Course Updates

http://www.phys.hawaii.edu/~varner/PHYS272-Spr10/physics272.html

Reminders:

1) Assignment #11 → due Wednesday

2) This week: Electromagnetic Waves + …

3) In the home stretch… [review schedule]

Maxwell's Equations (integral form)

Name	Equation	Description
Gauss' Law for Electricity	$\int \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$	Charge and electric fields
Gauss' Law for Magnetism	$\int \vec{B} \cdot d\vec{A} = 0$	Magnetic fields
Faraday's Law	$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$	Electrical effects from changing B field
Ampere's Law (modified by Maxwell)	$\oint \vec{B} \cdot dl = \mu_0 \left(i_c + \varepsilon_0 \frac{d\Phi_E}{dt} \right)$	Magnetic effects from current and Changing E field

Maxwell's Equations

James Clerk Maxwell (1831-1879)

- generalized Ampere's Law
- made equations symmetric:
  - a changing magnetic field produces an electric field
  - a changing electric field produces a magnetic field
- Showed that Maxwell's equations predicted electromagnetic waves and $c = 1/\sqrt{\varepsilon_0\mu_0}$
- Unified electricity and magnetism and light.

All of electricity and magnetism can be summarized by Maxwell's Equations.

Electromagnetic Waves in free space

A remarkable prediction of Maxwell's eqns is electric & magnetic fields can propagate in vacuum.

Examples of electromagnetic waves include; radio/TV waves, light, x-rays, and microwaves.

On to Waves!!

- Note the symmetry now of Maxwell's Equations in free space, meaning when no charges or currents are present

$$\int \vec{E} \cdot d\vec{A} = 0 \qquad \int \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \qquad \oint \vec{B} \cdot dl = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

- Combining these equations leads to wave equations for E and B, e.g.,

$$\boxed{\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2}}$$

- Do you remember the wave equation???

$$\frac{\partial^2 h}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 h}{\partial t^2}$$

h is the variable that is changing in space (x) and time (t). v is the velocity of the wave.

Review of Waves from Physics 170

- The one-dimensional wave equation: $$\frac{\partial^2 h}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 h}{\partial t^2}$$

  has a general solution of the form:

  $$h(x,t) = h_1(x - vt) + h_2(x + vt)$$

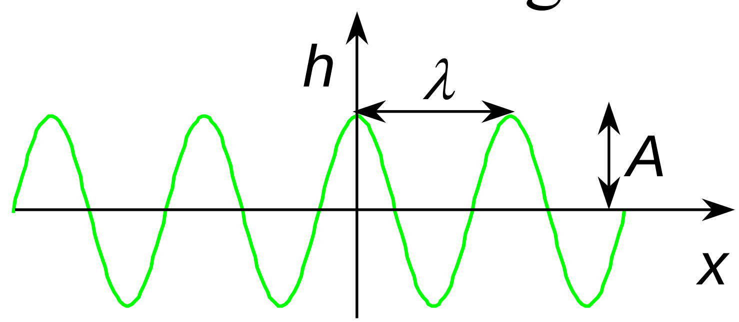
  where h_1 represents a wave traveling in the $+x$ direction and h_2 represents a wave traveling in the $-x$ direction.

- A specific solution for harmonic waves traveling in the $+x$ direction is:

$$h(x,t) = A\cos(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda} \quad \omega = 2\pi f = \frac{2\pi}{T}$$

$$v = \lambda f = \frac{\omega}{k}$$

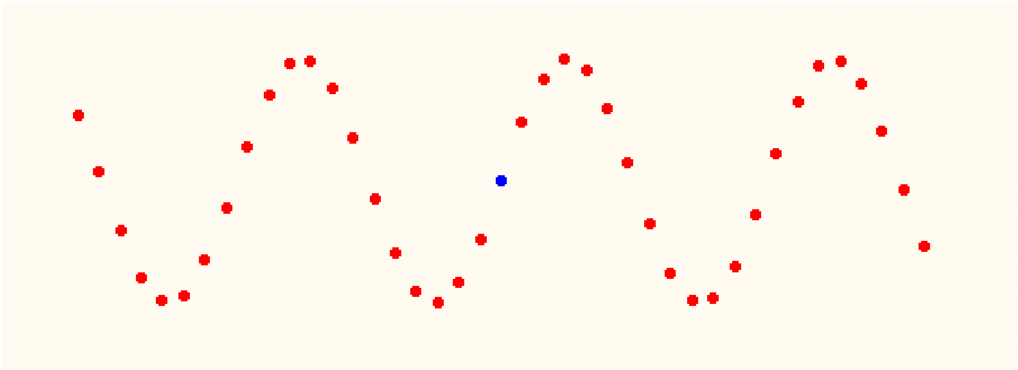


A = amplitude
λ = wavelength
f = frequency
v = speed
k = wave number

Motion of wave (e.g. to right)

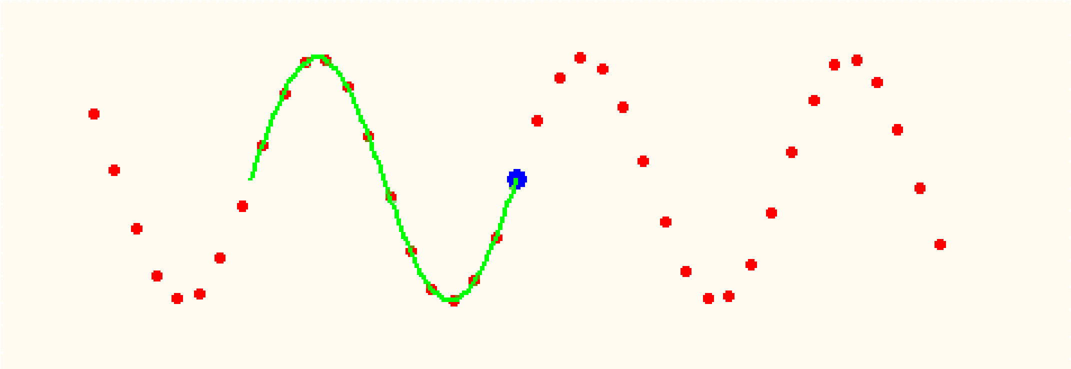
Transverse Wave:

- A note how the wave pattern moves.
- Any particular point (look at the blue one) just moves transversely (i.e., up and down) to the direction of the wave.



Wave Velocity:

- The wave velocity is defined as the wavelength divided by the time it takes a wavelength (green) to pass by a fixed point (blue).



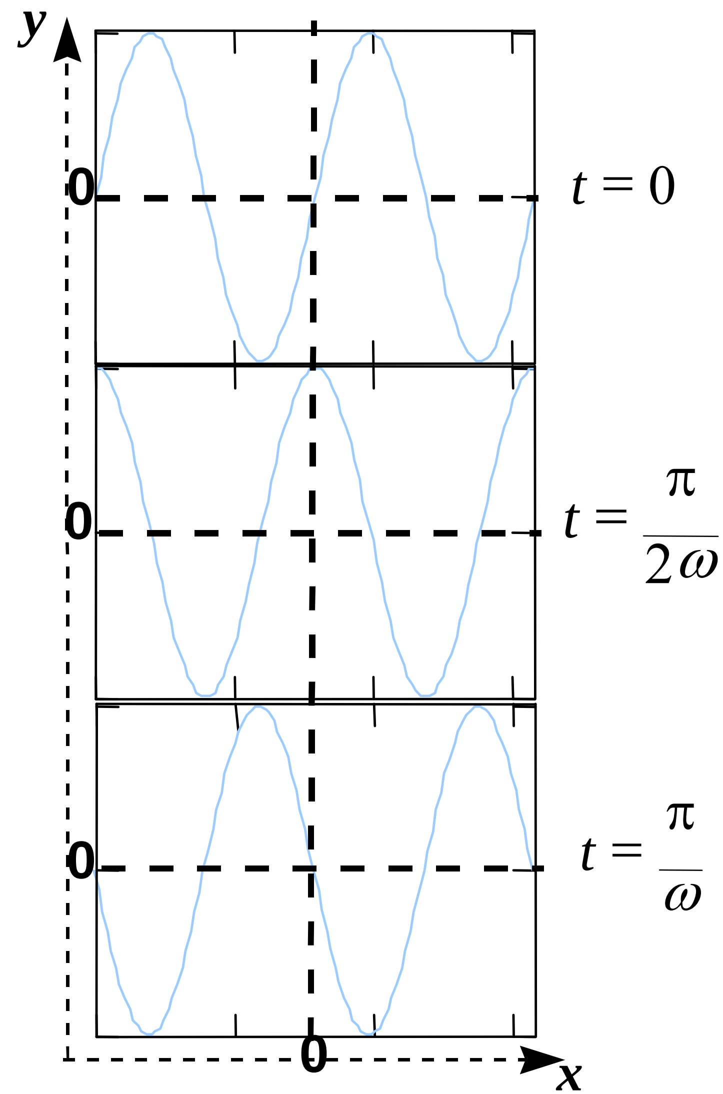
Question 1

- Snapshots of a wave with angular frequency ω are shown at 3 times:
  - Which of the following expressions describes this wave?

(a) $y = \sin(kx-\omega t)$

(b) $y = \sin(kx+\omega t)$

(c) $y = \cos(kx+\omega t)$



Question 1

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$\boxed{\text{(b) } y = \sin(kx + \omega t)}$

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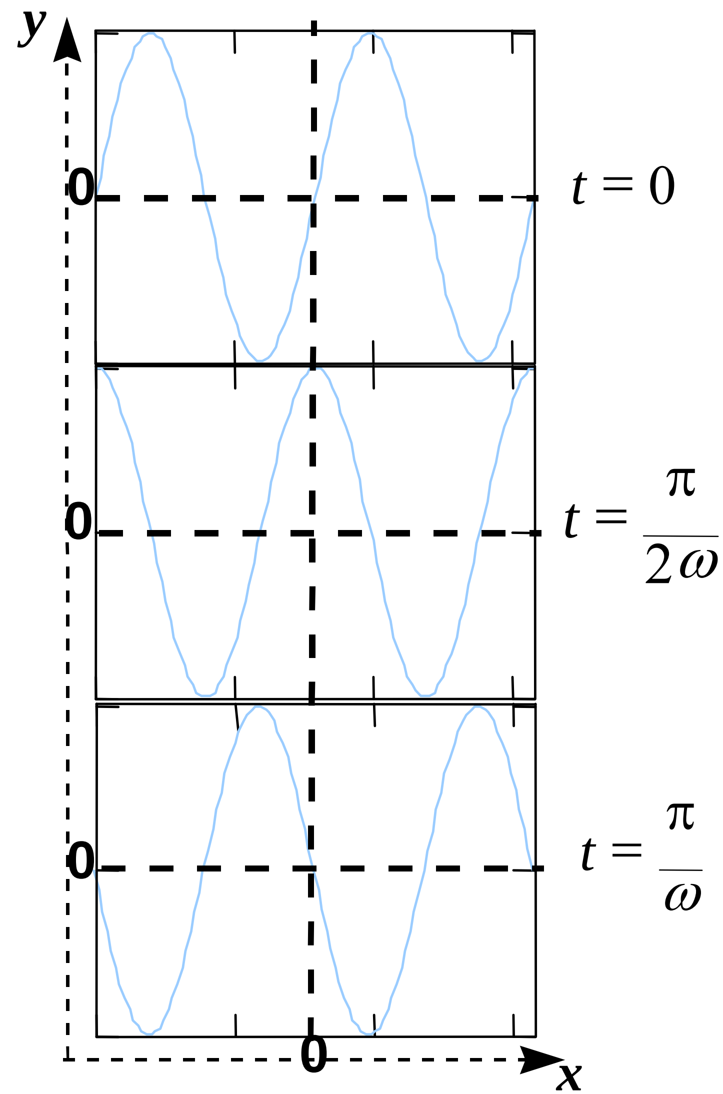
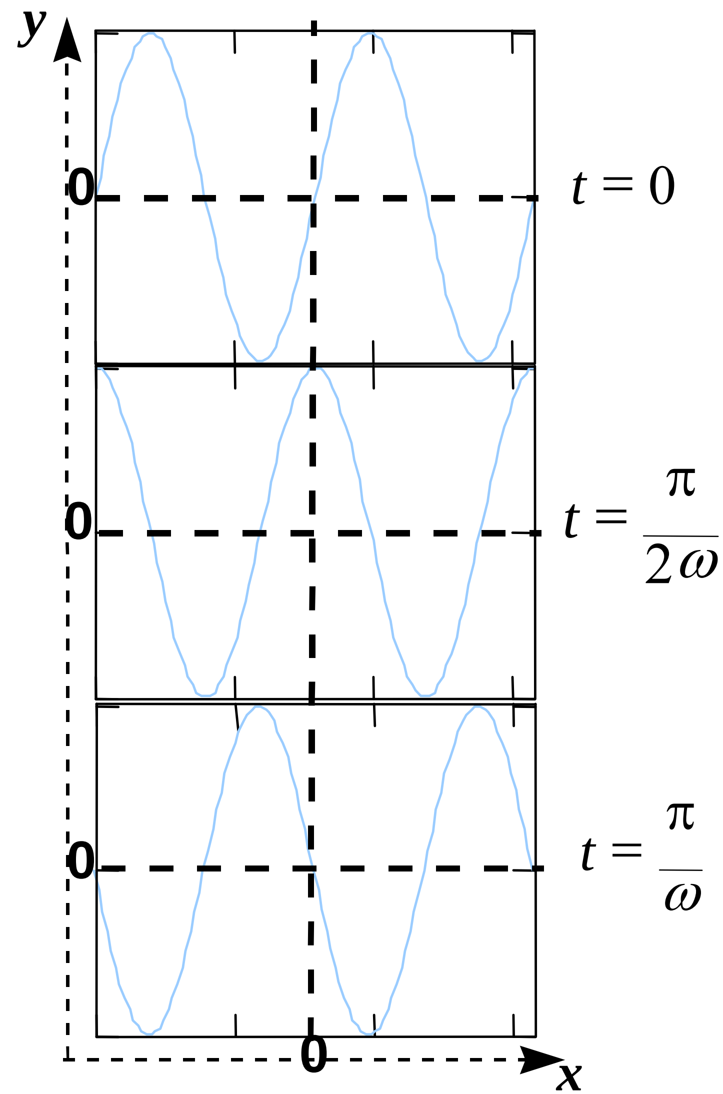


- **The $t = 0$ snapshot $\Rightarrow$ at $t = 0$, $y = \sin kx$**
- **At $t = \pi/2\omega$ and $x = 0$, (a) $\Rightarrow y = \sin(-\pi/2) = -1$**
- **At $t = \pi/2\omega$ and $x = 0$, (b) $\Rightarrow y = \sin(+\pi/2) = +1$**

Question 2

- Snapshots of a wave with angular frequency ω are shown at 3 times:



- **In what direction is this wave traveling?**

(a) +*x* direction **(b) -*x* direction**

Question 2

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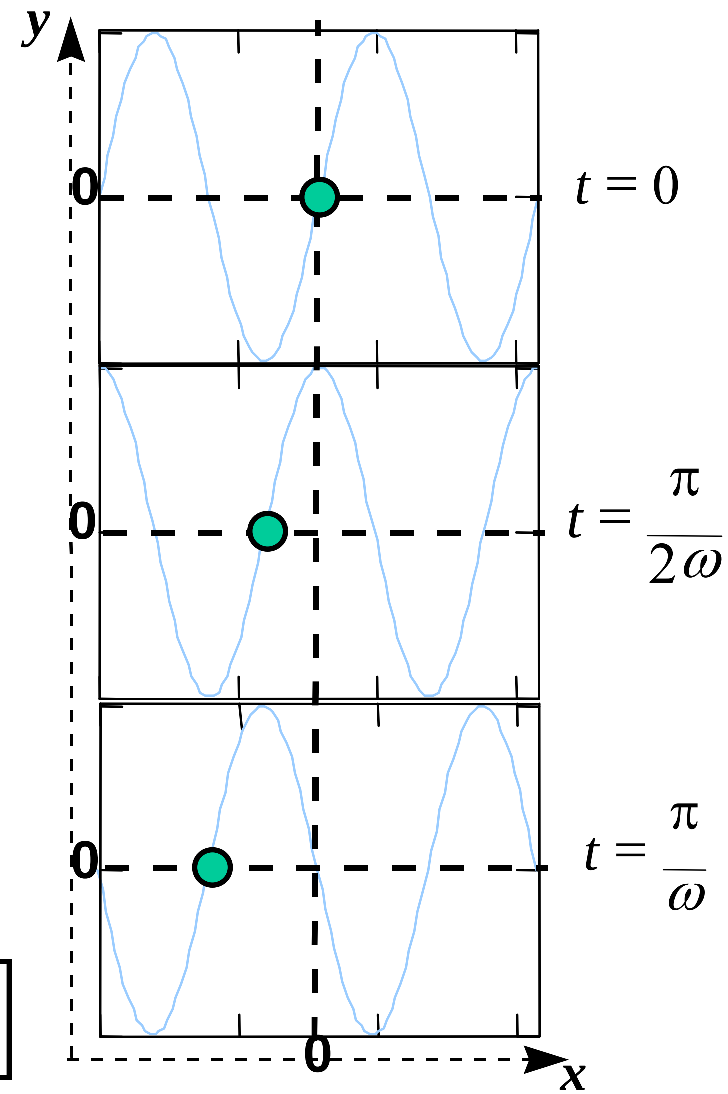


- In what direction is this wave traveling?

(a) $+x$ direction

$\boxed{\text{(b) } -x \text{ direction}}$

- We claim this wave moves in the $-x$ direction.
- The orange dot marks a point of constant phase.
- It thus moves in the $-x$ direction as time increases!!

Velocity of Electromagnetic Waves

- We derived the wave equation for E_x (Maxwell did it first, in ~1865!):

$$\boxed{\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2}}$$

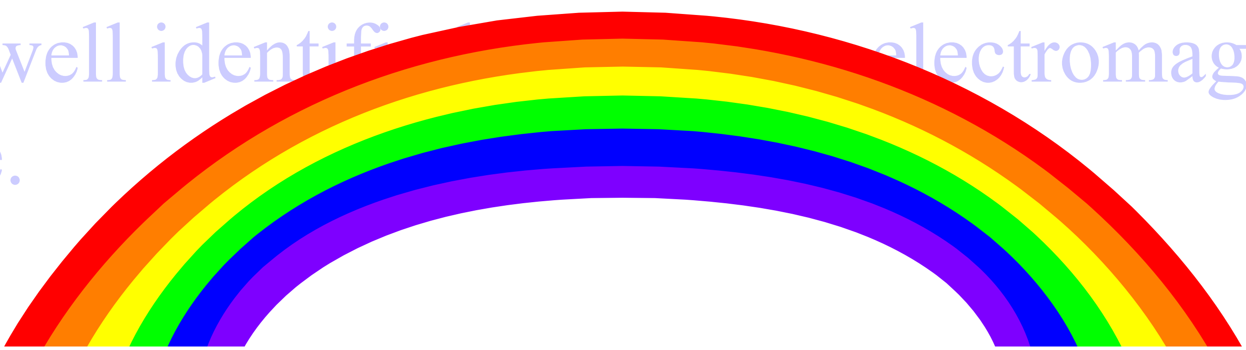
- Comparing to the general wave equation:

$$\frac{\partial^2 h}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 h}{\partial t^2}$$

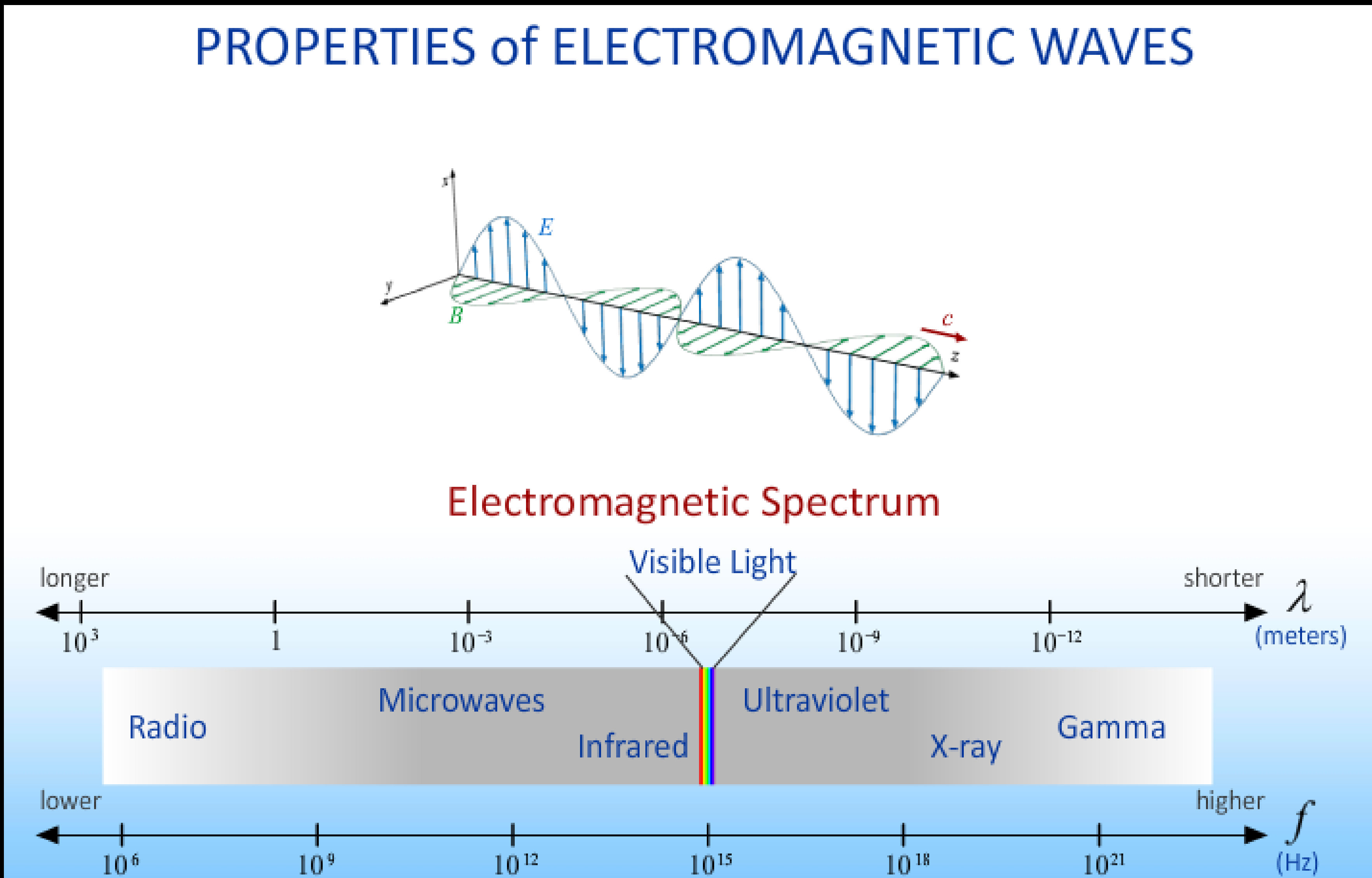
we have the velocity of electromagnetic waves in free space:

$$\boxed{v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3.00 \times 10^8 \text{ m/s} \equiv c}$$

- This value is essentially identical to the speed of light measured by Foucault in 1860!
  - Maxwell identif… electromagnetic wave.



The ElectroMagnetic Spectrum

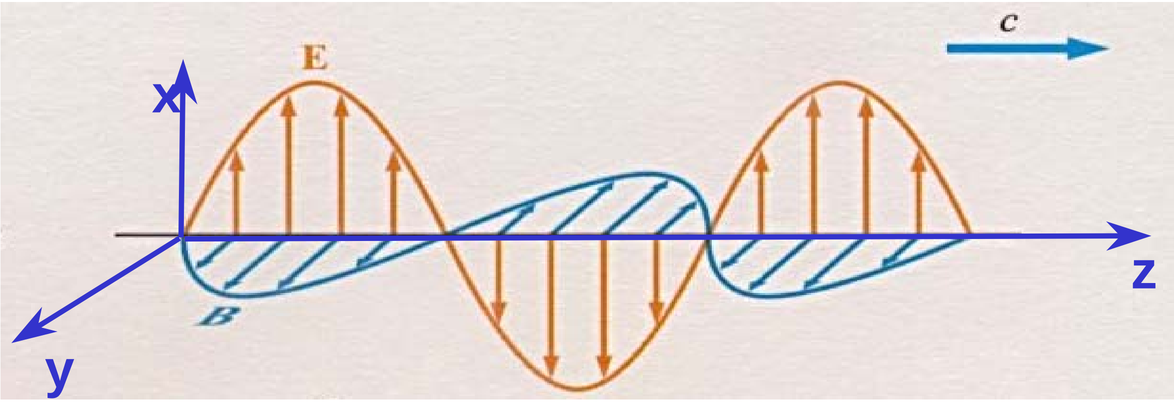


E & B in Electromagnetic Wave

- Plane Harmonic Wave:

$$E_x = E_0 \sin(kz - \omega t)$$
$$B_y = B_0 \sin(kz - \omega t)$$

where $\omega = kc$



➢ $\boxed{\boldsymbol{B_y} \text{ is in phase with } \boldsymbol{E_x}}$

➢ $\boxed{B_0 = E_0 / c}$

➢**The direction of propagation** $\hat{s}$ **is given by the cross product**

$$\boxed{\hat{s} = \hat{e} \times \hat{b}}$$

where $(\hat{e}, \hat{b})$ **are the unit vectors in the (E,B) directions.**

Nothing special about (E_x, B_y); e.g., could have (E_y, -B_x)

Question 3

- Suppose the electric field in an e-m wave is given by:

$$\vec{E} = -\hat{y}\, E_0 \cos(-kz + \omega t)$$

In what direction is this wave traveling ?

(a) + z direction (b) - z direction

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$$\vec{E} = -\hat{y}\, E_0 \cos(-kz + \omega t)$$

  – In what direction is this wave traveling ?

(a) + z direction (b) - z direction

- To determine the direction, set phase = 0: $-kz + \omega t = 0 \Rightarrow z = +\dfrac{\omega}{k} t$
- Therefore wave moves in + z direction!
- Another way: Relative signs opposite means + direction

Question 4

- Suppose the electric field in an e-m wave is given by:

$$\vec{E} = -\hat{y}\, E_0 \cos(-kz + \omega t)$$

Which of the following expressions describes the magnetic field associated with this wave?

(a) $B_x = -(E_o/c)\cos(kz + \omega t)$

(b) $B_x = +(E_o/c)\cos(kz - \omega t)$

(c) $B_x = +(E_o/c)\sin(kz - \omega t)$

Question 4

- Suppose the electric field in an e-m wave is given by:

$$\vec{E} = -\hat{y}\, E_0 \cos(-kz + \omega t)$$

- **Which of the following expressions describes the magnetic field associated with this wave?**

(a) $B_x = -(E_o/c) \cos(kz + \omega t)$

$\boxed{\text{(b) } B_x = +(E_o/c) \cos(kz - \omega t)\ )}$

(c) $B_x = +(E_o/c) \sin(kz - \omega t)$

- **B is in phase with E and has direction determined from:** $\hat{b} = \hat{s} \times \hat{e}$
- **At t=0, z=0, $E_y = -E_o$**
- **Therefore at t=0, z=0,** $\hat{b} = \hat{s} \times \hat{e} = \hat{k} \times (-\hat{j}) = \hat{i}$

$$\boxed{\vec{B} = +\hat{i}\, \frac{E_0}{c} \cos(kz - \omega t)}$$

Question 5

An electromagnetic wave is travelling along the x-axis, with its electric field oscillating along the y-axis. In what direction does the magnetic field oscillate?

a) along the x-axis

b) along the z-axis

c) along the y-axis

Question 5

An electromagnetic wave is travelling along the x-axis, with its electric field oscillating along the y-axis. In what direction does the magnetic field oscillate?

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b) along the z-axis
c) along the y-axis

Note: the direction of propagation $\hat{s}$ is given by the cross product

$$\boxed{\hat{s} = \hat{e} \times \hat{b}}$$

where $(\hat{e}, \hat{b})$ are the unit vectors in the (E,B) directions.

In this case, direction of $\hat{S}$ is x and direction of $\hat{e}$ is y

$$\boxed{\hat{x} = \hat{y} \times \hat{z}}$$

The fields must be perpendicular to each other and to the direction of propagation.

Properties of electromagnetic waves (e.g., light)

Speed: in vacuum, always 3•10^8 m/s, no matter how fast the source is moving (there is no "aether"!). In material, the speed can be reduced, usually only by ~1.5, but in 1999 to 17 m/s!

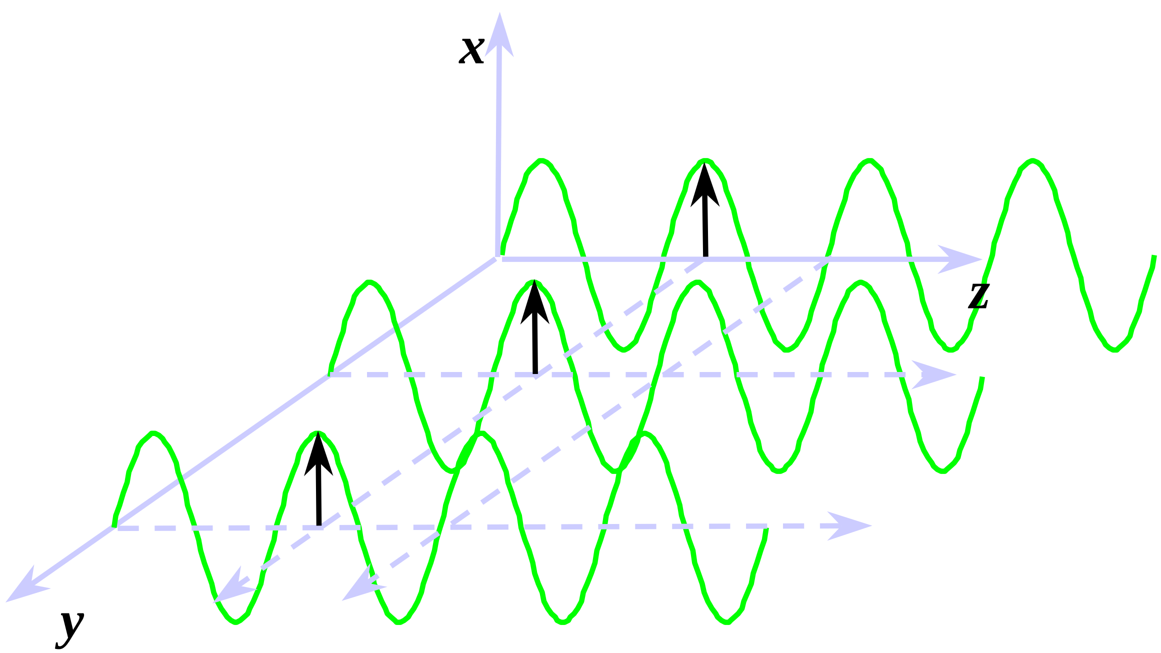
Direction: The wave described by cos(kx-ωt) is traveling in the $+\hat{x}$ direction. This is a "plane" wave—extends infinitely in $\hat{y}$ and $\hat{z}$.

In reality, light is often somewhat localized transversely (e.g., a laser) or spreading in a spherical wave (e.g., a star).

A plane wave can often be a good approximation (e.g., the wavefronts hitting us from the sun are nearly flat).

Plane Waves

- For any given value of z, the magnitude of the electric field is uniform everywhere in the x-y plane with that z value.



Waves Carry Energy

Total Energy Density

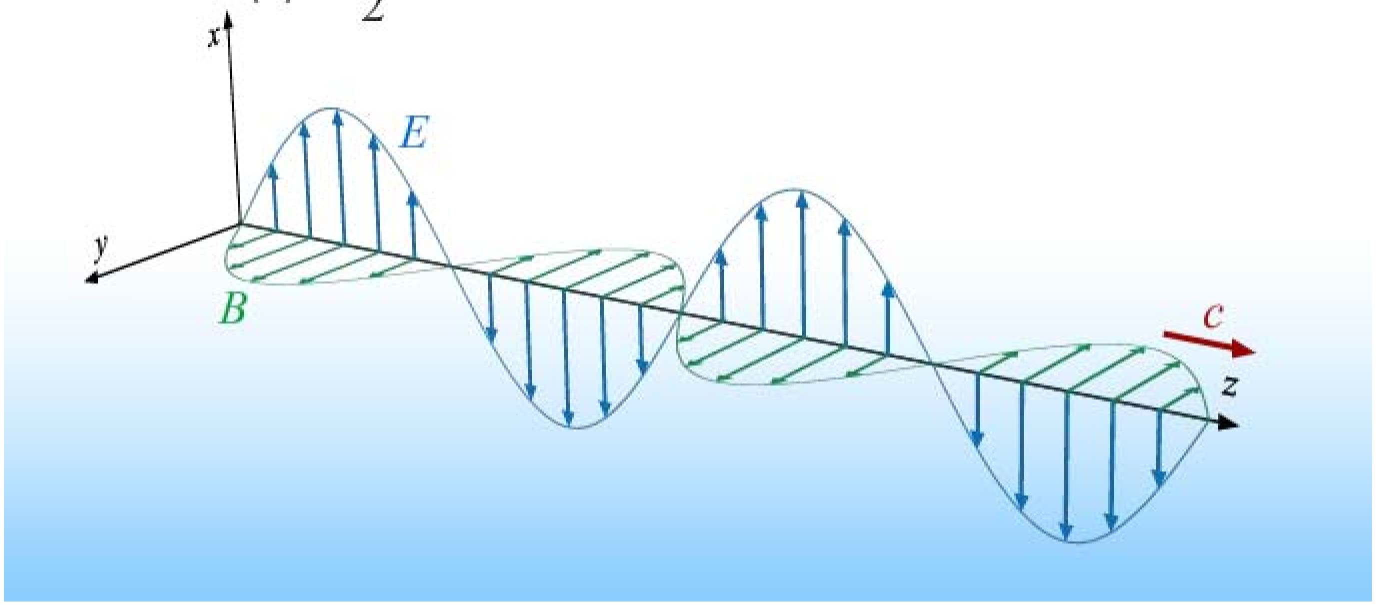
$$u = \varepsilon_o E^2$$

Average Energy Density

$$\langle u \rangle = \frac{1}{2}\varepsilon_o E_o^{\,2}$$

Intensity

$$\boxed{I = \frac{1}{2} c\,\varepsilon_o E_o^{\,2} = c\,\langle u \rangle}$$



Energy Density

Previously, we demonstrated the energy density existed in E fields in Capacitor and in B fields in inductors. We can sum these energies,

[redacted]

Since, *E=cB* and [redacted] **then *u* in terms of E,**

[redacted]

=> EM wave has energy and can transport energy at speed c

Waves Carry Energy

Total Energy Density

$$u = \varepsilon_o E^2$$

Average Energy Density

$$\langle u \rangle = \frac{1}{2} \varepsilon_o E_o^{\ 2}$$

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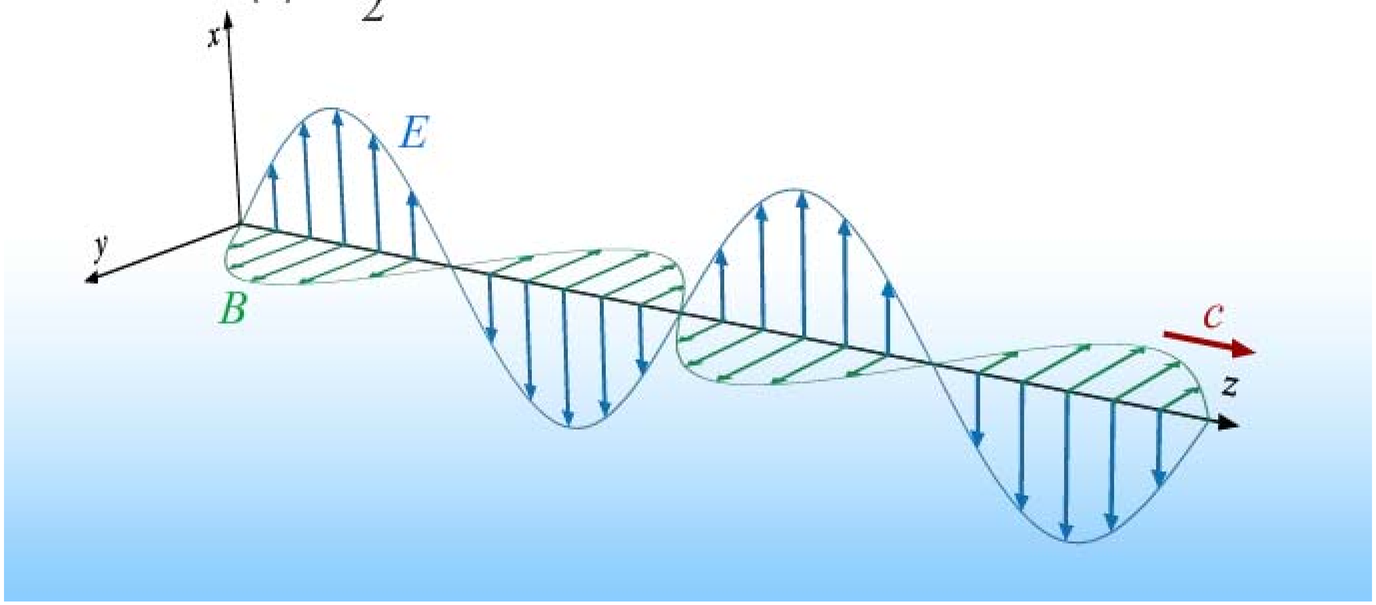
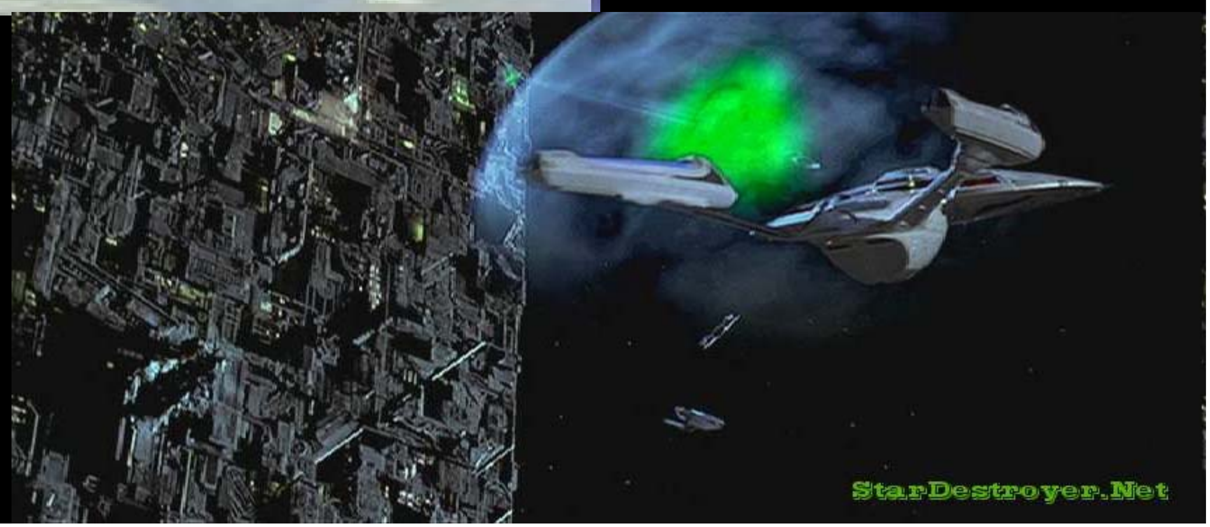
$$\boxed{I = \frac{1}{2} c\, \varepsilon_o E_o^{\ 2} = c \langle u \rangle}$$



As you already probably know…



PHOTONS

We believe the energy in an e-m wave is carried by photons

Question: What are Photons?

Answer: Photons are Photons.

Photons possess both wave and particle properties

Particle:

Energy and Momentum localized

Wave:

They have definite frequency & wavelength ($f\lambda = c$)

Connections seen in equations:

$$E = hf$$

$$p = h/\lambda$$

Planck's constant

$h = 6.63e^{-34}$ J-s

Question: How can something be both a particle and a wave?

Answer: It can't (when we observe it)

What we see depends on how we choose to measure it !

The mystery of quantum mechanics: More on this in PHYS 274

For next time

- Homework #11 due Wed.
- One more session on EM Waves, and then on to optics