Course Updates

http://www.phys.hawaii.edu/~varner/PHYS272-Spr10/physics272.html

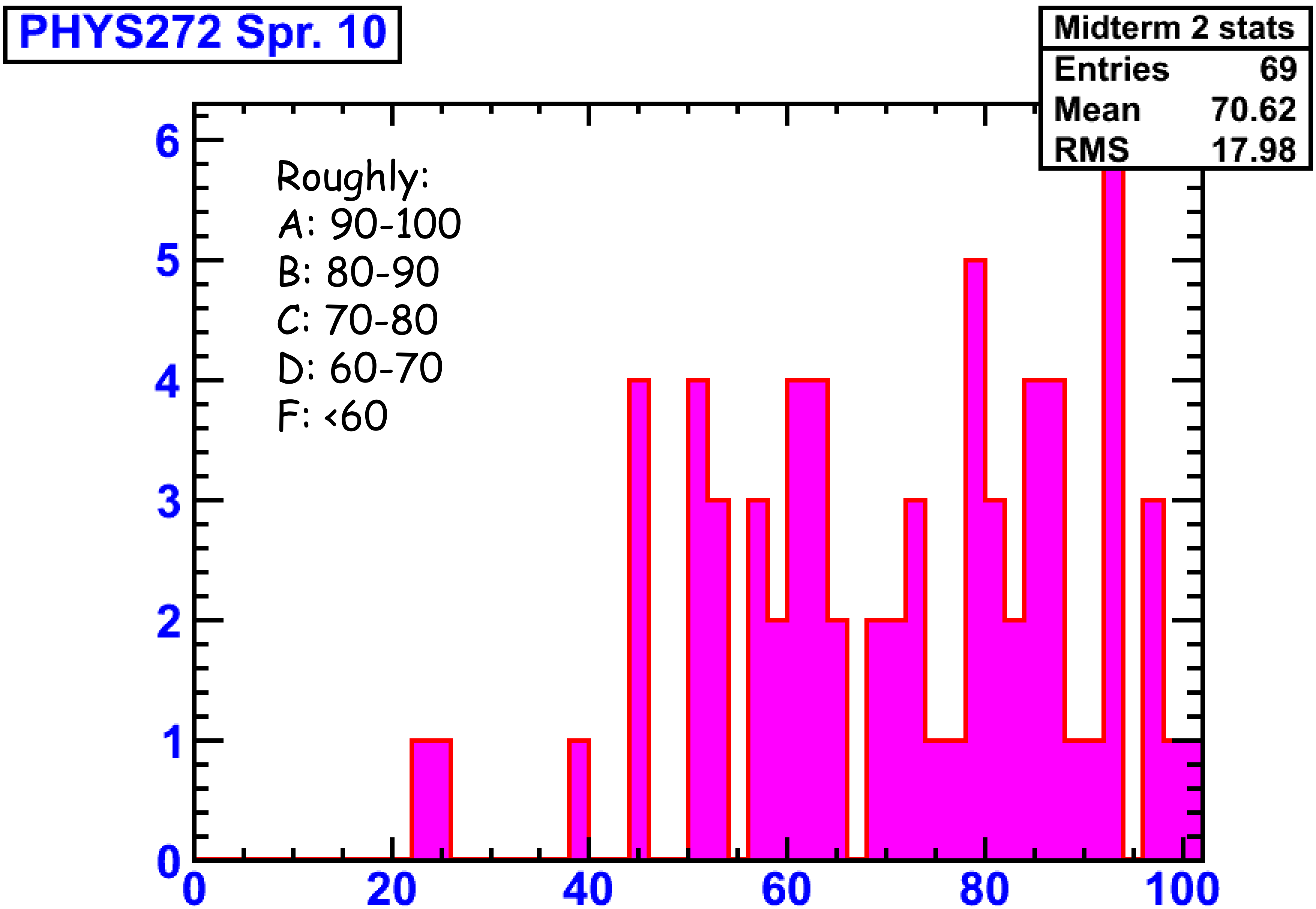
Reminders:

1) Assignment #11 → due today

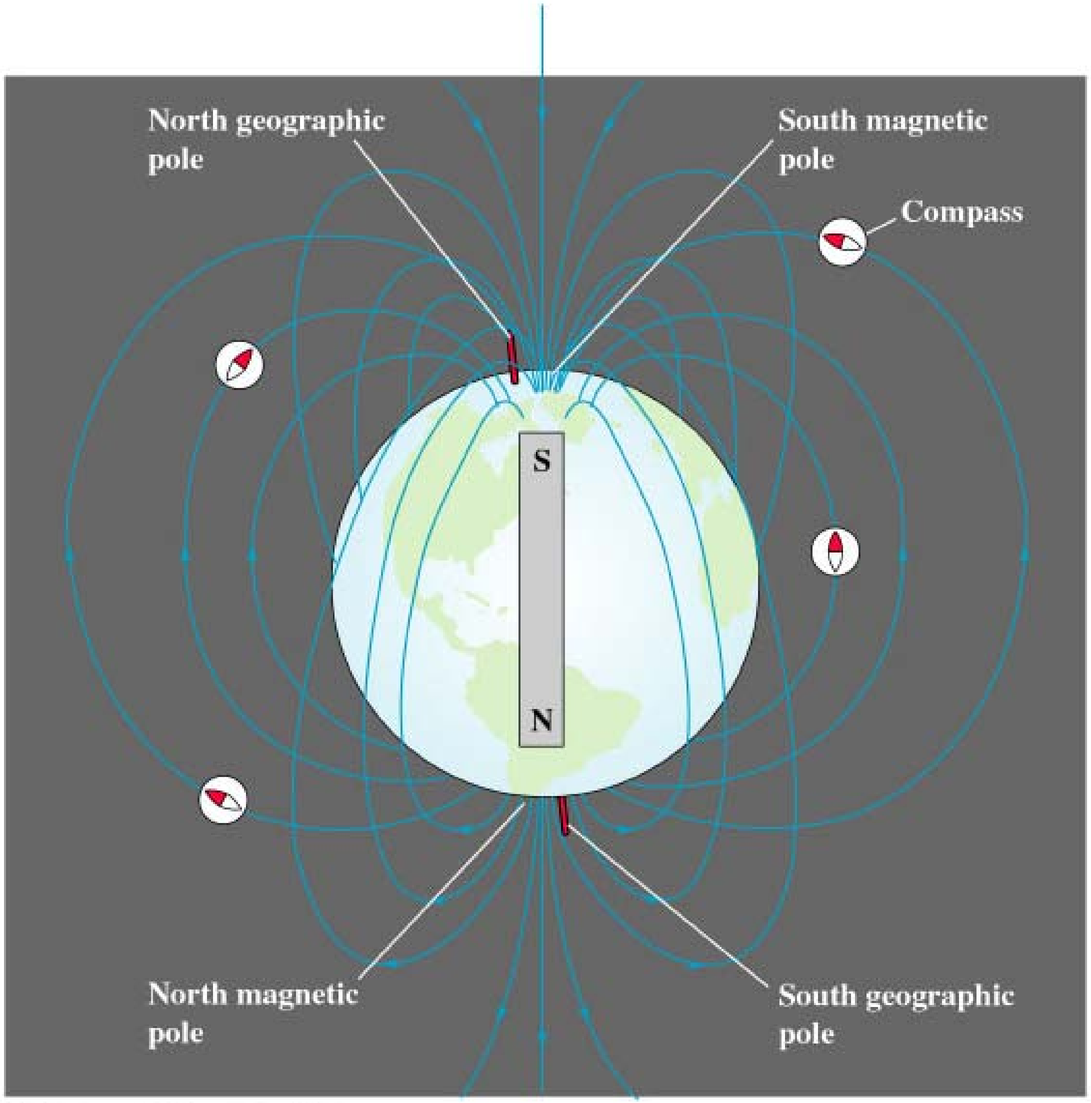
2) Complete Electromagnetic Waves

3) First – updates …

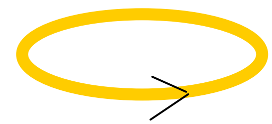
Midterm 2 summary

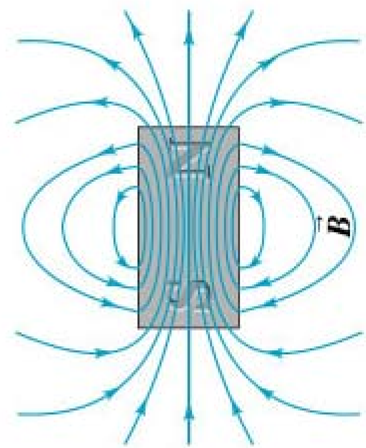


Problem problems



Const
Current I

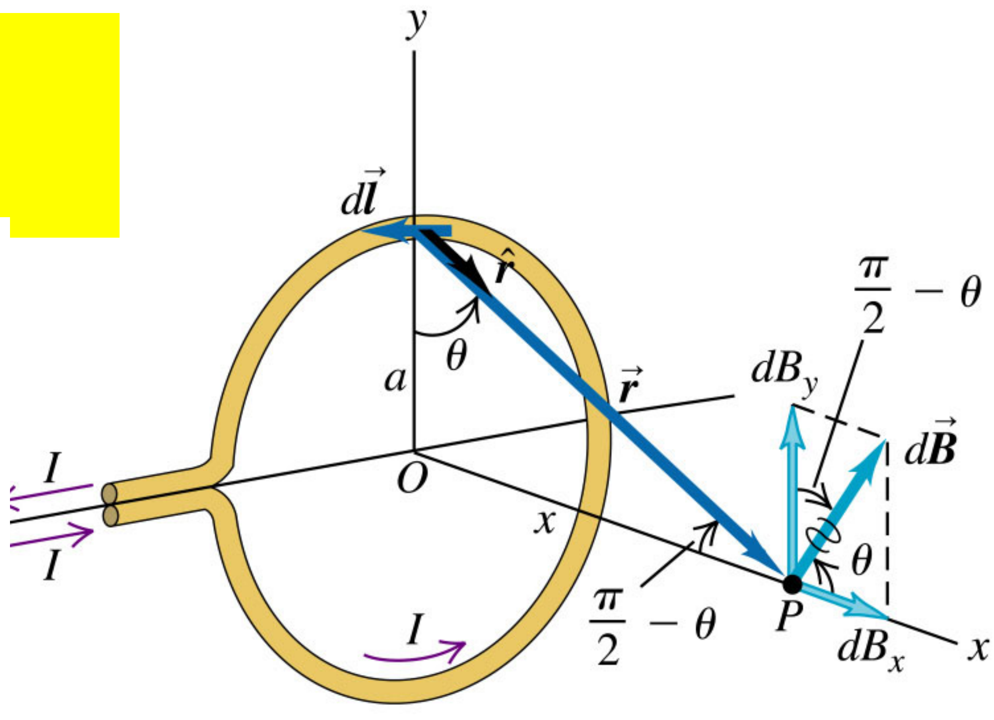




stationary magnet

Magnetic Field of a wire loop

Suppose a wire loop is centered at the origin in the y-z plane. What is the B field along the center line axis (x-axis)? By symmetry, the net B field must be only along the x-axis as the y and z components will cancel.



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Idl \times \hat{r}}{r^2}$$

$$dB_x = dB\cos\theta = dB\frac{a}{R}$$

$$dB = \frac{\mu_0 Idl}{4\pi r^2}$$

$$B_x = \frac{\mu_0 I}{4\pi}\int\left(\frac{1}{r^2}\right)\left(\frac{a}{r}\right)dl = \frac{\mu_0 I}{4\pi}\frac{a}{r^3}\int dl$$

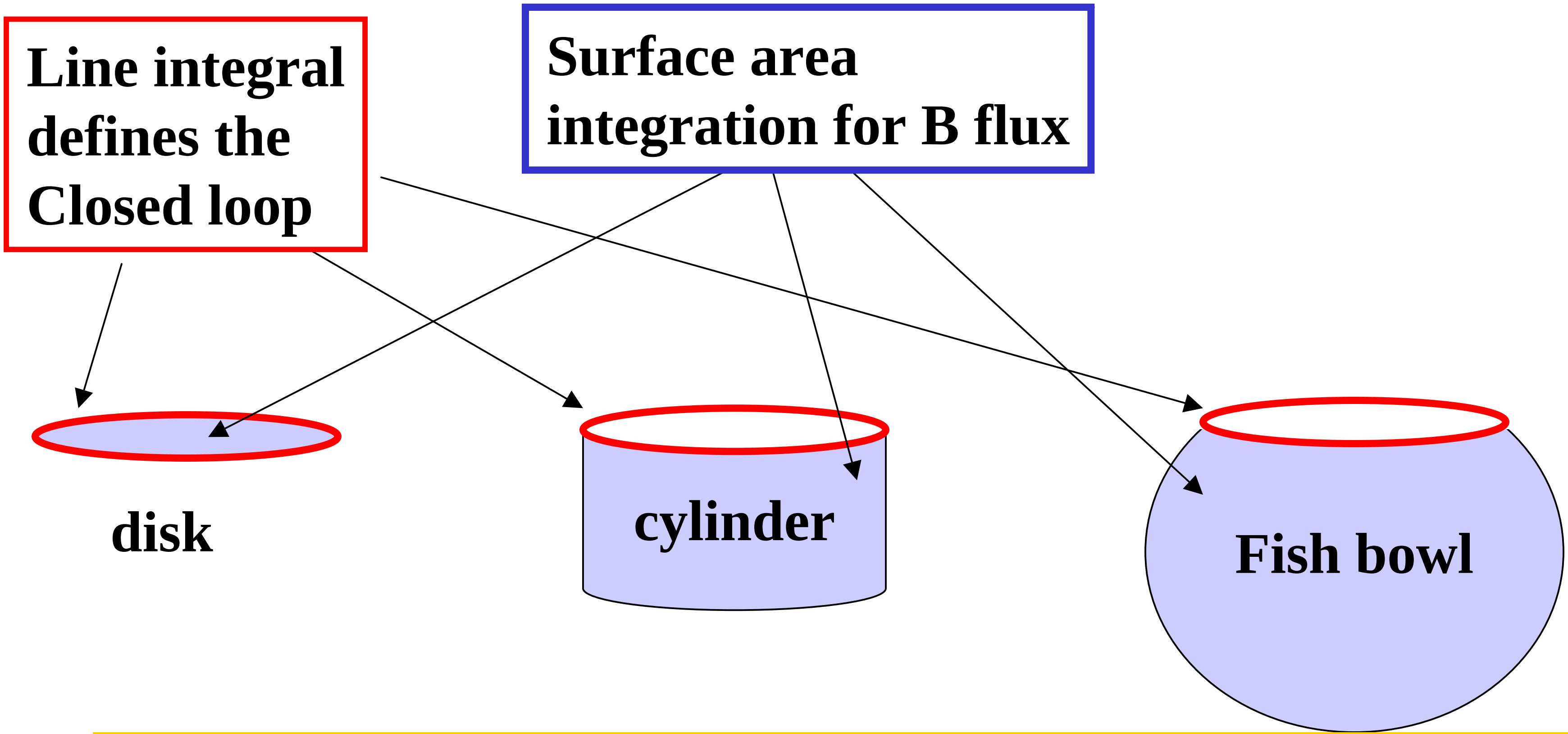
$$= \frac{\mu_0 I}{4\pi}\frac{a}{r^3}2\pi a = \boxed{\frac{\mu_0 I a^2}{2\sqrt{x^2+a^2}^{\,3}}}$$

<u>Problem 4 c</u>: a=1200km, x=6360km, B = 0.5×10^{-4} T, solve for I

Remarks on Faraday's Law with different attached surfaces

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d \int \vec{B} \cdot d\vec{A}}{dt}$$

Faraday's Law works for any closed Loop and <u>ANY</u> attached surface area

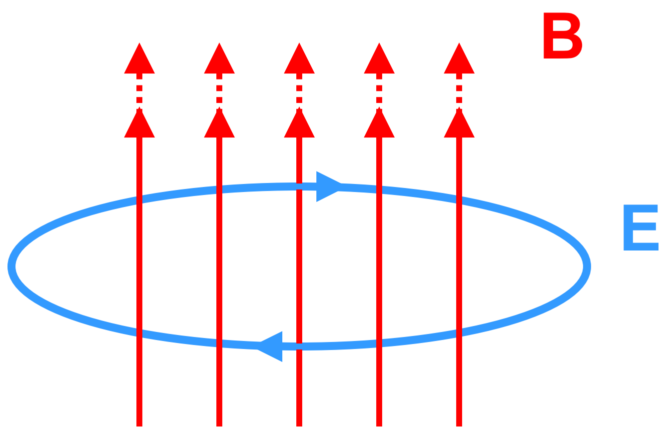


This is proved in Vector Calculus with Stoke's Theorem

Summary of Faraday's Law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

If we form any closed loop, the line integral of the electric field equals the time rate change of magnetic flux through the surface enclosed by the loop.

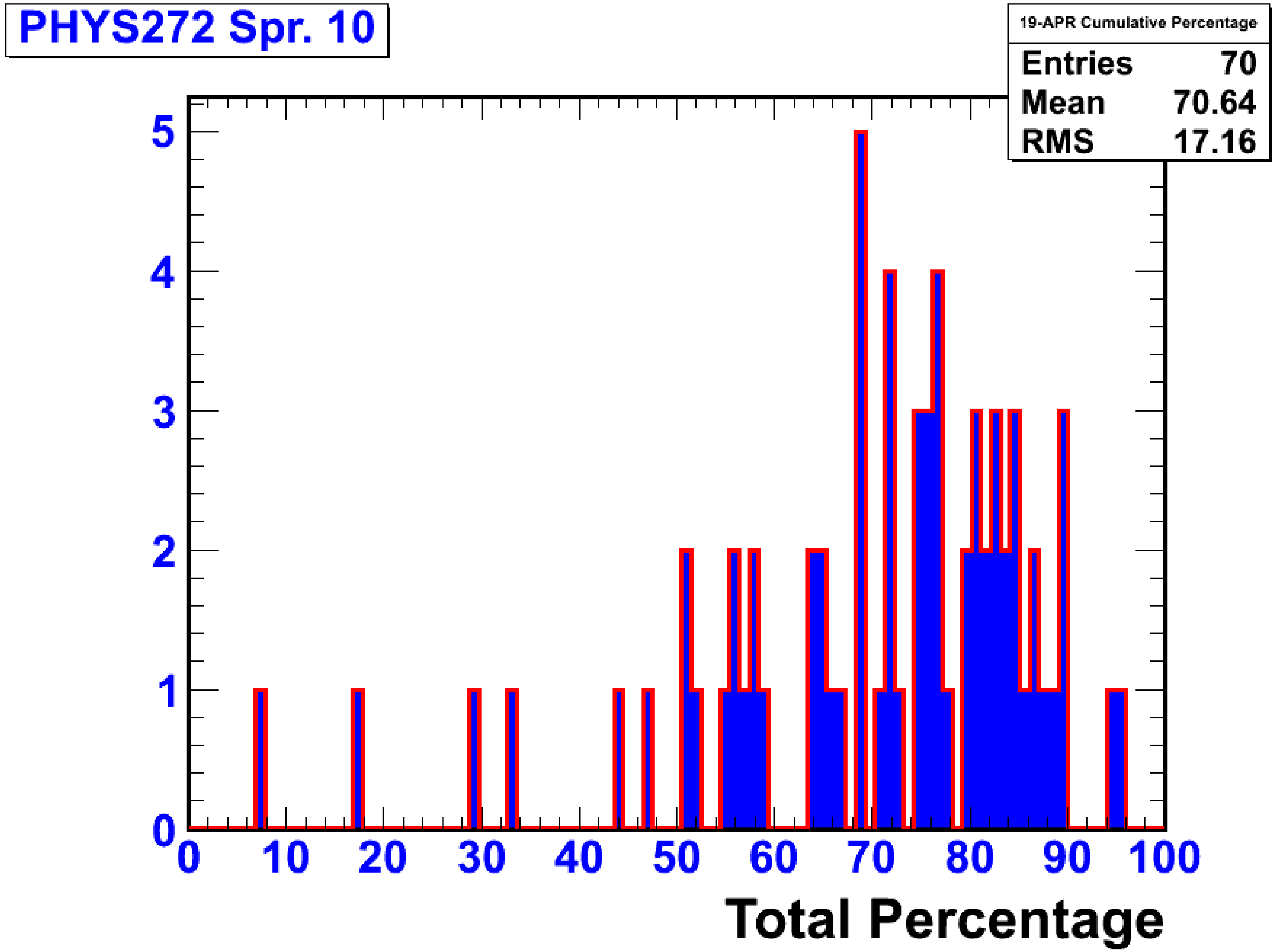


If there is a changing magnetic field, then there will be electric fields induced in closed paths. The electric fields direction will tend to reduce the changing B field.

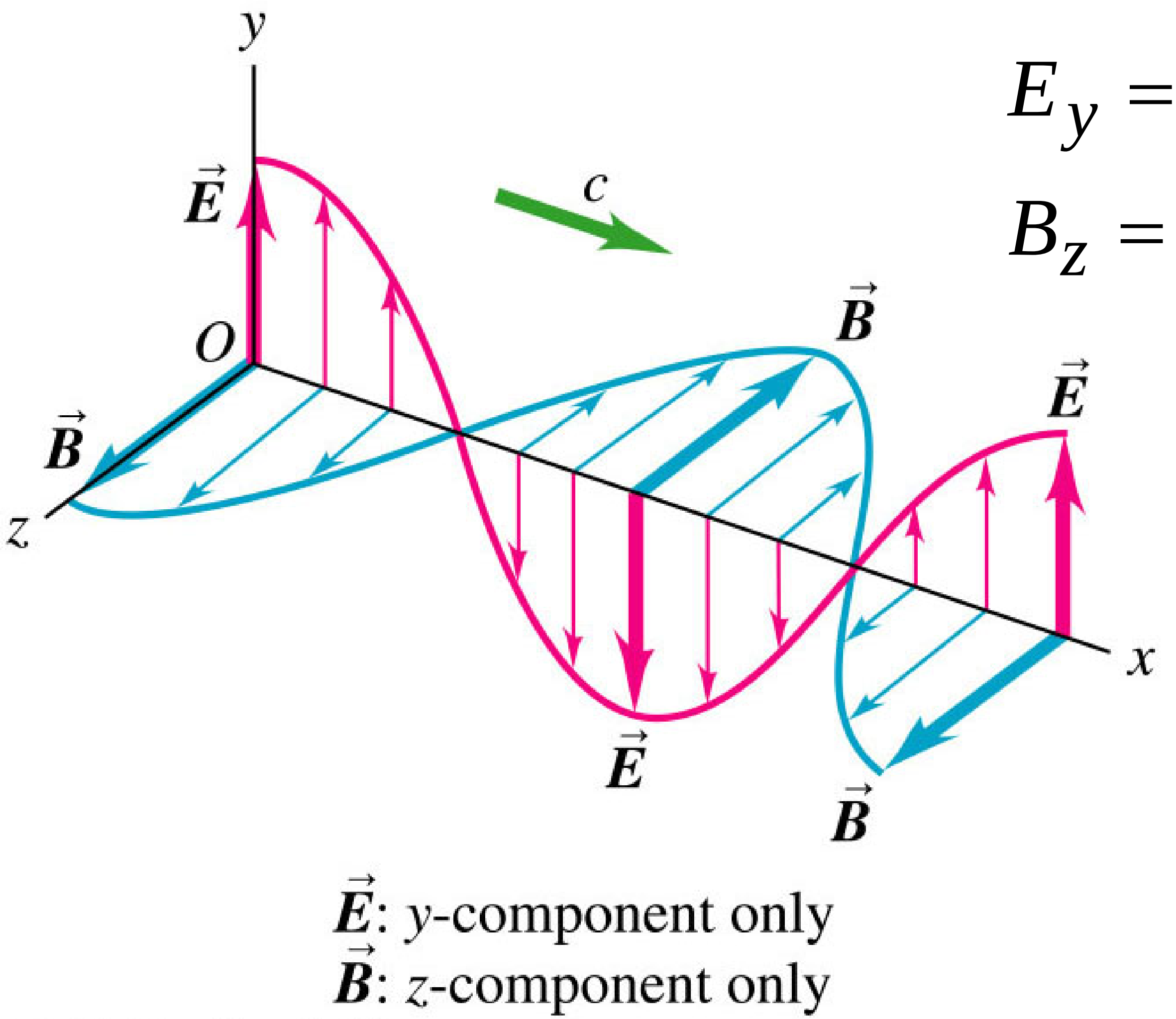
<u>Problem 4 d</u>: R=6360km,
B = 0.5×10^{-4} T, solve for $d\Phi_B/dt$

dt = 1s
$E(2\pi r) = -B\times A$
$A = \pi r^2$

Course summary



sinusoidal EM wave solutions; moving in +x



$$E_y = E_{max}\cos(kx - \omega t)$$

$$B_z = B_{max}\cos(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda}$$

$$\omega = 2\pi f$$

$$\lambda f = c = \frac{\omega}{k}$$

EM wave; energy momentum, angular momentum

Previously, we demonstrated the energy density existed in E fields in Capacitor and in B fields in inductors. We can sum these energies,

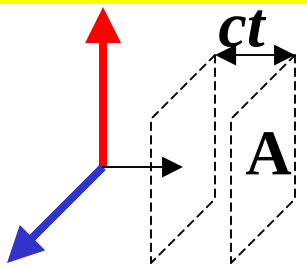
$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\mu_0 B^2$$

Since, *E=cB* and $c = 1/\sqrt{\mu_0\varepsilon_0}$ **then *u* in terms of E,**

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\mu_0\left(\frac{E}{c}\right)^2 = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\varepsilon_0 E^2 = \varepsilon_0 E^2$$

=> EM wave has energy and can transport energy at speed c

EM wave; energy flow & Poynting Vector



Suppose we have transverse E and B fields moving, at velocity c, to the right. If the energy density is

$$u = \varepsilon_0 E^2$$

and the field is moving through area A at velocity c covers volume *Act*, that has energy, *uAct*. Hence the <u>amount of energy flow per unit time per unit surface area</u> is,

$$\frac{1}{A}\frac{dU}{dt} = \frac{1}{At}(uAct) = uc = \varepsilon_0 cE^2$$

This quantity is called, *S*. We can write this in terms of E&B fields,

$$S = \varepsilon_0 cE^2 = \varepsilon_0 cEcE / c = \varepsilon_0 cEcB = EB / \mu_0$$

We can also define the energy flow with direction, called Poynting vector, $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$**, which provides direction of propagation**

Comment on Poynting Vector

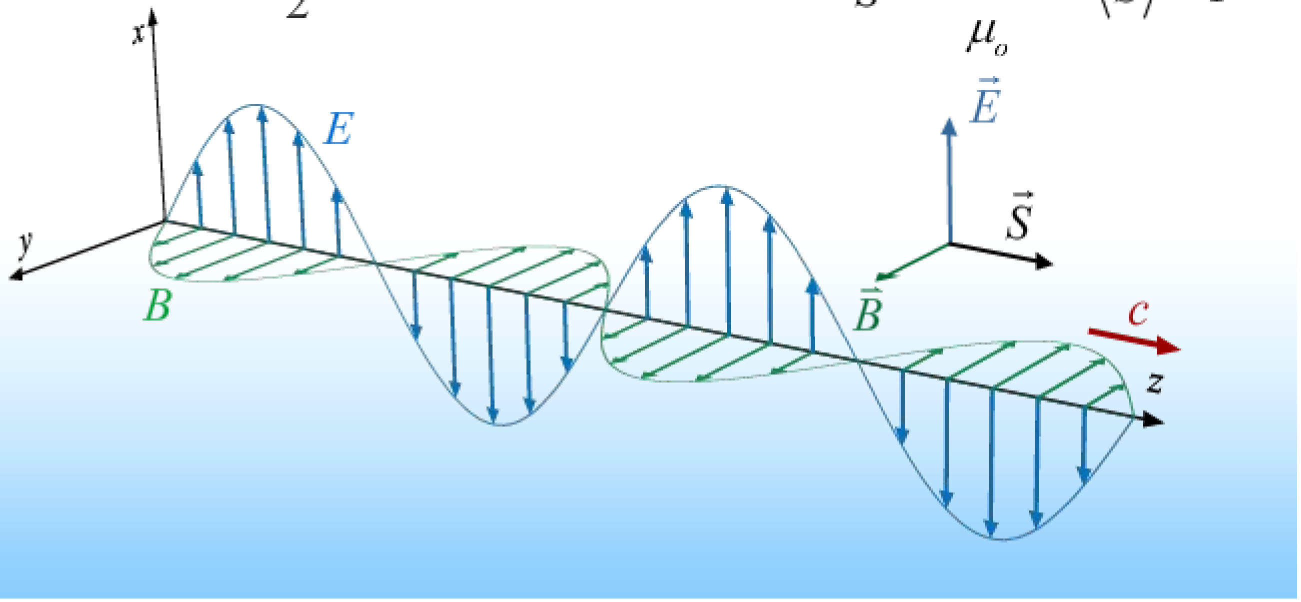
Just another way to keep track of all this
- Its magnitude is equal to I
- Its direction is the direction of propagation of the wave

Average Energy Density

$$\langle u \rangle = \frac{1}{2}\varepsilon_o E_o^{\,2}$$

Poynting Vector

$$\vec{S} \equiv \frac{\vec{E}\times\vec{B}}{\mu_o} \qquad \langle S \rangle = I$$



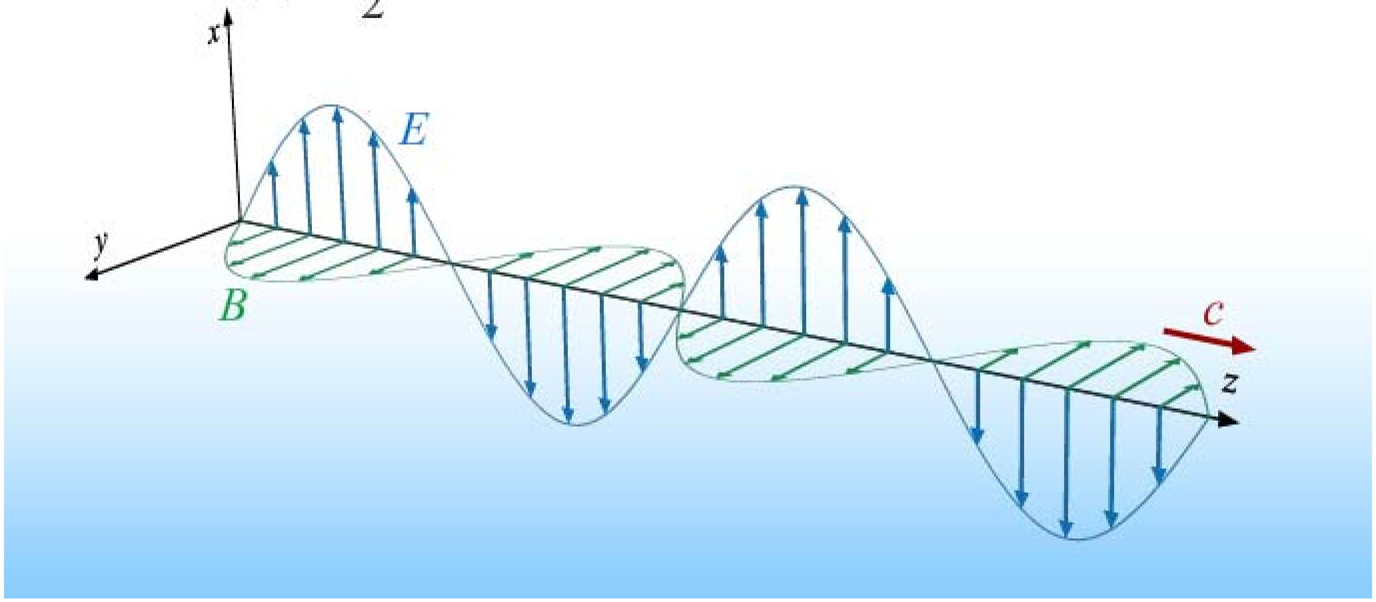
Waves Carry Energy

Total Energy Density

$$u = \varepsilon_o E^2$$

Average Energy Density

$$\langle u \rangle = \frac{1}{2}\varepsilon_o E_o^{\,2}$$



EM wave; Intensity

The intensity, I, is the time average of the magnitude of the Poynting vector and represents the <u>energy flow per unit area</u>.

$$I = \left\langle \vec{S}(x,y,z,t) \right\rangle = \frac{1}{\mu_0} \left\langle \vec{E}(x,y,z,t) \times \vec{B}(x,y,z,t) \right\rangle$$

In Y&F, section 32.4, the average for a sinusoidal plane wave \(squared) is worked out,

$$I = \left\langle \vec{S}(x,y,z,t) \right\rangle = \frac{E_{\max} B_{\max}}{2\mu_0} \left\langle 1 + \cos(2kx - 2\omega t) \right\rangle = \frac{E_{\max} B_{\max}}{2\mu_0}$$

The can be rewritten as,

$$I = \left\langle \vec{S}(x,y,z,t) \right\rangle = \frac{E_{\max} B_{\max}}{2\mu_0} = \frac{\varepsilon_0 c}{2} E_{\max}^2 = \frac{\varepsilon_0}{2c} B_{\max}^2$$

Waves Carry Energy

Total Energy Density

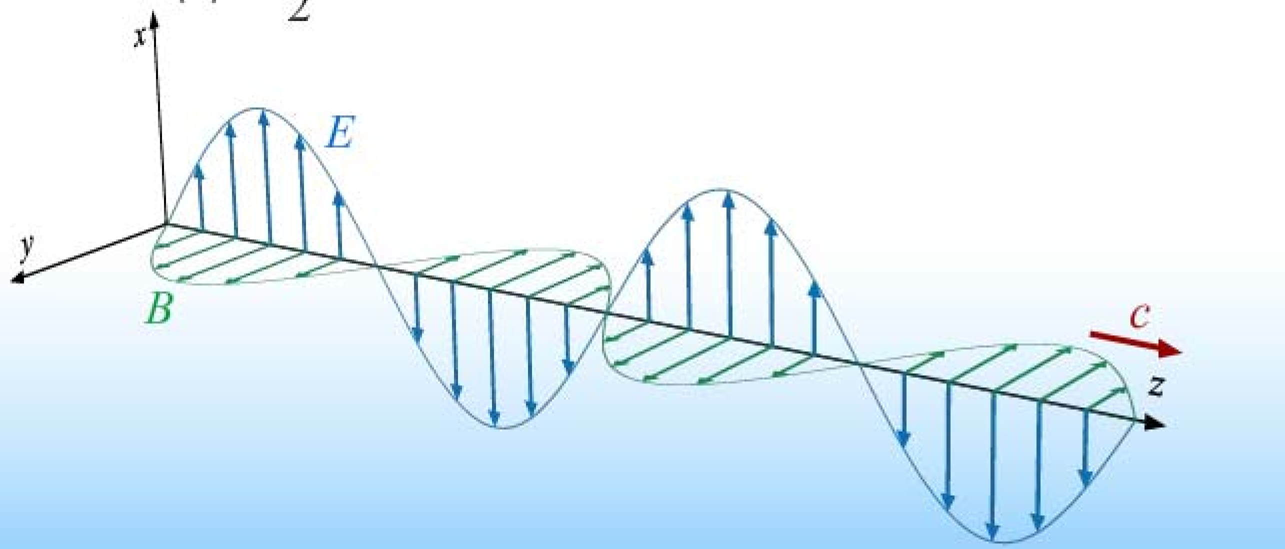
$$u = \varepsilon_o E^2$$

Average Energy Density

$$\langle u \rangle = \frac{1}{2}\varepsilon_o E_o{}^2$$

Intensity

$$\boxed{I = \frac{1}{2} c\, \varepsilon_o E_o{}^2 = c \,\langle u \rangle}$$



Cell Phone Example

A sinusoidal electromagnetic wave emitted by a cell phone has a wavelength of 35.4 cm and an electric field amplitude of 5.4×10^{-2} V/m at a distance of 250 m from the antenna.

(a) What is the magnetic field amplitude?

a.) $E = c\,B$; $B = E/c = 5.4 \times 10^{-2}\ (\mathrm{V/m}) / 3 \times 10^{8}\ \mathrm{m/s}$

$B = 1.8 \times 10^{-10}\ \mathrm{T}$

Cell Phone Example

A sinusoidal electromagnetic wave emitted by a cell phone has a wavelength of 35.4 cm and an electric field amplitude of 5.4×10^{-2} V/m at a distance of 250 m from the antenna.

(a) What is the magnetic field amplitude?

(b) The intensity?

(C) The total average power?

Cell Phone Example

A sinusoidal electromagnetic wave emitted by a cell phone has a wavelength of 35.4 cm and an electric field amplitude of 5.4×10^{-2} V/m at a distance of 250 m from the antenna. (a) What is the magnetic field amplitude? (b) The intensity? (C) The total average power?

b.)

$$I = \frac{1}{2}\varepsilon_0 c E^2;$$

$$I = \frac{1}{2}(8.85\times10^{-12} C^2/Nm^2)(3\times10^8 m/s)(5.4\times10^{-2} V/m)$$

$$I = 3.87\times10^{-6} W/m^2$$

Cell Phone Example

A sinusoidal electromagnetic wave emitted by a cell phone has a wavelength of 35.4 cm and an electric field amplitude of 5.4×10^{-2} V/m at a distance of 250 m from the antenna.

(C) The total average power?

Cell Phone Example

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c.) Assume isotropic: $I=P/A$;

$P=4\pi r^2 I = 4\pi (250m)^2 I$

$P = 3$ W

From last time... EM waves carry energy...

Lasers = Coherence!

Even reducing divergence: sun through magnifying glass

Light has Momentum!

If it has energy and its moving, then it also has momentum:

Analogy from mechanics:

$$E = \frac{p^2}{2m}$$

$$\frac{dE}{dt} = \frac{\cancel{2}p}{\cancel{2}m}\frac{dp}{dt} = \frac{\cancel{m}v}{\cancel{m}}\frac{dp}{dt} = vF$$

For E-M waves:

$$\frac{dE}{dt} \rightarrow \frac{dU}{dt} = IA$$

$$v \rightarrow c$$

$$IA = cF$$

$$\frac{I}{c} = \frac{F}{A}$$ pressure

$$P = \frac{I}{c}$$

Radiation pressure

Light Pressure

Energy transport → momentum

$$\text{Momentum} = \frac{\textit{total}\ \text{energy carried by wave}}{\text{speed}} = \frac{U}{c}$$

$$\text{Intensity} = \frac{\text{energy}}{\text{time-area}} \rightarrow \frac{I}{c} = \frac{\text{energy}/c}{\text{time-area}} = \frac{\text{momentum}}{\text{time-area}}$$

$$\text{Force} = \frac{\text{momentum}}{\text{time}} \Rightarrow \boxed{\frac{I}{c} = \frac{\text{Force}}{\text{area}} \equiv \text{Radiation Pressure}}$$

Light pressure, though "light", has noticeable effects → comet's tail pushed away from the sun*.

***Note: The dust tail is pushed away by radiation; the ion tail is pushed away by the solar wind!**

Question 1:

An absorbing black disk of radius r, mass m, is hanging by a thread. A laser beam with radius a, intensity I, and frequency ω, is incident on the disk (centered on it) from the left. If we increase ____ (keeping all other parameters the same), the light force on the disk will increase.

Increasing I definitely increases force.
Intensity is independent of frequency.

a. disk radius r

b. disk mass m

c. laser beam radius a

d. laser beam intensity I

e. laser frequency ω

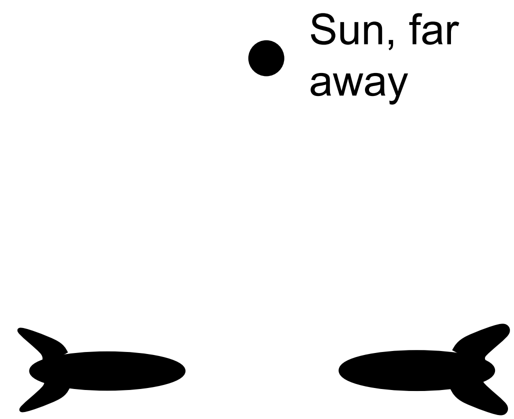
force = pressure x area
pressure = I/c

$r < a$: increasing $r \rightarrow$ more of the beam is intercepted $\rightarrow$ more force.
increasing a does nothing
(if I is kept constant)

$r > a$: increasing $a \rightarrow$ more of disk is hit by the beam $\rightarrow$ more force.
increasing r does nothing

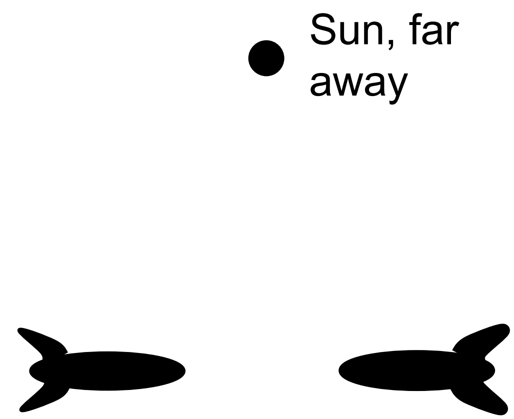
Question 2

- Two identical spaceships are sitting at equal distances from the sun. One ship is colored black, the other is silver.
  - Which ship will experience the greater force from the light pressure?



(a) **black ship** (b) **silver ship** (c) **same**

Question 2



- Two identical spaceships are sitting at equal distances from the sun. One ship is colored black, the other is silver.
  - Which ship will experience the greater force from the light pressure?

(a) **black ship** (b) $\boxed{\textbf{silver ship}}$ (c) **same**

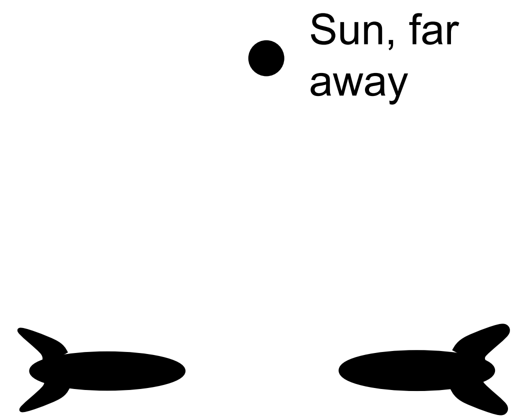
- **The ships are the same distance from the source, so the intensity of light hitting each is the same.**

- **They are the same size and shape and (as drawn) have the same orientation. Therefore, they present the same cross-sectional area and experience the same total *incident* flux of light.**

- **BUT…the black ship absorbs *all* the momentum from the light, while the silver ship *reflects* it. Therefore, the silver ship will experience twice the change in momentum, or twice the force.**

Question 3

- Two identical spaceships are sitting at equal distances from the sun. One ship is colored black, the other is silver.

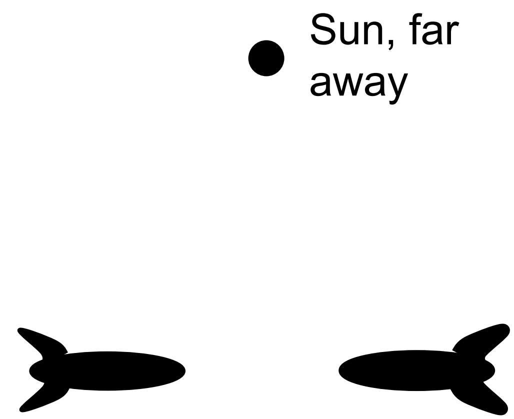


- If the distance from the silver ship to the sun is doubled, the pressure will be reduced by a factor of

(a) 1 (b) 2 (c) 4

Question 3

- Two identical spaceships are sitting at equal distances from the sun. One ship is colored black, the other is silver.



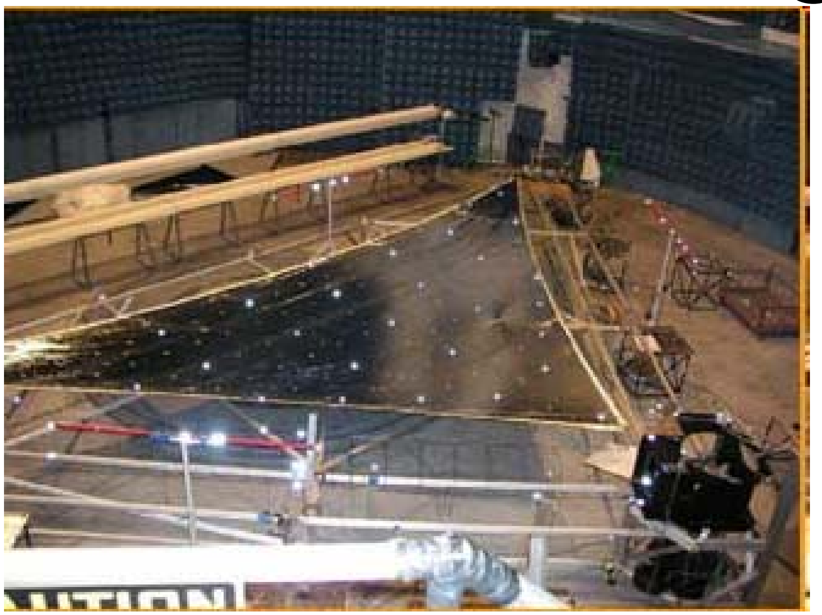
- **If the distance from the silver ship to the sun is doubled, the pressure will be reduced by a factor of**

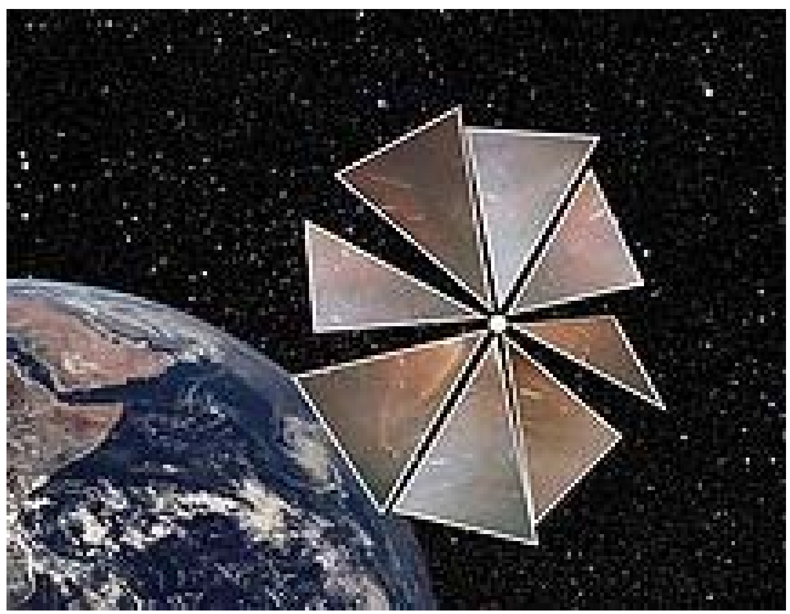
(a) 1 **(b) 2** **(c) 4**

- **Because the sun is assumed to be far away, the intensity at a distance r is simply the total power divided by the surface area of the sphere: $4\pi r^2$.**

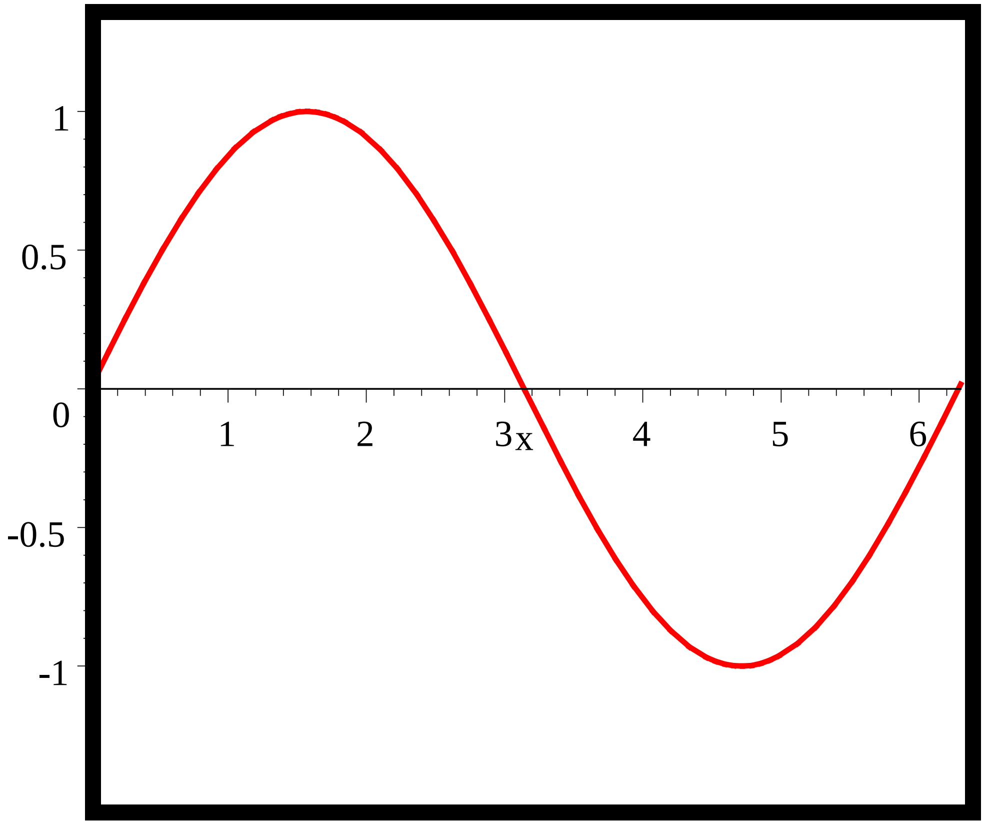
- **Therefore, doubling r increases the surface area by 4, and reduces the intensity by 4. Thus the pressure on the ship is similarly reduced by 4.**

Solar sail





Standing Waves or EM waves in a metal box



X=0 X=L

A sinusoidal E&B wave can exist in a metal box if the E field is ZERO on the boundary of the box. This requires the wave to be zero at x=0,L. This is possible if,

$$E = E_{max} \sin(n\pi x / L)$$

Where n = 1,2,3… Since

$$E = E_{max} \sin(2\pi x/ \lambda)$$

We have for standing waves, fixed wavelengths of,

$$\lambda_n = \frac{2L}{n}$$

Examples; Klystron in microwave oven, waveguides

For next time

- Homework #11 due.
- Homework #12 posted → due <u>Monday</u>

- Start optics/optical phenomenon