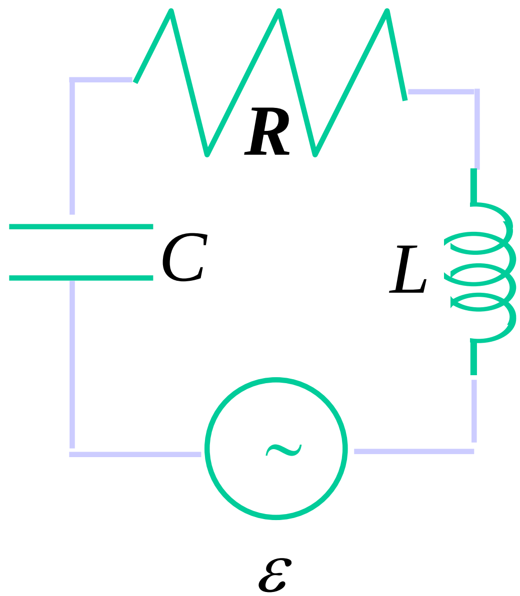
Course Updates

http://www.phys.hawaii.edu/~varner/PHYS272-Spr10/physics272.html

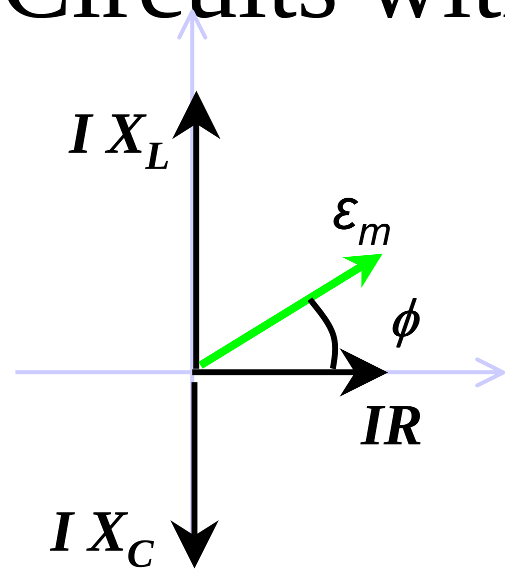
Reminders:

1) Assignment #11 → posted, due next Wednesday
2) Quiz # 5 today (Chap 29, 30)
3) Complete AC Circuits

Last time: LRC Circuits with phasors…



$\Rightarrow$

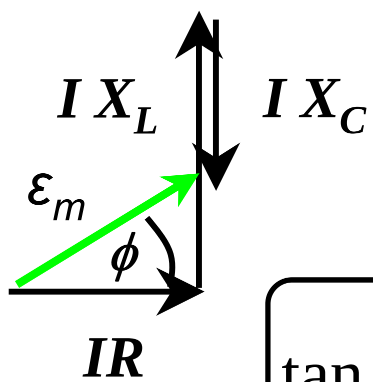


where . . .

$$\boxed{X_L \equiv \omega L}$$

$$\boxed{X_C \equiv \frac{1}{\omega C}}$$

The phasor diagram gives us graphical solutions for ϕ and I:



$$\boxed{\tan\phi = \frac{X_L - X_C}{R}}$$

$$\varepsilon_m^2 = I^2\left(R^2 + \left(X_L - X_C\right)^2\right)$$

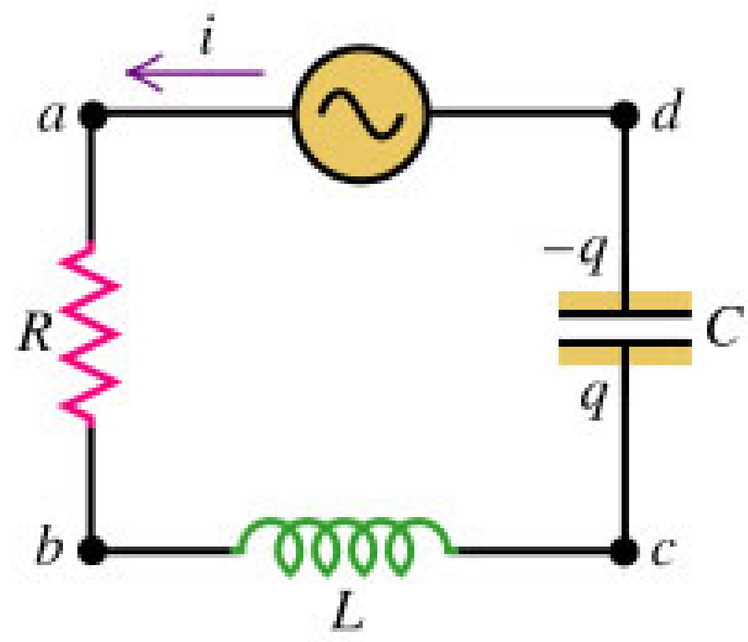
$$\Downarrow$$

$$\boxed{\varepsilon_m = I\sqrt{R^2 + \left(X_L - X_C\right)^2} = IZ}$$

$$\boxed{Z \equiv \sqrt{R^2 + \left(X_L - X_C\right)^2}}$$

LRC series circuit; Summary of instantaneous Current and voltages

$$V_R = IR$$
$$V_L = IX_L$$
$$V_C = IX_C$$



$$i(t) = I\cos(\omega t)$$
$$v_R(t) = IR\cos(\omega t)$$
$$v_C(t) = IX_C\cos(\omega t - 90) = I\frac{1}{\omega C}\cos(\omega t - 90)$$
$$v_L(t) = IX_L\cos(\omega t + 90) = I\omega L\cos(\omega t + 90)$$
$$\varepsilon(t) = v_{ad}(t) = IZ\cos(\omega t + \phi) = \varepsilon_m\cos(\omega t + \phi)$$

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{\omega L - 1/\omega C}{R}$$

$$Z = \sqrt{(X_R)^2 + (X_L - X_C)^2}$$

Lagging & Leading

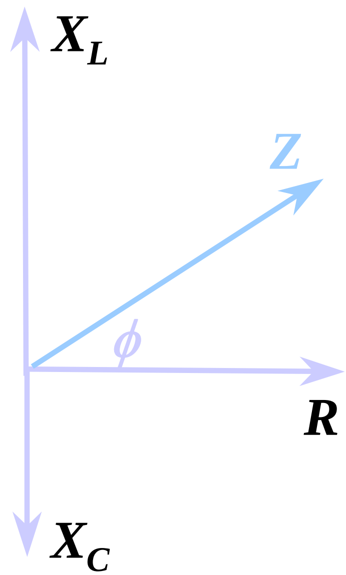
The phase ϕ between the current and the driving emf depends on the relative magnitudes of the inductive and capacitive reactances.

$$I = \frac{\varepsilon_m}{Z}$$

$$\tan\phi = \frac{X_L - X_C}{R}$$

$$X_L \equiv \omega L$$

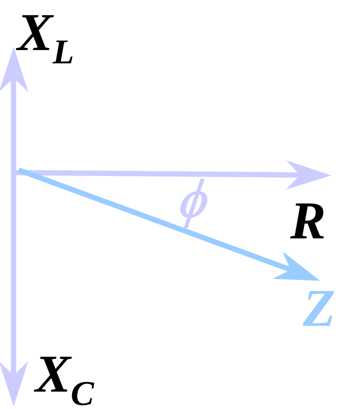
$$X_C \equiv \frac{1}{\omega C}$$



$X_L > X_C$

$\phi > 0$

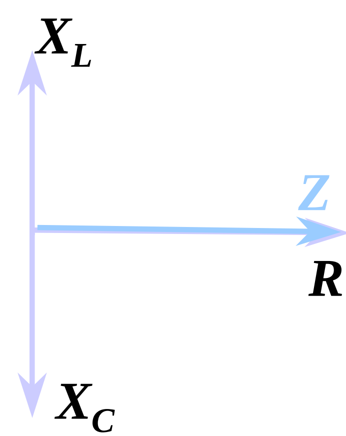
current
LAGS
applied voltage



$X_L < X_C$

$\phi < 0$

current
LEADS
applied voltage



$X_L = X_C$

$\phi = 0$

current
IN PHASE WITH
applied voltage

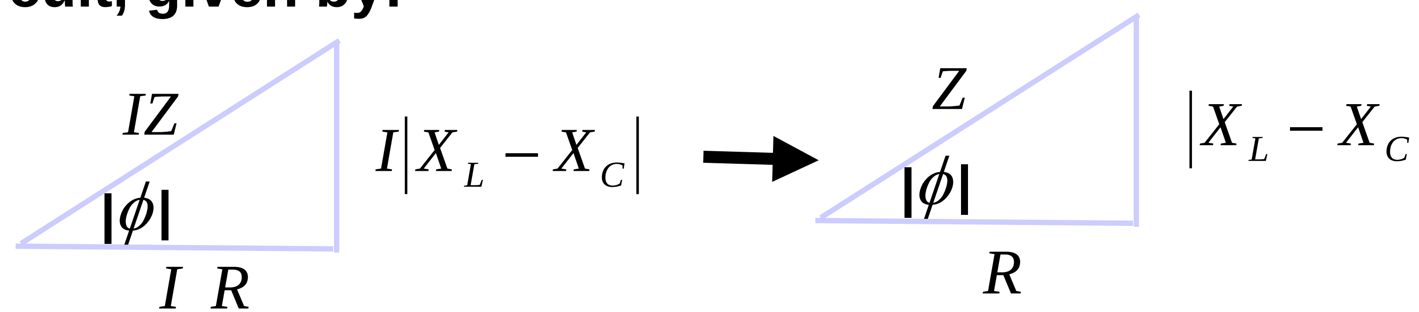
Impedance, Z

• **From the phasor diagram we found that the current amplitude I was related to the drive voltage amplitude ε_m by**

$$\varepsilon_m = I\ Z$$

• ***Z*** **is known as the "impedance", and is basically the frequency dependent equivalent resistance of the series LRC circuit, given by:**

" Impedance Triangle"



$$\boxed{Z \equiv \frac{\varepsilon_m}{I} = \sqrt{R^2 + \left(X_L - X_C\right)^2}}$$

or

$$\boxed{Z = \frac{R}{\cos(\phi)}}$$

• **Note that *Z* achieves its minimum value (*R*) when $\phi = 0$. Under this condition the maximum current flows in the circuit.**

Resonance

- For fixed R, C, L the current I will be a maximum at the resonant frequency ω which makes the impedance Z purely resistive ($Z = R$). i.e.,

$$I = \frac{\varepsilon_m}{Z} = \frac{\varepsilon_m}{\sqrt{R^2 + \left(X_L - X_C\right)^2}}$$

reaches a maximum when:

$$\boxed{X_L = X_C}$$

This condition is obtained when:

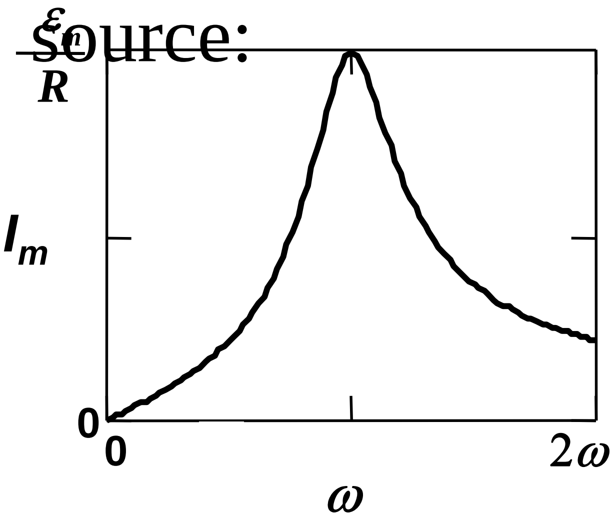
$$\omega L = \frac{1}{\omega C} \quad \Rightarrow \quad \boxed{\omega = \frac{1}{\sqrt{LC}}}$$

- **Note that this resonant frequency is identical to the natural frequency of the LC circuit by itself!**
- **At this frequency, the current and the driving voltage are in phase:**

$$\tan\phi = \frac{X_L - X_C}{R} = 0$$

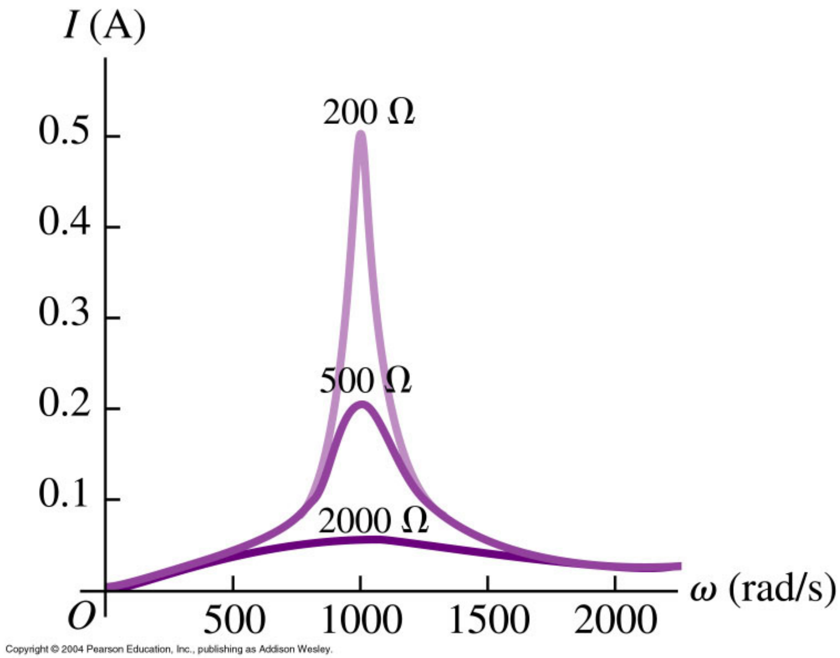
Resonance

Plot the current versus ω, the frequency of the voltage source:

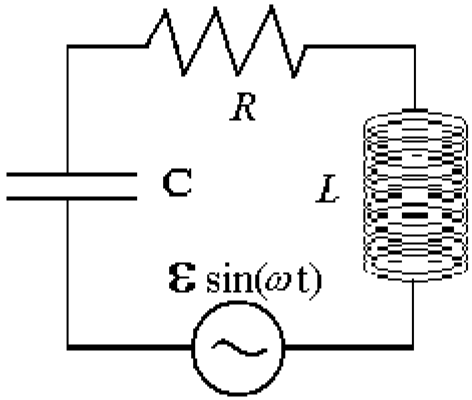


- **For ω very large, $X_L >> X_C$, $\phi \rightarrow 90°$, $I_m \rightarrow 0$**
- **For ω very small, $X_C >> X_L$, $\phi \rightarrow -90°$, $I_m \rightarrow 0$**

Example: vary R
V=100 v
ω=1000 rad/s
R=200, 500, 2000 ohm
L=2 H
C=0.5 μC



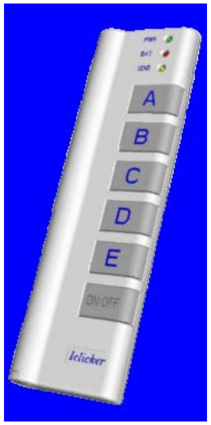
Question 1



Consider a general AC circuit containing a resistor, capacitor, and inductor, driven by an AC generator.

As the frequency of the circuit is either raised above or lowered below the resonant frequency, the impedance of the circuit _______________.

a) always increases
b) only increases for lowering the frequency below resonance
c) only increases for raising the frequency above resonance



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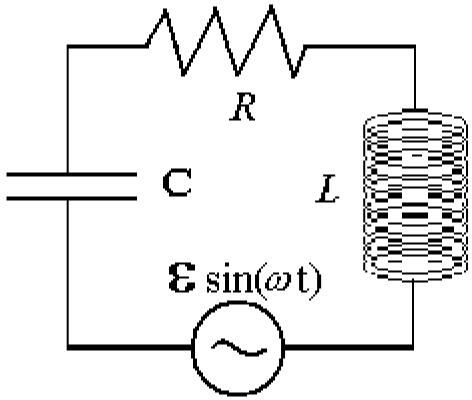

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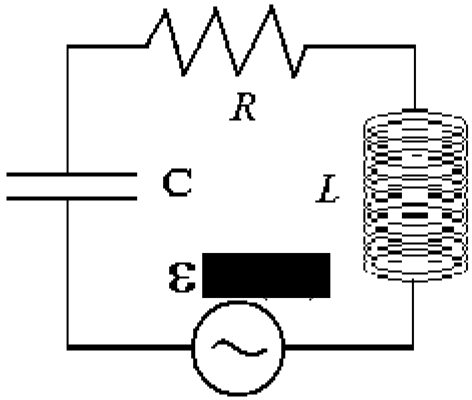
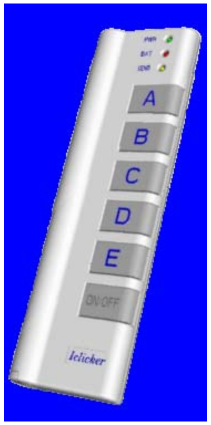


Consider a general AC circuit containing a resistor, capacitor, and inductor, driven by an AC generator.

At the resonant frequency, which of the following is true?

a) The current leads the voltage across the generator.
b) The current lags the voltage across the generator.
c) The current is in phase with the voltage across the generator.



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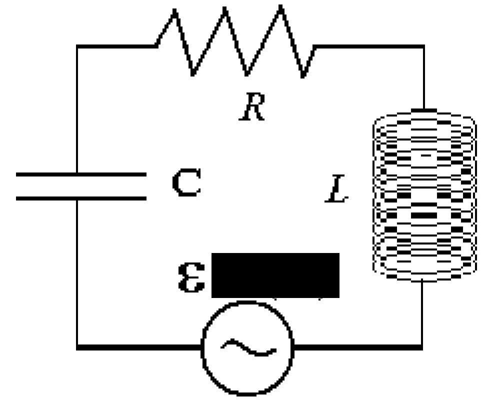

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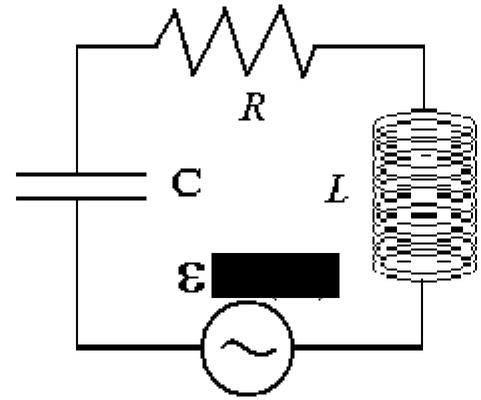
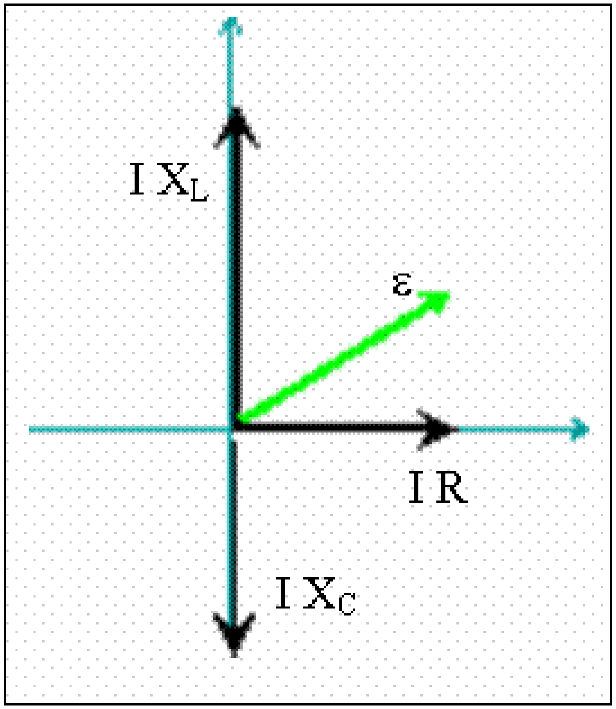


- Impedance = $Z = \mathrm{sqrt}(R^2 + (X_L - X_C)^2)$
- At resonance, $(X_L - X_C) = 0$, and the impedance has its minimum value: $Z = R$

- As frequency is changed from resonance, either up or down, $(X_L - X_C)$ no longer is zero and Z must therefore increase.

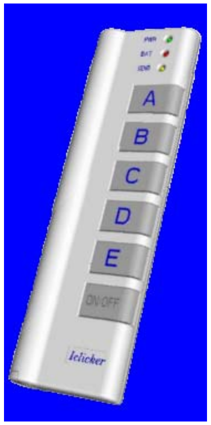
Changing the frequency away from the resonant frequency will change both the reductive and capacitive reactance such that $X_L - X_C$ is no longer 0. This, when squared, gives a positive term to the impedance, increasing its value. By definition, at the resonance frequency, I_{max} is at its greatest and the phase angle is 0, so the current is in phase with the voltage across the generator.

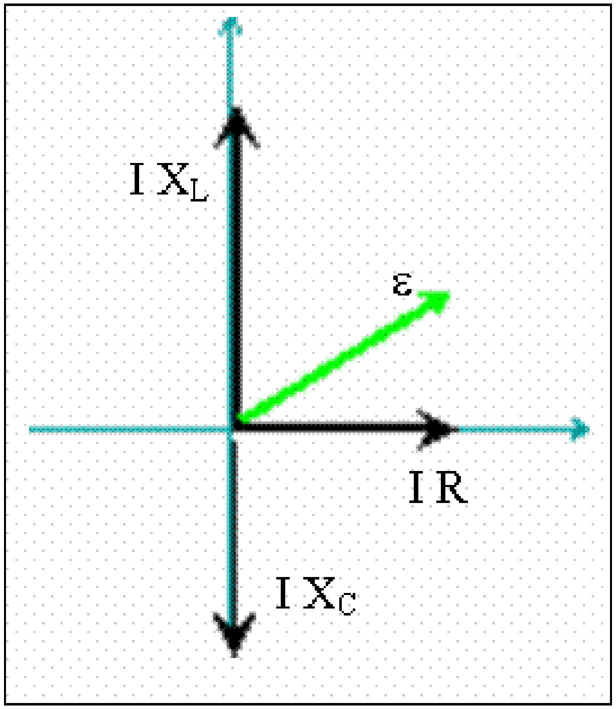


Question 3

Fill in the blank. This circuit is being driven ___________ its resonance frequency.

a) above
b) below
c) exactly at





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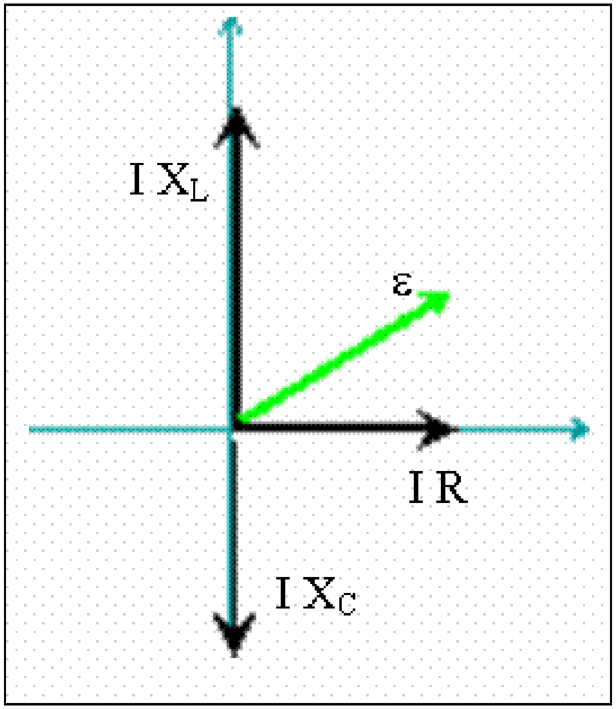


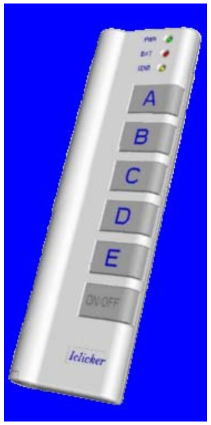
Question 4

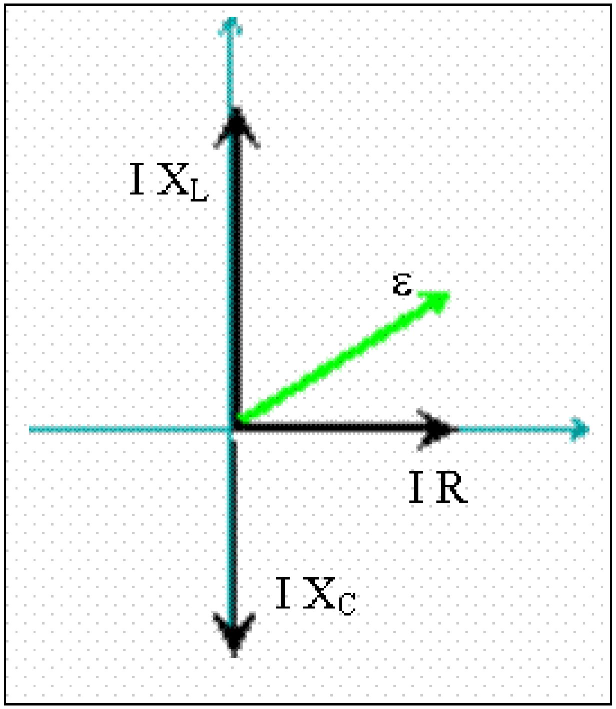
The generator voltage ________________ the current.

a) leads

b) lags

c) is in phase with





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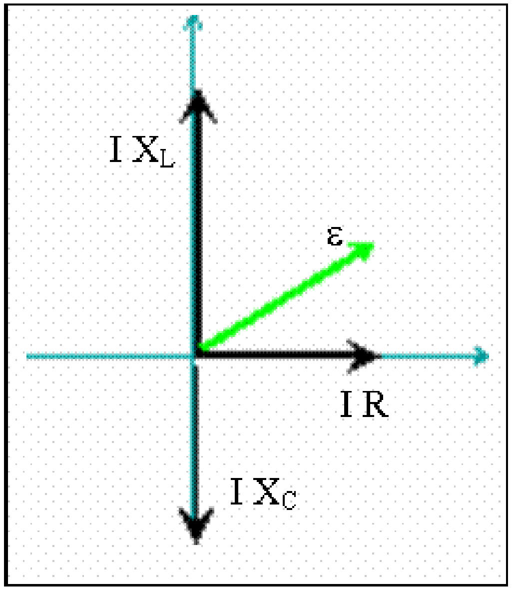
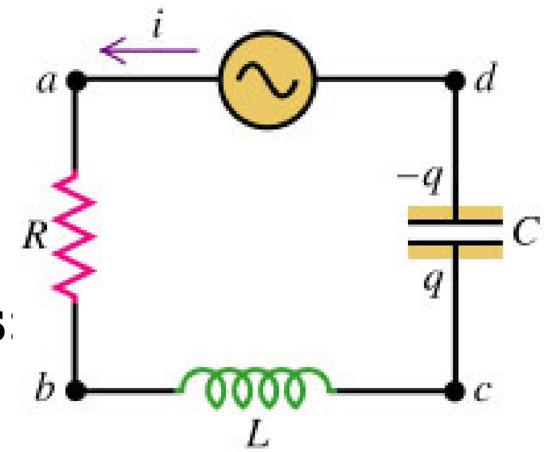


- At resonance, $X_L = X_C$.
- Here, $X_L > X_C$
- Therefore, need to reduce $X_L = \omega L$ and increase $X_C = 1/(\omega C)$
- Therefore, lower ω !!

- From diagram, ε leads IR (rotation = ccw)

Power in LRC circuit



$$i(t) = I(t)\cos(\omega t); \qquad v_{ad}(t) = V\cos(\omega t + \phi)$$

The instantaneous power delivered to L-R-C is:

$$P(t) = i(t)v_{ad}(t) = V\cos(\omega t + \phi) I\cos(\omega t)$$

We can use trig identities to expand the above to,

$$P(t) = V[\cos(\omega t)\cos(\phi) - \sin(\omega t)\sin(\phi)] I\cos(\omega t)$$

$$= VI\cos^2(\omega t)\cos(\phi) - VI\sin(\omega t)\cos(\omega t)\sin(\phi)$$

$$P_{ave} = \langle P(t)\rangle = VI\langle\cos^2(\omega t)\rangle\cos(\phi) - VI\langle\sin(\omega t)\cos(\omega t)\rangle\sin(\phi)$$

$$= VI\langle\cos^2(\omega t)\rangle\cos(\phi) = VI\left(\frac{1}{2}\right)\cos(\phi)$$

$$P_{ave} = \langle P(t)\rangle = \frac{1}{2}VI\cos(\phi) = \frac{V}{\sqrt{2}}\frac{I}{\sqrt{2}}\cos(\phi)$$

$$= V_{RMS} I_{RMS}\cos(\phi)$$

Power in LRC circuit, continued

$$P_{ave} = \langle P(t) \rangle = \frac{1}{2} VI \cos(\phi) = V_{RMS} I_{RMS} \cos(\phi)$$

General result. V_{RMS} is voltage across element, I_{RMS} is current through element, and φ is phase angle between them.

Example; 100Watt light bulb plugged into 120V house outlet, Pure resistive load (no L and no C), $\varphi = 0$.

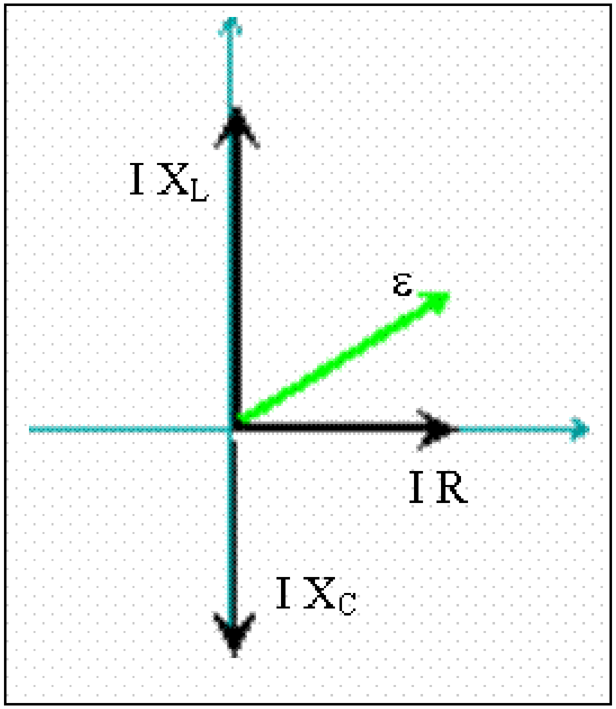
$$P = I_{rms} V_{rms} = \frac{V_{rms}^2}{R}$$

$$R = \frac{V_{rms}^2}{P_{ave}} = \frac{120^2}{100} = 144\Omega$$

$$I_{rms} = \frac{P_{ave}}{V_{rms}} = \frac{100}{120} = 0.83A$$

Note; 120V house voltage is rms and has peak voltage of $120\sqrt{2} = 170V$

Question: What is P_{AVE} for an inductor or capacitor?



Question 5

If you wanted to increase the power delivered to this *RLC* circuit, which modification(s) would work?

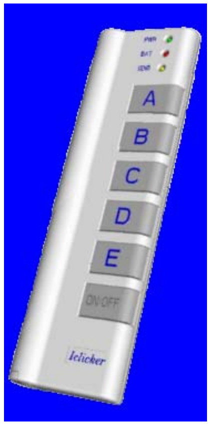
a) increase *R*

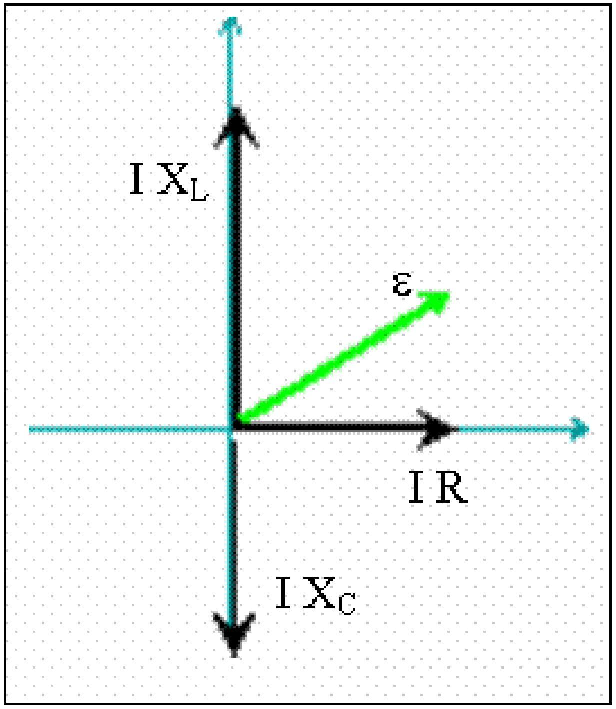
b) increase *C*

c) increase C,*L*

d) decrease *R*

e) decrease *C*,*L*





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If you wanted to increase the power delivered to this *RLC* circuit, which modification(s) would work?

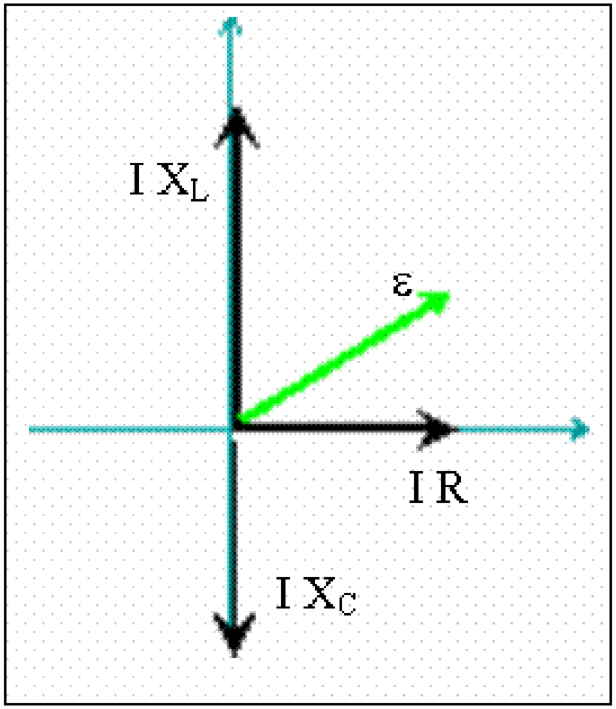
a) increase *R*

b) increase *C or L*

c) increase C,*L*

d) decrease *R*

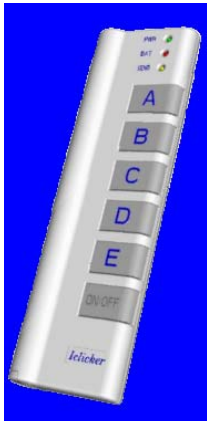
e) decrease *C,L*

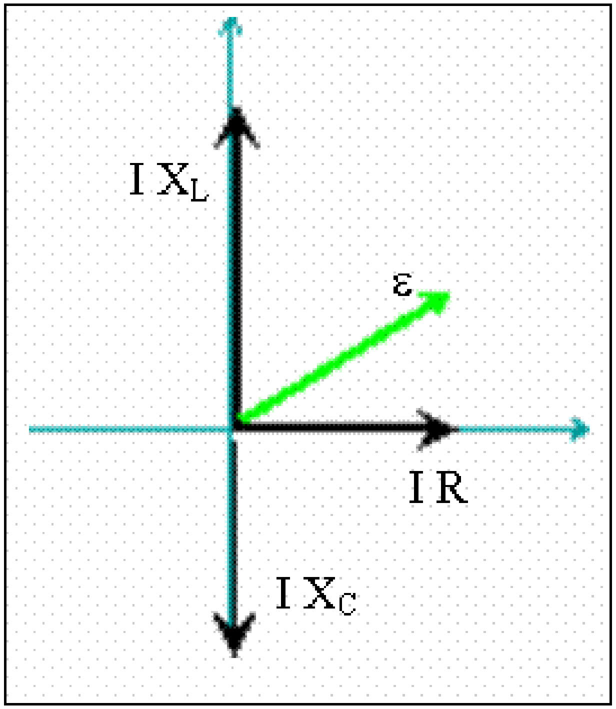


Question 6

Would using a larger resistor increase the current?

a) yes b) no





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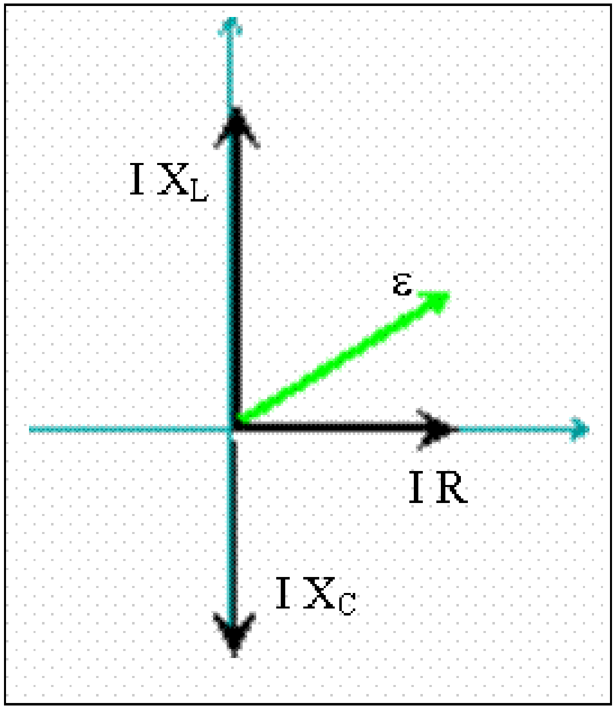
a) yes

b) no

- Power ~ $I\cos\phi$ ~ $(1/Z)(R/Z) = (R/Z^2)$

- To increase power, want Z to decrease:
    - L: decrease X_L $\Rightarrow$ decrease L
    - C: increase X_C $\Rightarrow$ decrease C
    - R: decrease Z $\Rightarrow$ decrease R



Since power peaks at the resonant frequency, try to get X_L and X_C to be equal. Power also depends inversely on R, so decrease R to increase Power.

Summary

- Power

"power factor"

$$\boxed{\begin{aligned}\langle P(t)\rangle &= \varepsilon_{rms} I_{rms} \underbrace{\cos\phi}_{\text{"power factor"}} \\ &= (I_{rms})^2 R\end{aligned}}$$

$$\boxed{\varepsilon_{rms} \equiv \frac{1}{\sqrt{2}}\varepsilon_m}$$

$$\boxed{I_{rms} \equiv \frac{1}{\sqrt{2}} I_m}$$

$$\boxed{Z \equiv \sqrt{R^2 + (X_L - X_C)^2}}$$

$$\boxed{\tan\phi = \frac{X_L - X_C}{R}}$$

- Driven Series LRC Circuit: $\boxed{\omega = \frac{1}{\sqrt{LC}}}$
  - Resonance condition
    - Resonant frequency

For next time

- Homework #11 posted, due next Wed.

- Quiz now: Faraday’s Law, Inductance and Inductors, RLC circuits