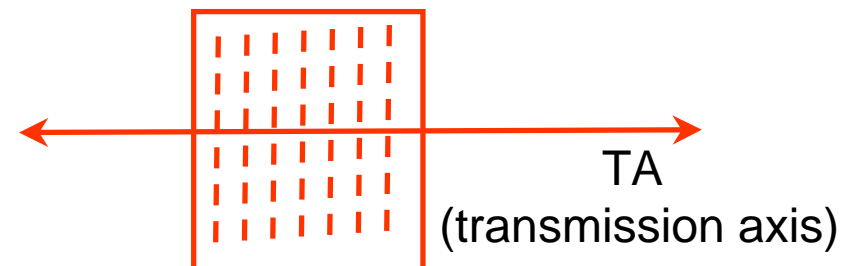2004 Pearson Education, Inc., publishing as Addison Wesley.

Most light sources are not polarized in a particular direction. This is called unpolarized light or radiation.
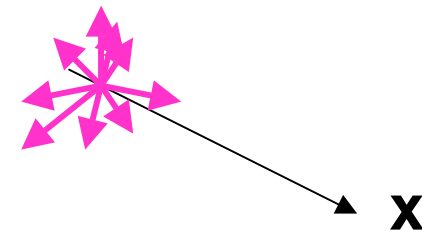
polaroid (sunglasses)

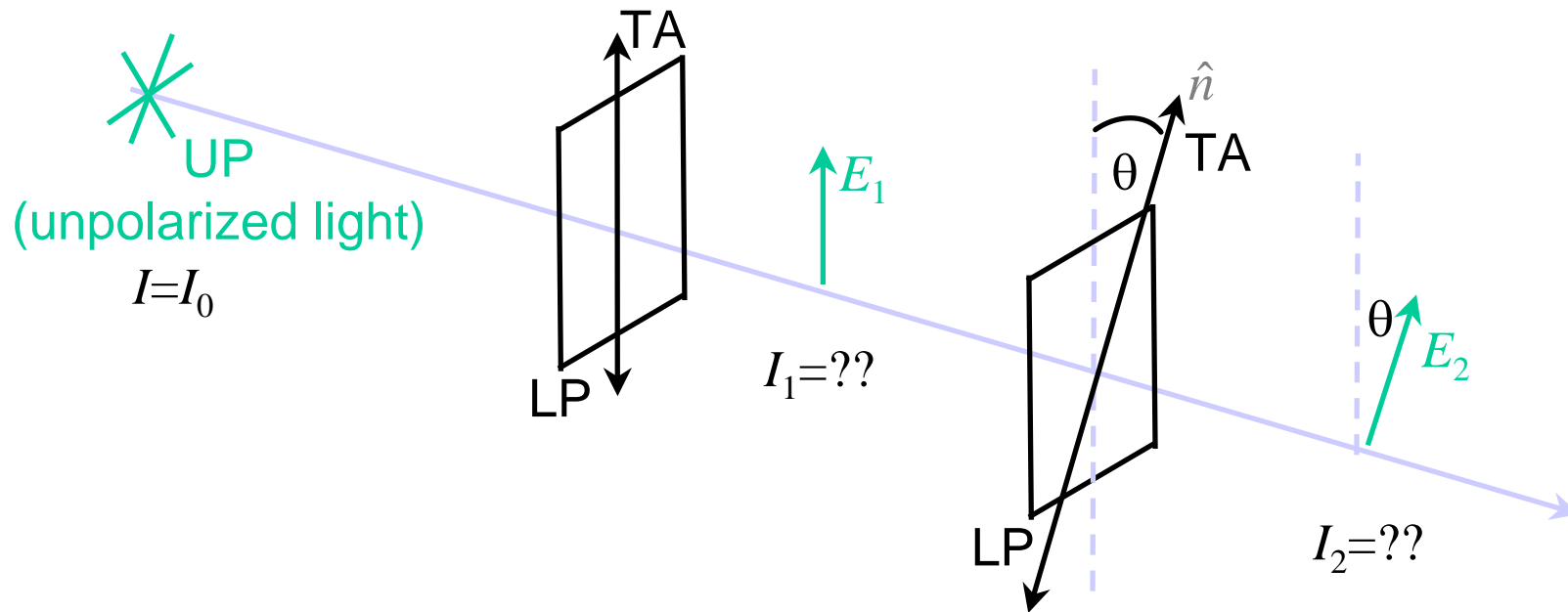Long molecules absorb E-field parallel to molecule.

TA
(transmission axis)

LP Intensity Reduction

- This set of two linear polarizers produces LP light. What is the final intensity?
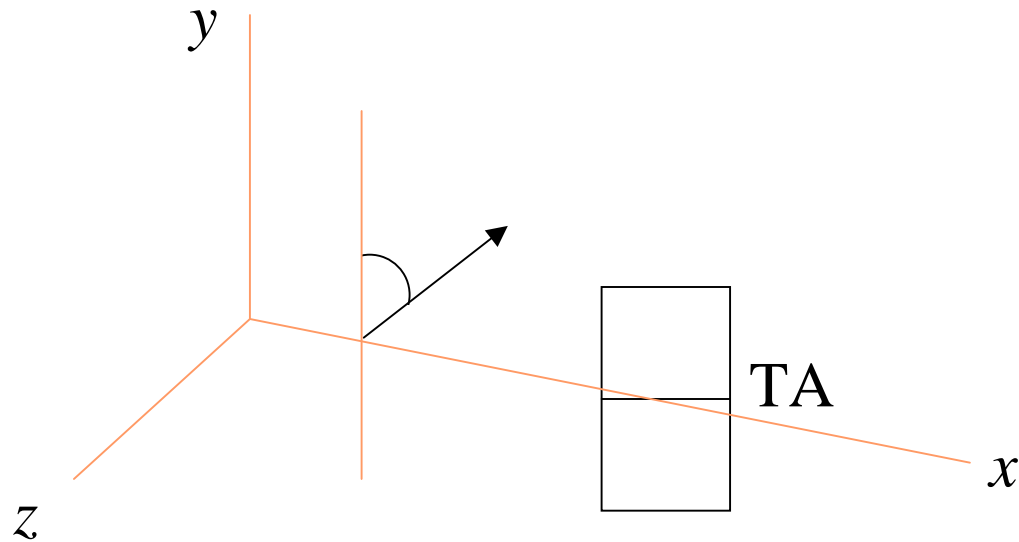 – First LP transmits 1/2 of the unpolarized light: $I_1 = 1/2$

  $$E_2 \stackrel{I_0}{=} E_1 \cos\theta \implies I_2 = I_1 \cos^2\theta \quad \textbf{Law of Malus}$$
  $$I \propto E^2$$

  – Second LP projects out the E-field component parallel to the TA:

y

z

TA
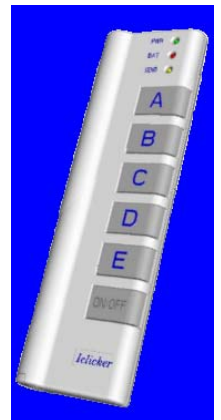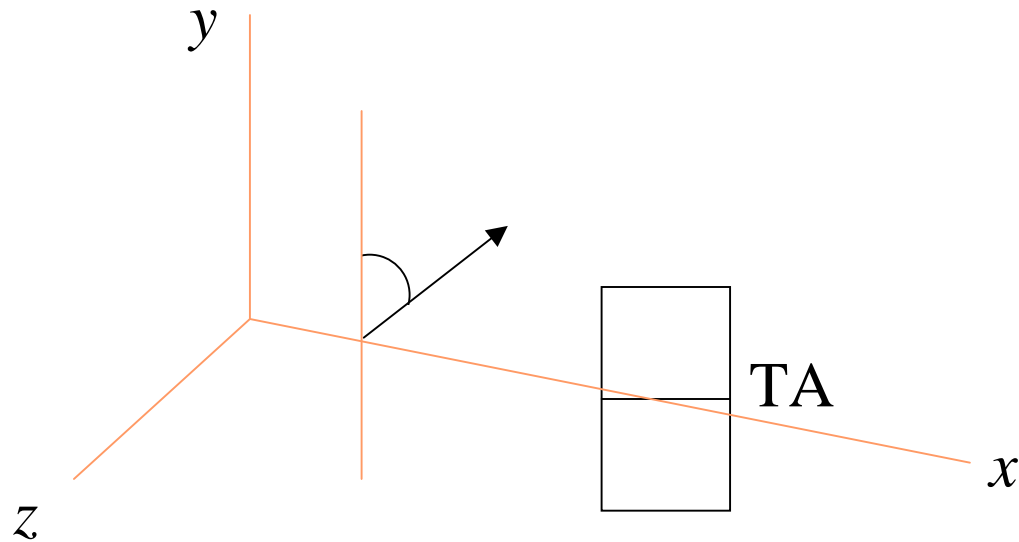
x

An EM wave is passed through a linear polarizer. Which component of the E-field is absorbed? The component of the E-field which is absorbed is _____ .

a) perpendicular to the transmission axis

b) parallel to the transmission axis

c) both components are absorbed

An EM wave is passed through a linear polarizer. Which component of the E-field is absorbed? The component of the E-field which is absorbed is _____ .
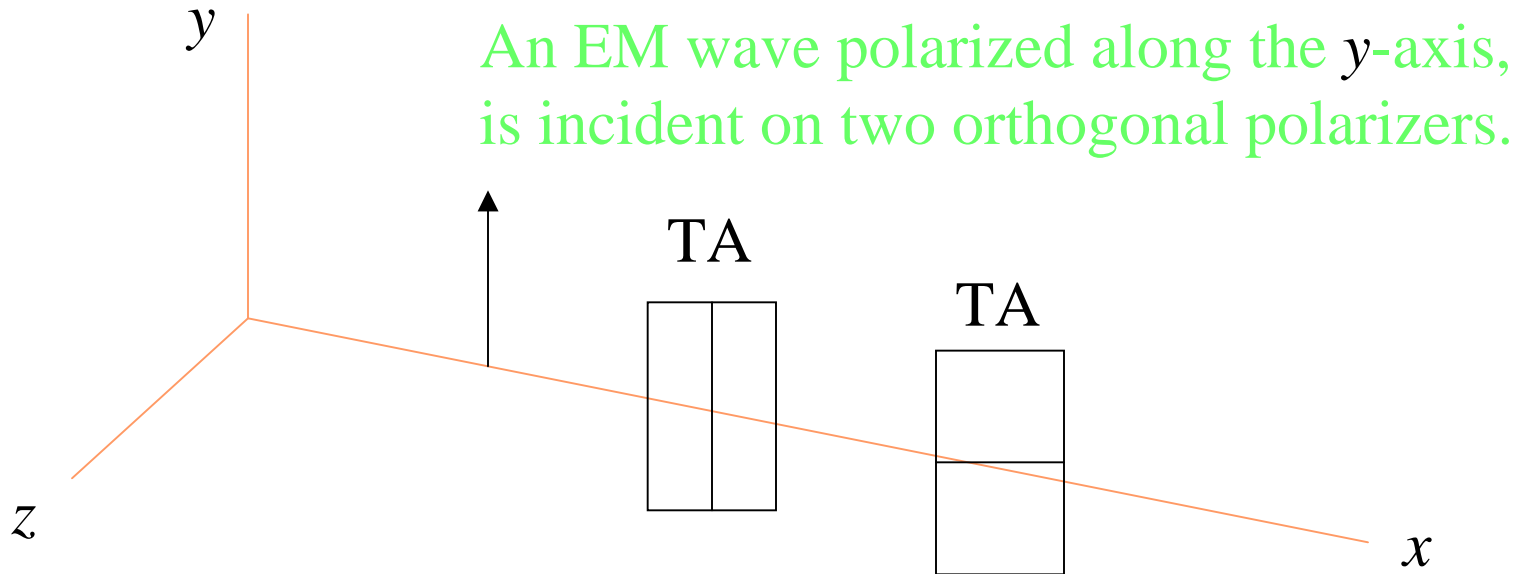
a) perpendicular to the transmission axis

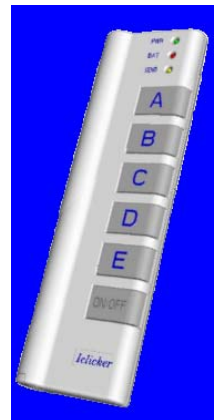b) parallel to the transmission axis

c) both components are absorbed

y

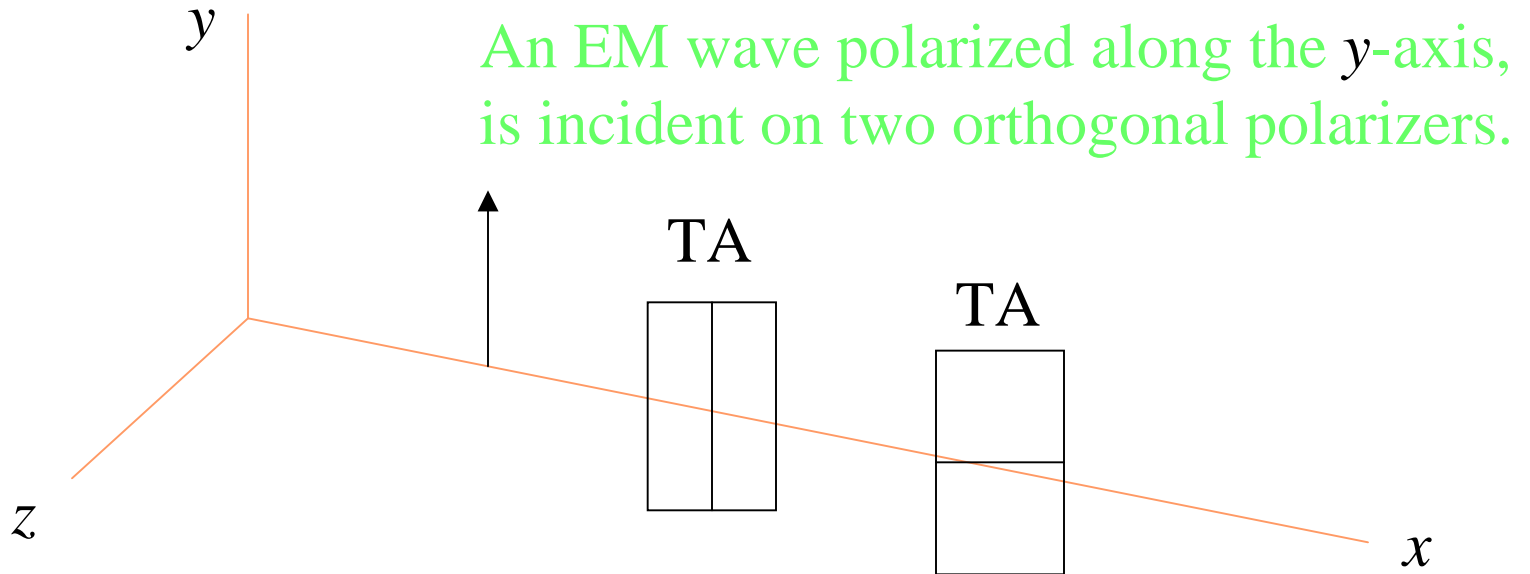An EM wave polarized along the y-axis, is incident on two orthogonal polarizers.

TA

TA

z

x

What percentage of the intensity gets through both polarizers?

a) 50%

b) 25%

c) 0%

y

An EM wave polarized along the y-axis, is incident on two orthogonal polarizers.

TA

TA

z

x

What percentage of the intensity gets through both polarizers?

a) 50%

b) 25%

c) 0%

y

An EM wave polarized along the *y*-axis,
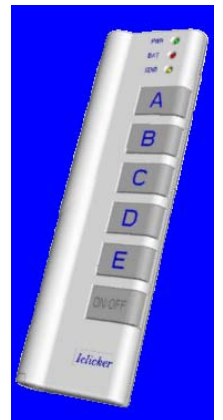is incident on two orthogonal polarizers.

TA

TA

z

x
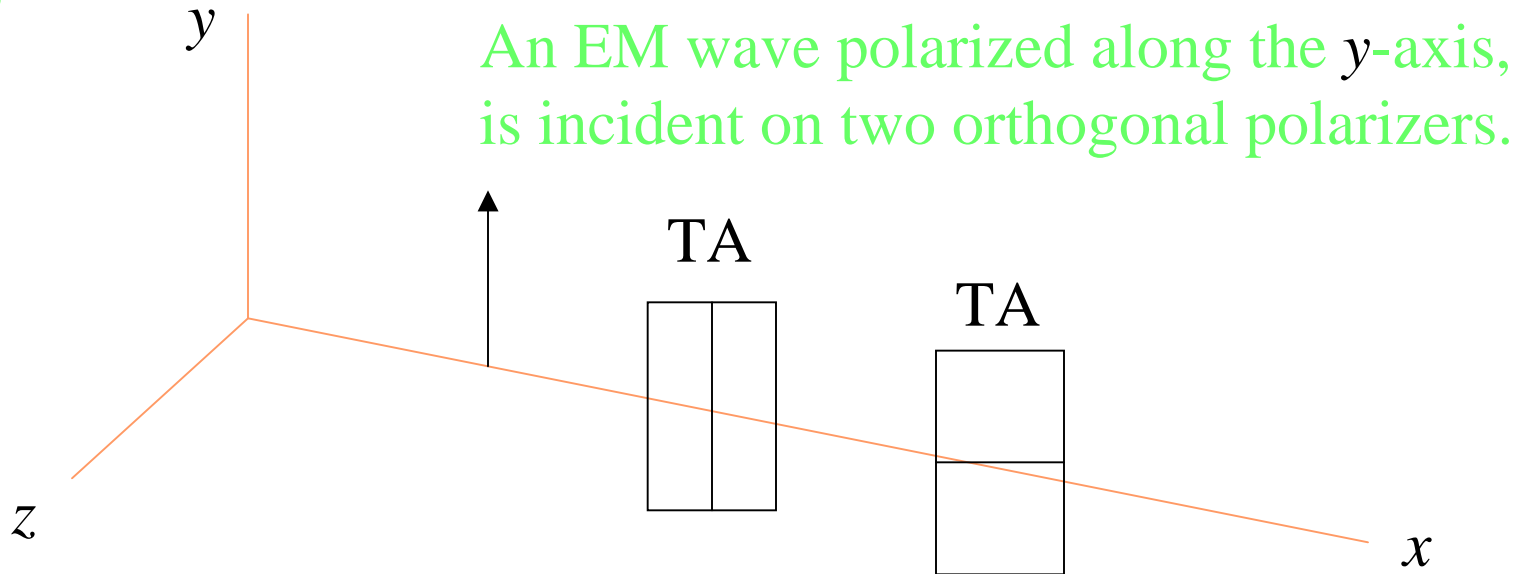
Is it possible to increase this percentage by inserting another polarizer
between the original two? (Explain.)

a) Yes

b) No

y

An EM wave polarized along the *y*-axis, is incident on two orthogonal polarizers.
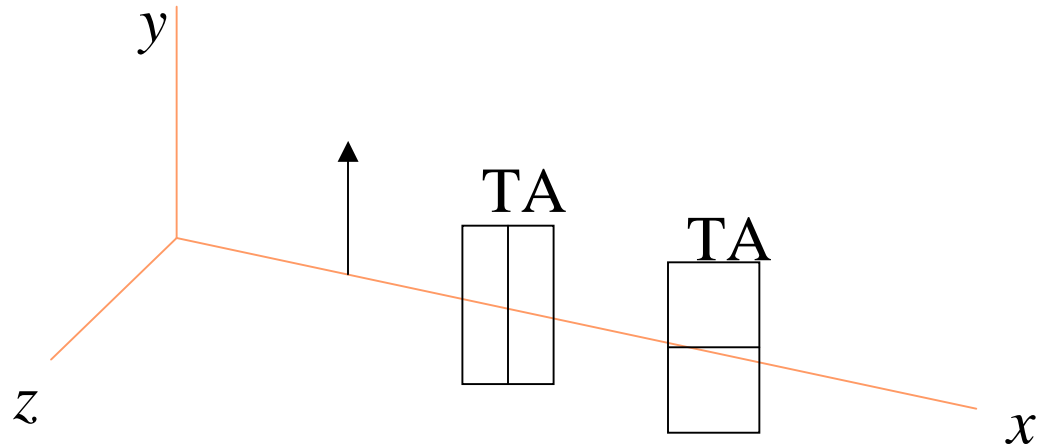
TA

TA

z

x

Is it possible to increase this percentage by inserting another polarizer between the original two? (Explain.)

a) Yes

b) No

What you said ??

• ZERO: the component that is passed by the first is blocked by the second.
• BUT, adding an intermediate polarizer will restore some light !!!

Question 7

- Light of intensity I_0, polarized along the x direction is incident on a set of 2 linear polarizers as shown.

  – Assuming $\theta = 45°$, what is I_{45}, the intensity at the exit of the 2 polarizers, in terms of I_0?
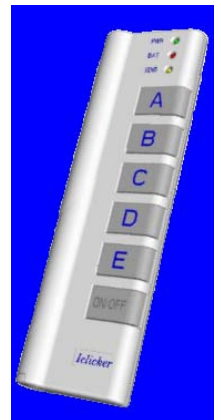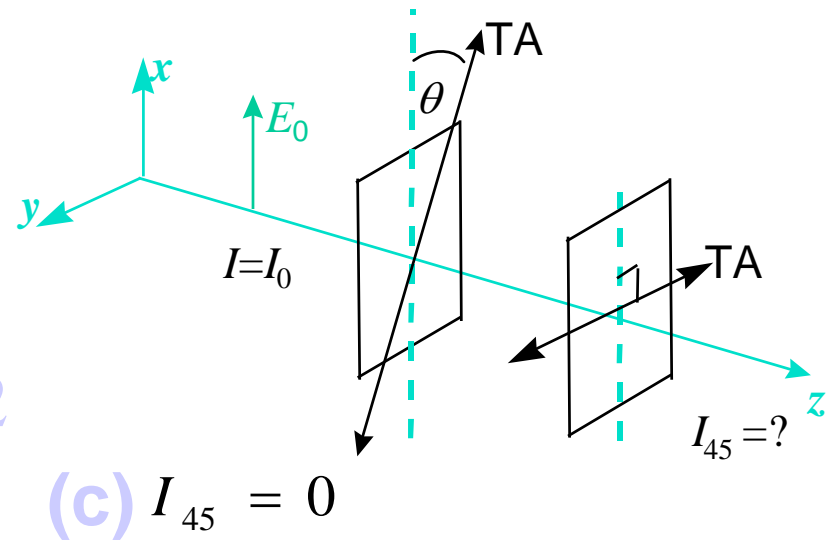
  (a) $I_{45} = \dfrac{1}{2}I_0$ (b) $I_{45} = \dfrac{1}{4}I_0$ (c) $I_{45} = 0$

Question 7

- Light of intensity I_0, polarized along the x direction is incident on a set of 2 linear polarizers as shown.

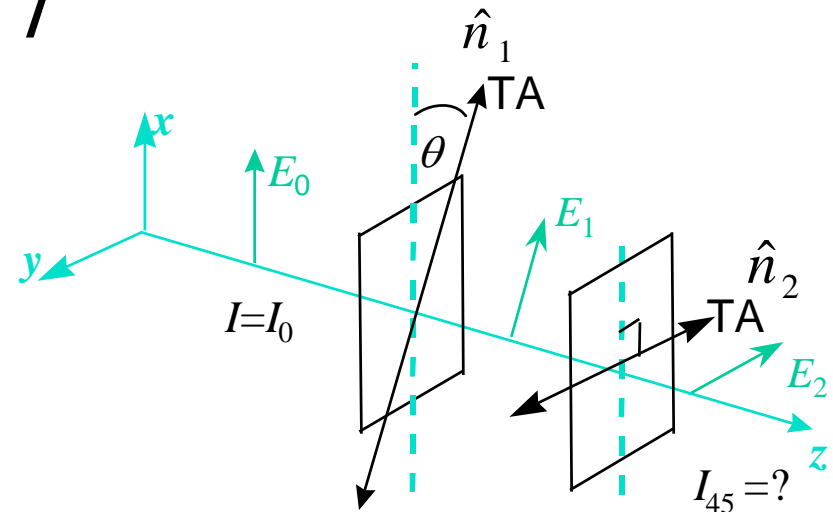 – Assuming $\theta = 45°$, what is I_{45}?
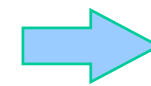
  (a) $I_{45} = \dfrac{1}{2} I_0$ **(b)** $I_{45} = \dfrac{1}{4} I_0$ (c) $I_{45} = 0$

  the intensity at the exit of the 2 polarizers, in terms of I_0?

- **We proceed through each polarizer in turn.**
  - **The intensity after the first polarizer is:** $I_1 = I_0 \cos^2(45 - 0) = \dfrac{I_0}{2}$
  - **The electric field after the first polarizer is LP at $\theta_1 = 45$.**
  - **The intensity after the second polarizer is:**

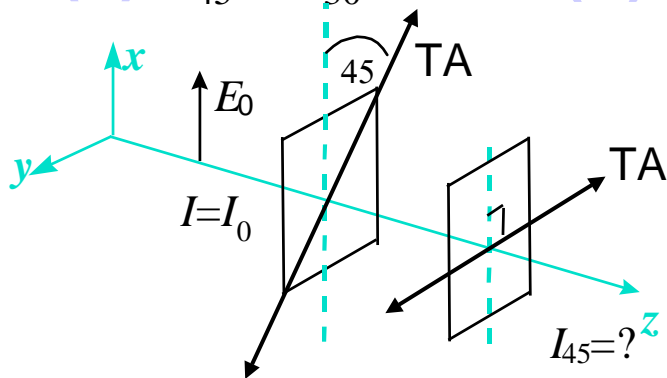$$I_{45} = I_1 \cos^2(90 - 45) = \dfrac{I_1}{2} \qquad \Rightarrow \qquad \boxed{I_{45} = \dfrac{1}{4} I_0}$$

Question 8

- Light of intensity I_0, polarized along the x direction is incident on a set of 2 linear polarizers as shown.
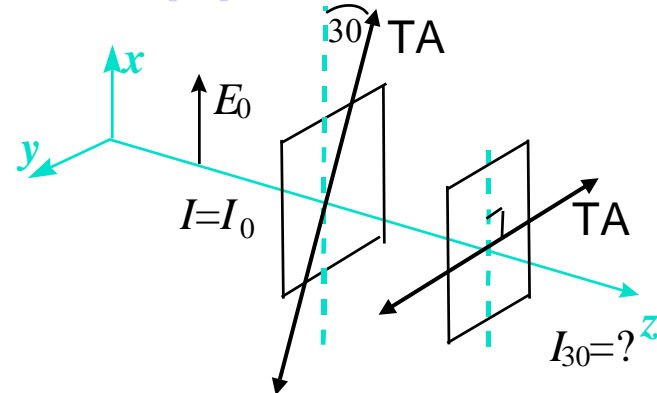
  - What is the relation between I_{45} and I_{30}, the final intensities in the situation above when the angle $\theta = 45°$ and $30°$, respectively?

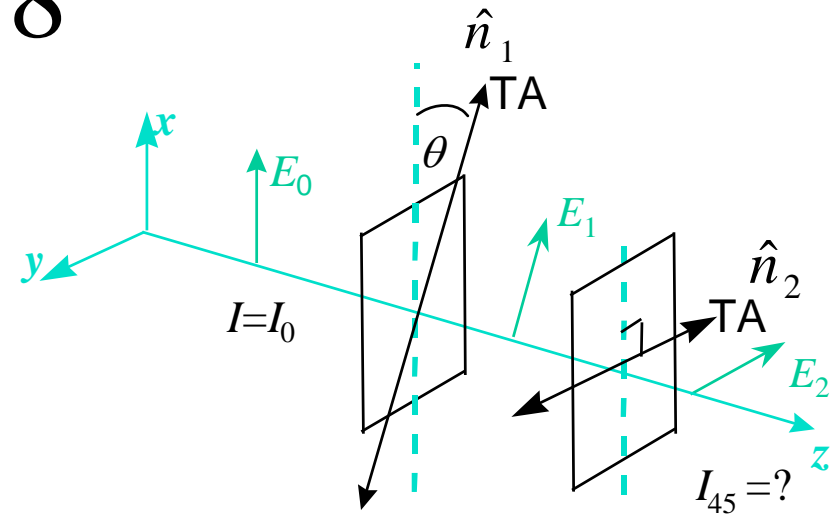**(a)** $I_{45} < I_{30}$ **(b)** $I_{45} = I_{30}$ **(c)** $I_{45} > I_{30}$

Question 8

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(a) $I_{45} < I_{30}$ (b) $I_{45} = I_{30}$ (c) $I_{45} > I_{30}$
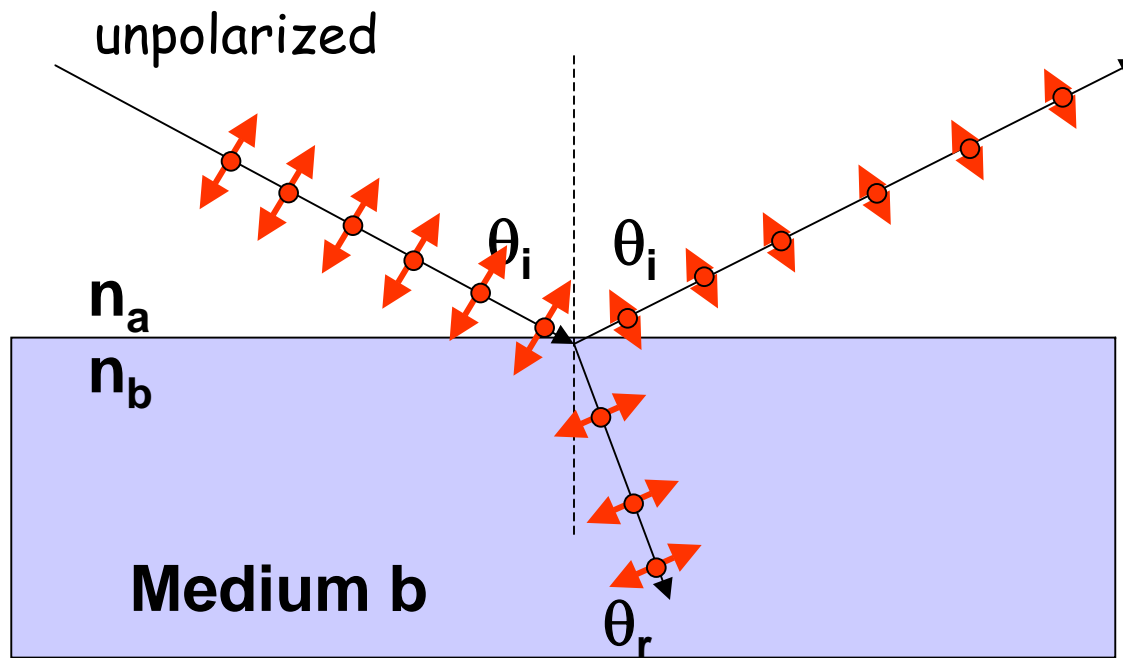
- In general, the first polarizer reduces the intensity by $\cos^2\theta$, while the second polarizer reduces it by an additional factor of $\cos^2(90 - \theta)$.
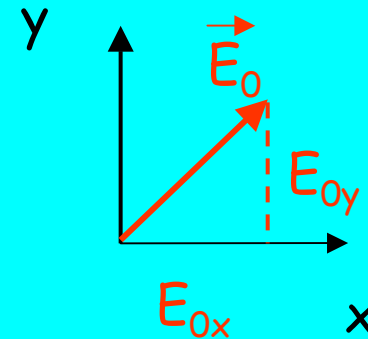- Thus, the final output intensity is given by:

$$I_{out} = I_0 \cos^2(\theta) \cos^2(90-\theta) = I_0 \cos^2(\theta) \sin^2(\theta) \propto \sin^2(2\theta)$$

This has a maximum when $\theta = 45°$.

Polarization by reflection

unpolarized

n_a

n_b

θ_i θ_i

Medium b

θ_r

y

$\vec{E_0}$

E_{0y}

E_{0x} x

Can always describe E in terms of components in two arbitrary directions. The components are equal for unpolarized light.

The reflected rays are partially polarized in the horizontal plane. The transmitted rays are also partially polarized.

Polarization by reflection

n_a

n_b

**Medium b**

θ_p θ_p

θ_r
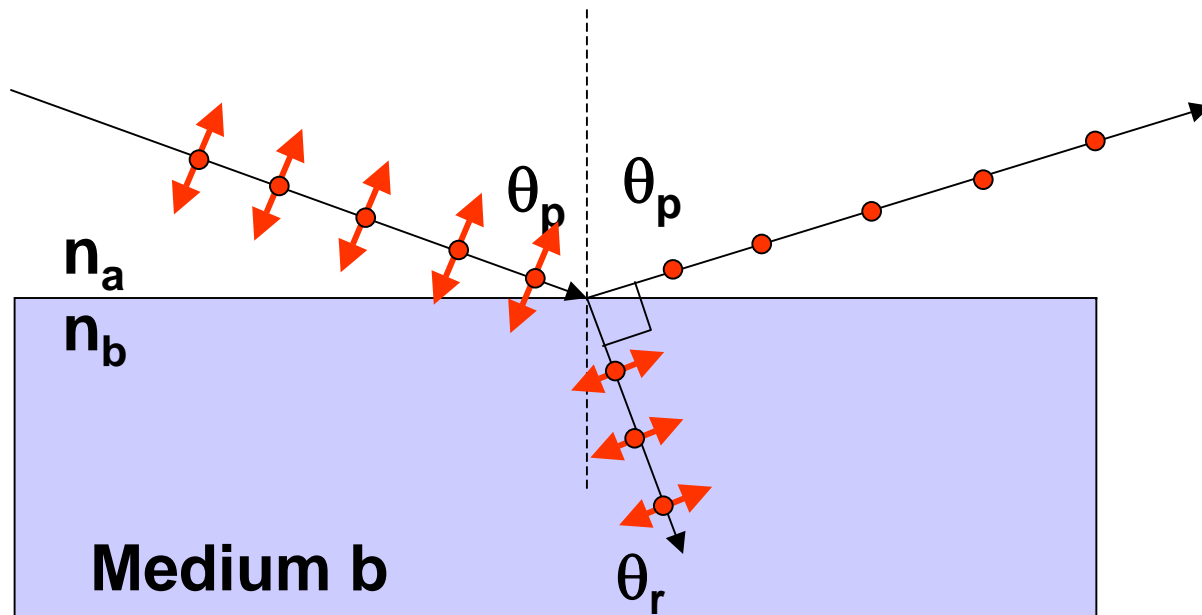
For a certain angle, the <u>Brewster angle</u>, the reflected light is completely polarized in the horizontal plane. This occurs when the angle between the refl. and refr. rays is 90°.

From Maxwell's eqn. it can be shown that Brewster's angle is given by
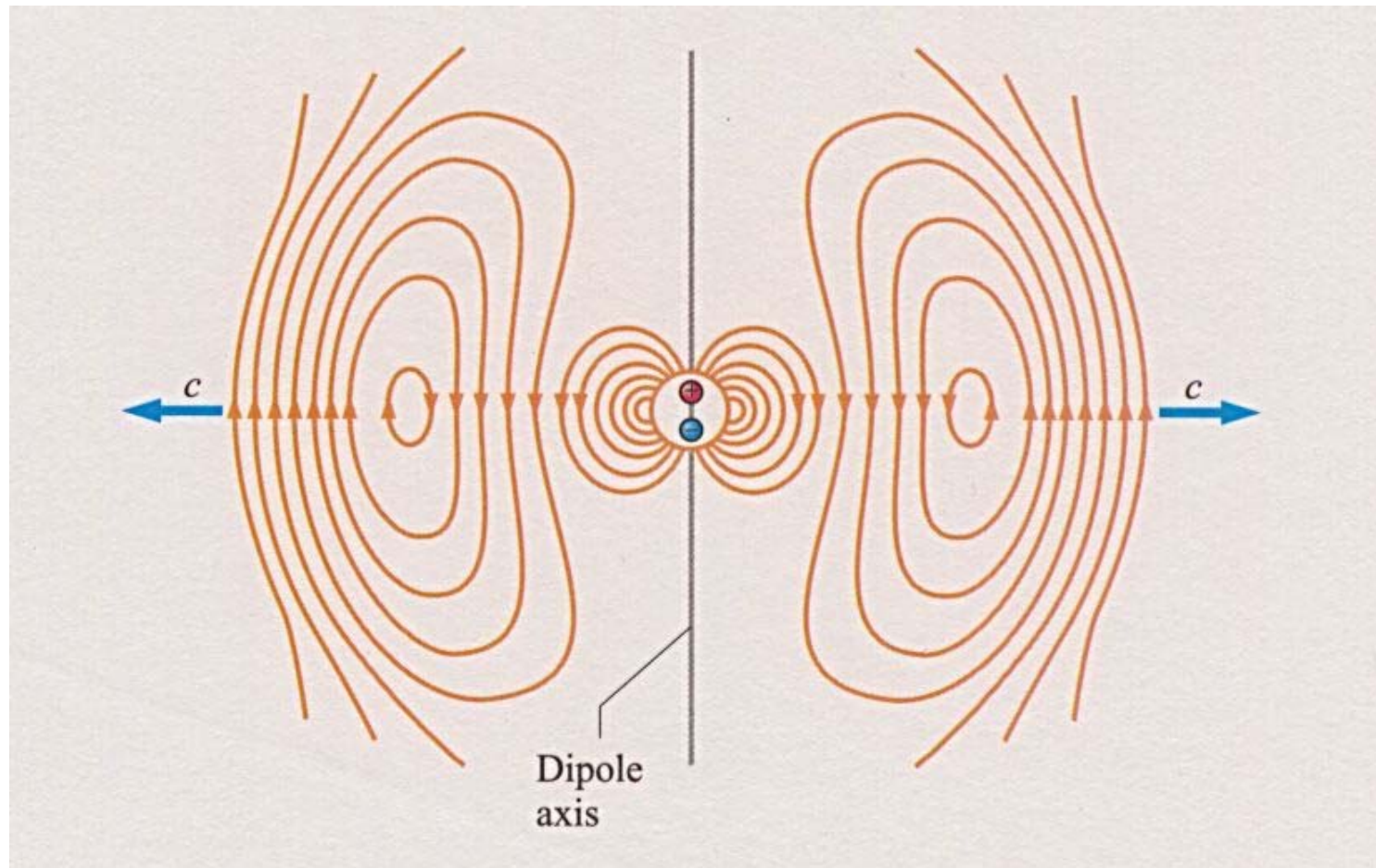
$$\tan \theta_p = \frac{n_b}{n_a}$$

Light reflected on dashboard to the windshield will be polarized in the horizontal plane. Using polaroid dark glasses with a vertical axis will remove most of reflected light
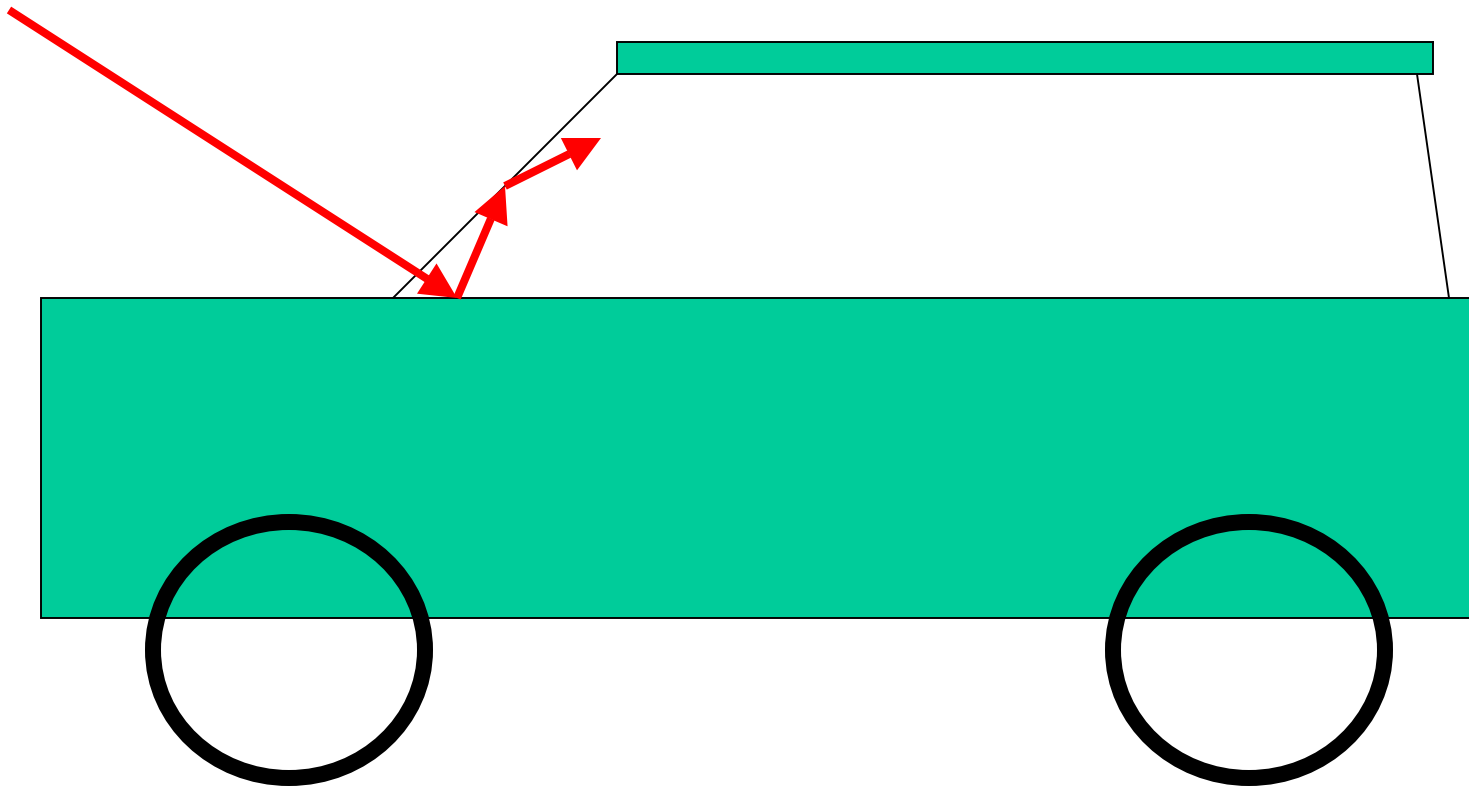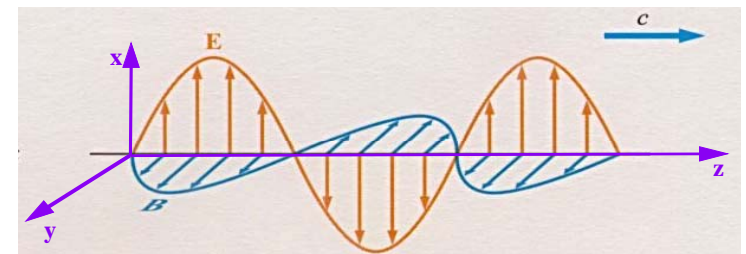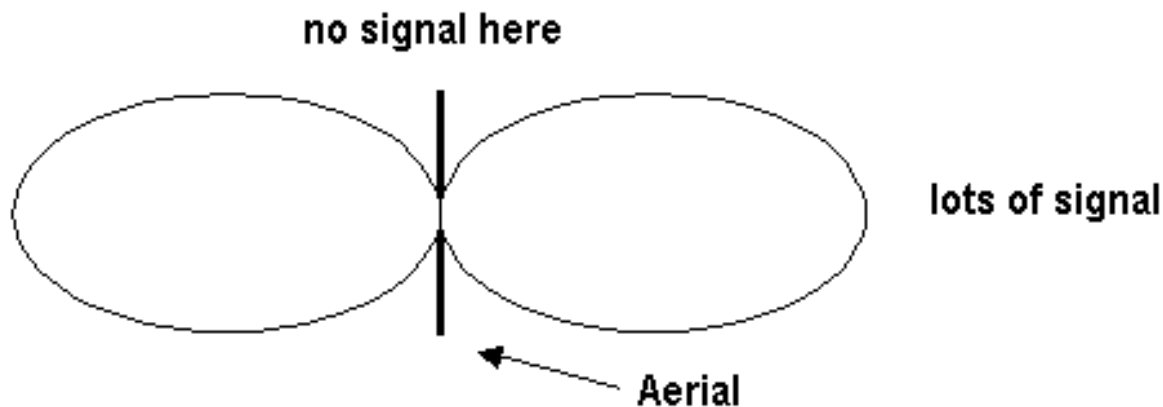
Electric field lines from oscillating dipole

computer simulation - a snapshot in time

Dipole radiation pattern

- Oscillating electric dipole generates e-m radiation that is **linearly polarized** in the direction of the dipole
- Radiation pattern is doughnut shaped & outward traveling
  - zero amplitude above and below dipole

For next time

- Homework #12 posted → due today

- The home stretch: optics/optical phenomenon

- Quiz #6 on Friday (E&M waves, refraction [Snell's Law])