# Course Updates

http://www.phys.hawaii.edu/~varner/PHYS272-Spr10/physics272.html

Reminders:

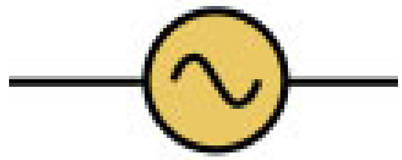
1) Assignment #10 → due Today
2) Quiz # 5 Friday (Chap 29, 30)
3) Start AC Circuits

# Alternating Currents (Chap 31)

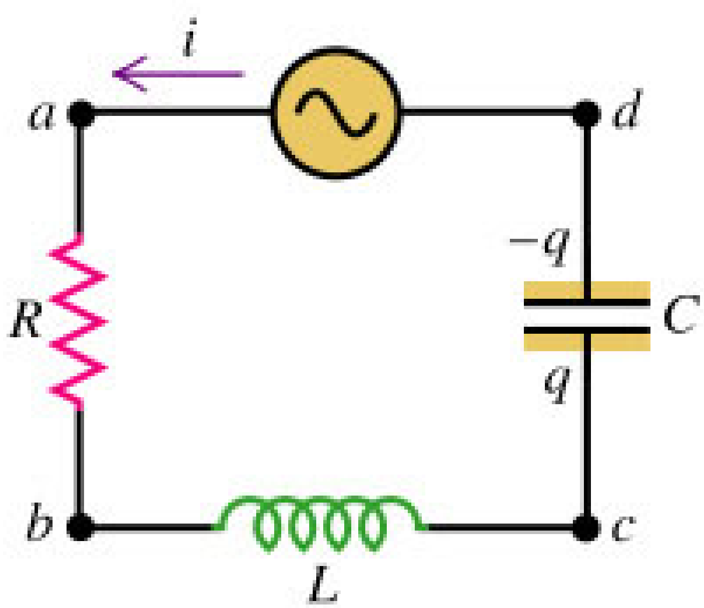
In this chapter we study circuits where the battery is replaced by a sinusoidal voltage or current source.

$$v(t) = V_0 \cos(\omega t) \quad \text{or} \quad i(t) = I_0 \cos(\omega t)$$

The circuit symbol is,



An example of an LRC circuit connected to sinusoidal source is,




Important:
I(t) is same throughout – just like DC case.

# Alternating Currents

Since the currents & voltages are sinusoidal, their values change over time and the averages are zero.

A more useful description of sinusoidal currents and voltages are given by considering the average of the square of this quantities.

We define the RMS (root mean square), which is the square root of the average of ,

$$i^2(t) = \left(I_0 \cos(\omega t)\right)^2$$

$$\left\langle i^2(t) \right\rangle = \left\langle \left(I_0 \cos(\omega t)\right)^2 \right\rangle = I_0^2 \frac{1}{2}\left(1 + \left\langle \cos(2\omega t) \right\rangle\right) = \frac{I_0^2}{2}$$

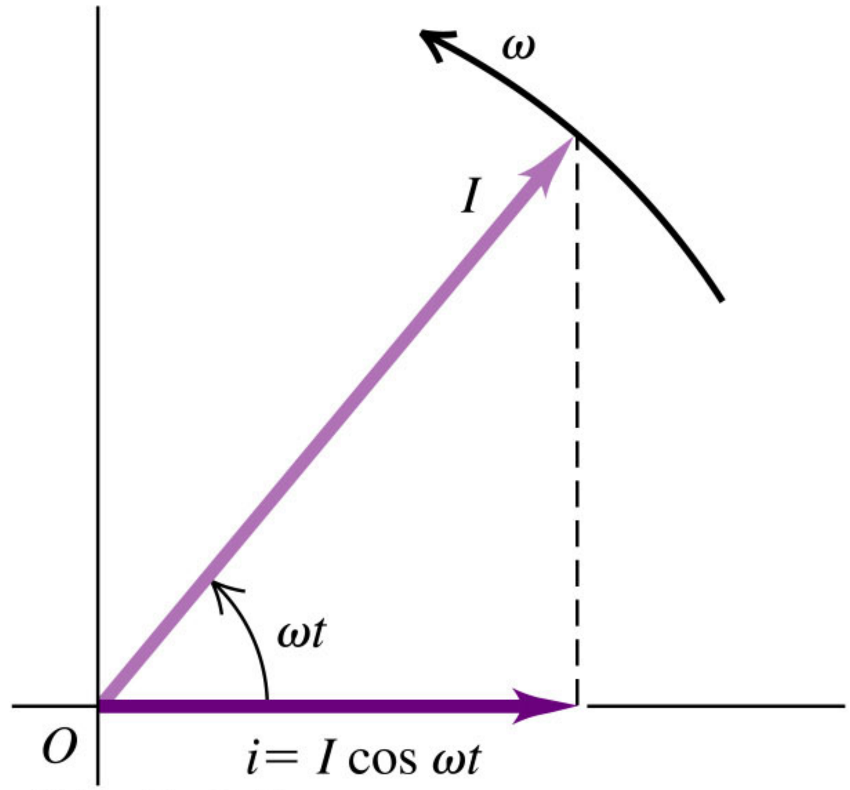
$$I_{RMS} = \sqrt{\left\langle i^2(t) \right\rangle} = \frac{I_0}{\sqrt{2}}$$

# Alternating Currents ; Phasors

A convenient method to describe currents and voltages in AC circuits is "Phasors". Since currents and voltages in circuits with capacitors & inductors have different phase relations, we introduce a phasor diagram. For a current, $i = I\cos(\omega t)$

We can represented this by a vector rotating about the origin. The angle of the vector is given by $\omega t$ and the magnitude of the current is its projection on the X-axis.

If we plot simultaneously currents & voltages of different components we can display different phases .

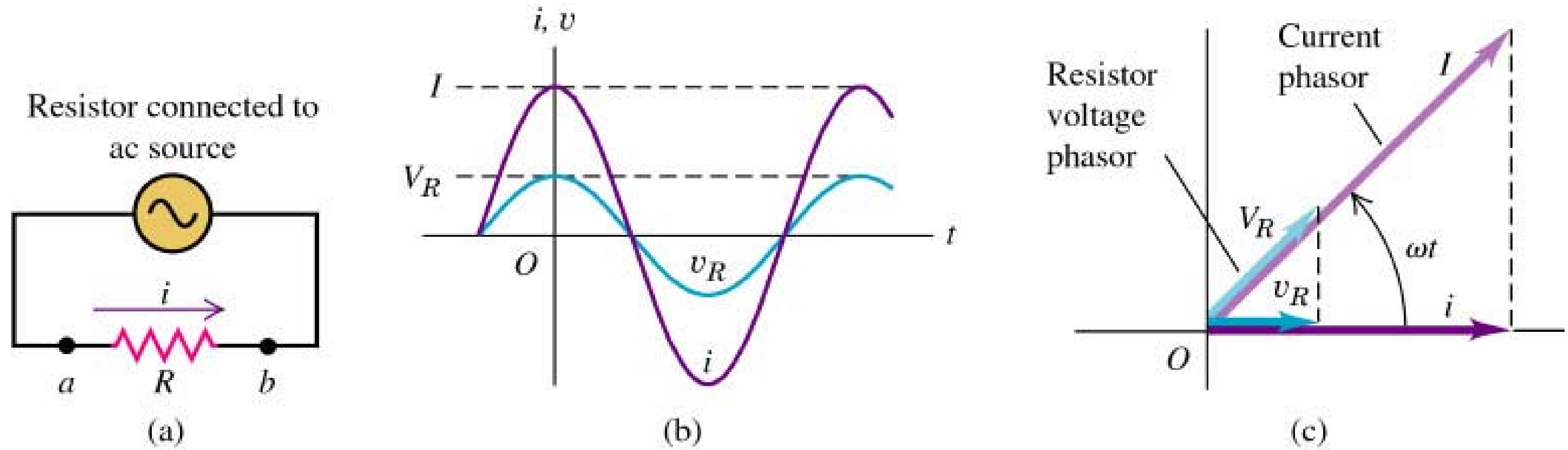





Note this method is equivalent to imaginary numbers approach where we take the real part (x-axis projection) for the magnitude

“Beam me up Scotty –
It ate my phasor!”

# Alternating Currents: Resistor in AC circuit



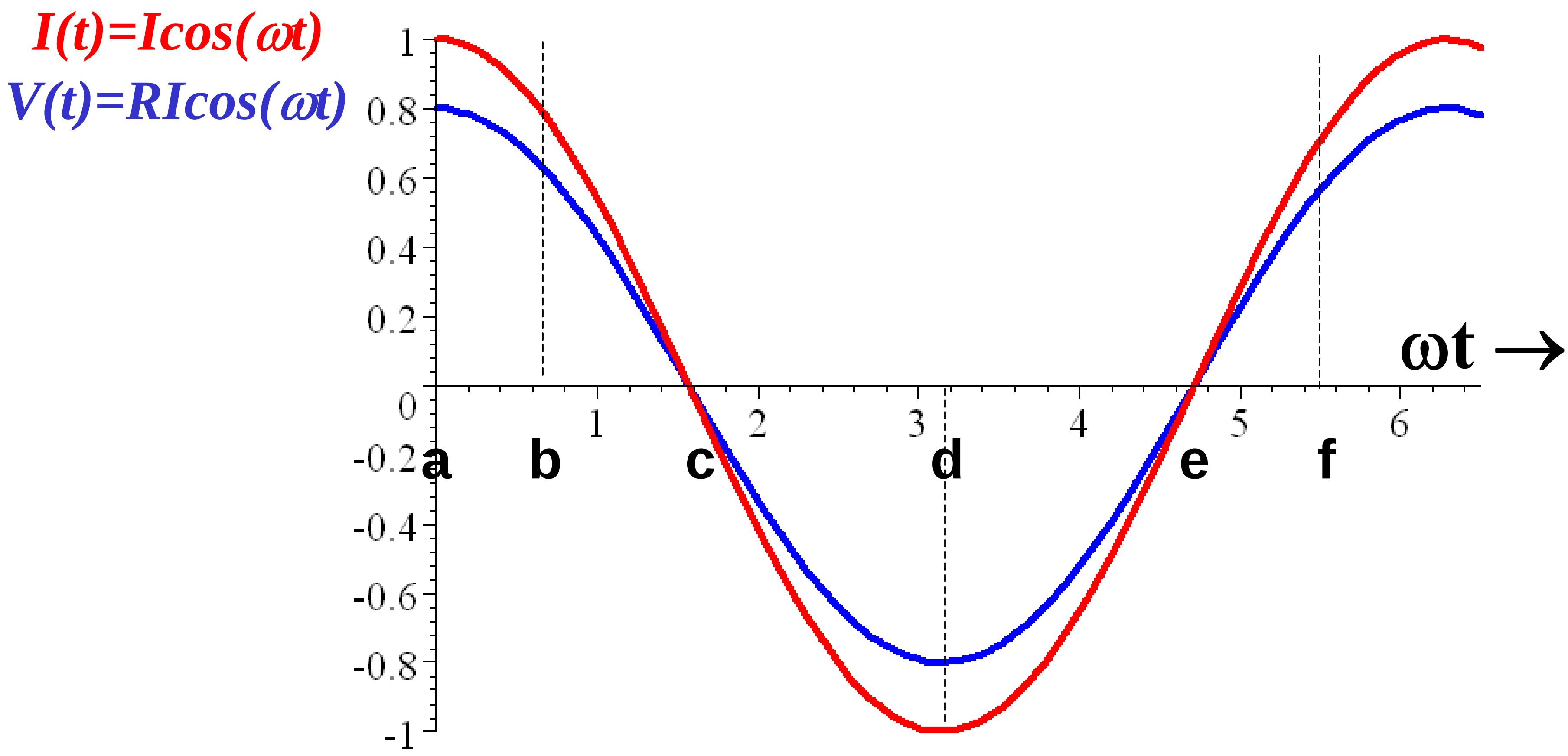

A resistor connected to an AC source will have the voltage, $v_R$, and the current across the resistor has the same phase. We can draw the current phasor and the voltage phasor with the same angle.

$$v_R = V_R \cos(\omega t) = iR = I\cos(\omega t)R$$

and $V_R = IR$ (just like DC case)

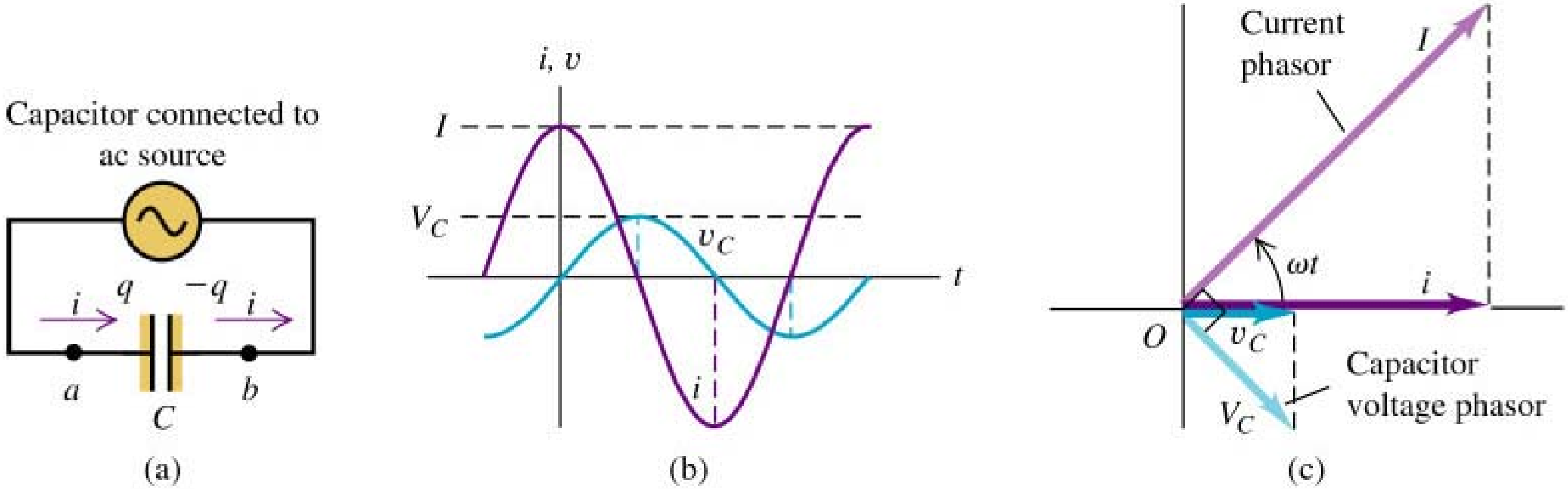
Phasors are rotating 2 dimensional vectors

# Resistor in AC circuit; I & V versus $\omega t$




Note: Voltage is in phase with current

# Alternating Currents: Capacitor in AC circuit




A capacitor connected to an AC current source will have the voltage lagging behind the current by 90 degrees. We can draw the current phasor and the voltage phasor <u>behind</u> the current by 90 degrees.

$$i = \frac{dq}{dt} = I\cos(\omega t)$$

$$q = \frac{I}{\omega}\sin(\omega t)$$

Find voltage: $$v = \frac{q}{C} = \frac{1}{C}\int i\,dt = \frac{I}{\omega C}\sin(\omega t)$$

$$V_{MAX} = I\left(\frac{1}{\omega C}\right)$$

# Alternating Currents ; Capacitor in AC circuit

We define the capacitive reactance, $X_C$, as $$X_C = \frac{1}{\omega C}$$

$$V_{cap}^{MAX} = \frac{I}{\omega C} = I\left(\frac{1}{\omega C}\right) = I\ X_C$$

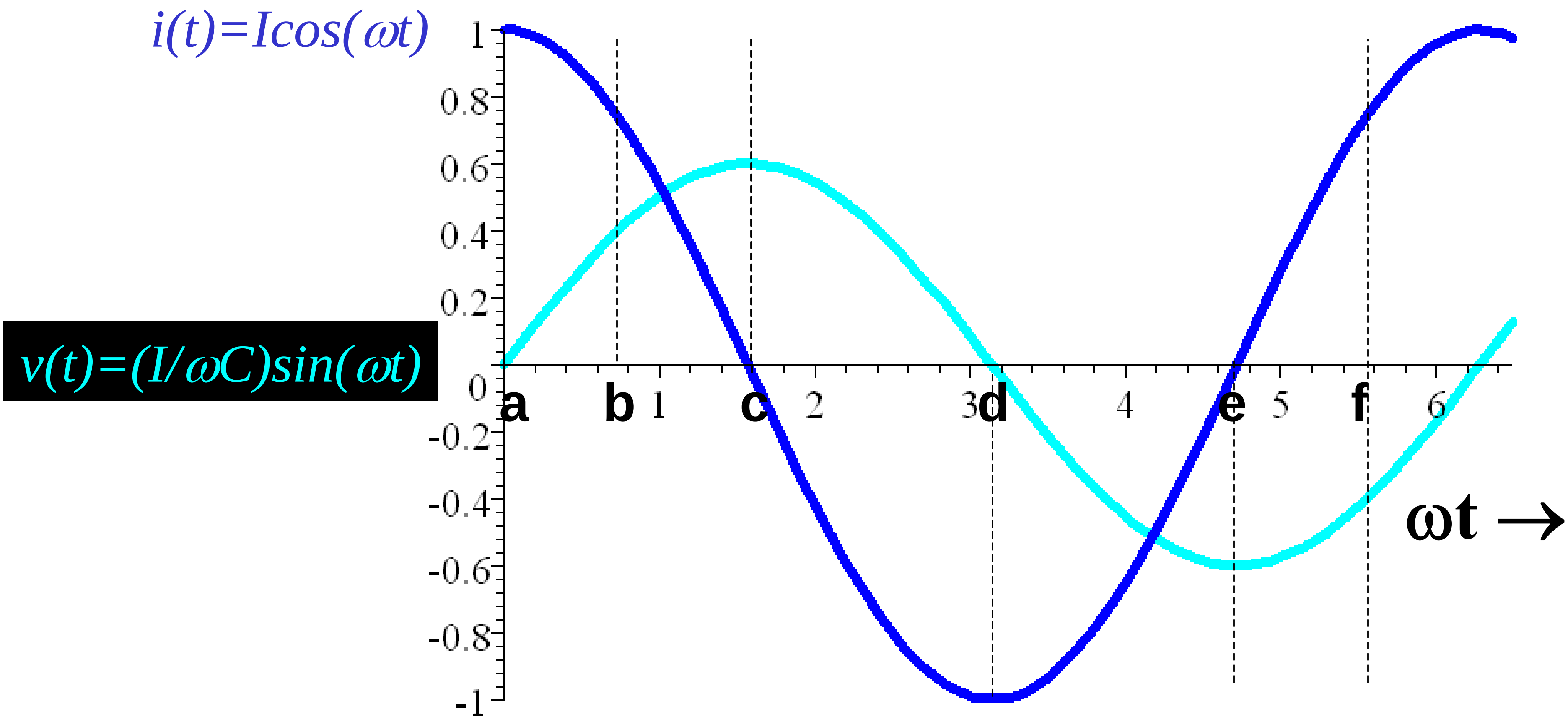
Like: $V_R = IR$

We stated that voltage lags by 90 deg., so equivalent solution is

$$v = \frac{I}{\omega C}\cos(\omega t - 90) = \frac{I}{\omega C}\left[\cos\omega t\cos 90 + \sin\omega t\sin 90\right]$$

$$= \frac{I}{\omega C}\sin\omega t$$

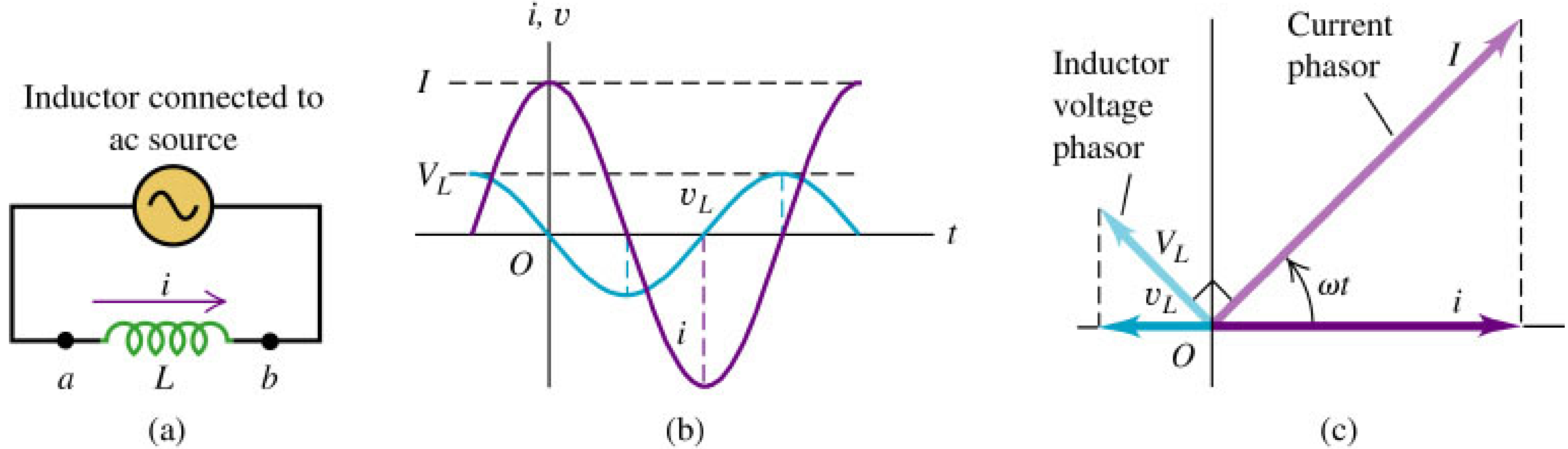
# Capacitor in AC circuit; I & V versus $\omega t$




Note voltage lags 90 deg. Behind current

$$V(t)=(I/\omega C)\sin(\omega t)= (I/\omega C)\cos(\omega t-\pi/2)$$

# Alternating Currents: Inductor in AC circuit




An inductor connected to an AC current source will have the voltage leading (ahead of) the current by 90 degrees. We can draw the current phasor and the voltage phasor <u>ahead</u> the current by 90 degrees.

$$i = I\cos(\omega t) \quad \text{and} \quad V = L\frac{di}{dt} = -IL\omega\sin(\omega t)$$

$$V_{MAX} = IL\omega$$

Define inductive reactance, $X_L$, as $\quad X_L = \omega L$

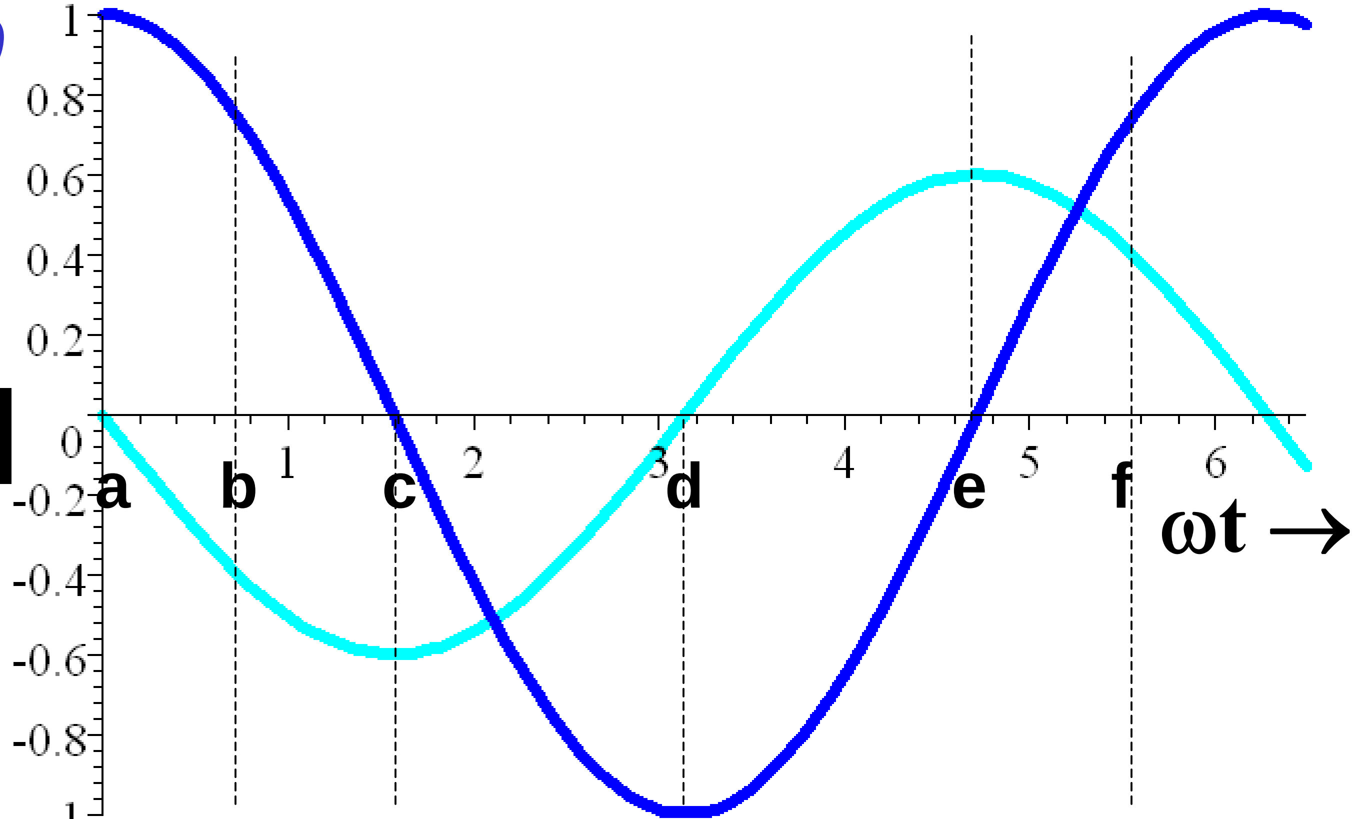
$$V_{ind}^{MAX} = I\omega L = I(\omega L) = I\ X_L$$

Like: $V_R = IR$

# Inductor in AC circuit; I & V versus $\omega t$

$i(t) = I\cos(\omega t)$

$v(t) = -IL\omega\sin(\omega t)$




**Draw phasor diagram for each point**

**Note voltage leads 90 deg. ahead current**

$$v(t) = IL\omega\sin(\omega t) = IL\omega\cos(\omega t + \pi/2)$$

# What is *reactance*?

**You can think of it as a frequency-dependent resistance.**

$$X_C = \frac{1}{\omega C}$$

For high $\omega$, $\chi_C \sim 0$

- Capacitor looks like a wire ("short")

For low $\omega$, $\chi_C \rightarrow \infty$

- Capacitor looks like a break

$$X_L = \omega L$$

For low $\omega$, $\chi_L \sim 0$

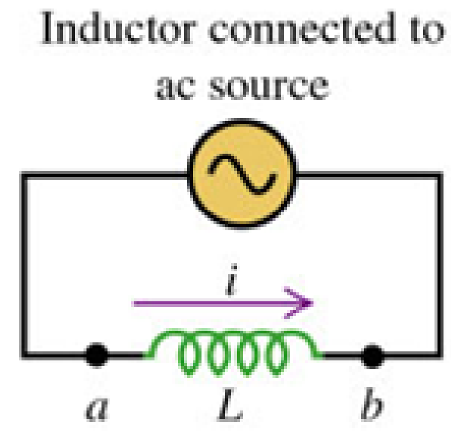
- Inductor looks like a wire ("short")

For high $\omega$, $\chi_L \rightarrow \infty$

- Inductor looks like a break
(inductors resist change in current)

$$("X_R" = R)$$

# Frequency Filtering with inductor circuit

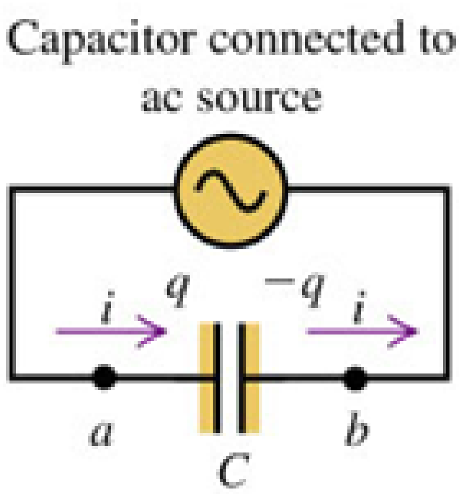



The voltage across the inductor is,

$$V_{ab} = L\frac{di}{dt} = -IL\omega\sin(\omega t)$$

and the magnitude depends on ω. So LOW frequencies are reduced or FILTERED out.

# Frequency Filtering with capacitor circuit

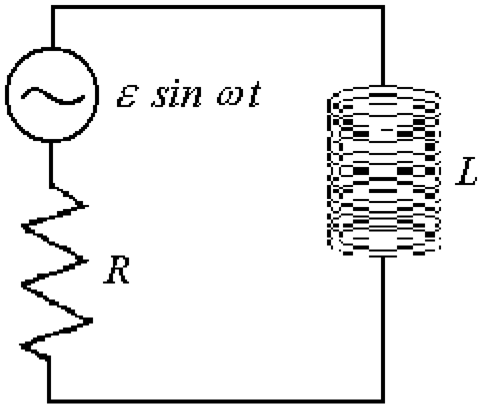



The voltage across the capacitor is,

$$V_{\mathrm{ab}} = \frac{I}{\omega C}\sin(\omega t)$$

and the magnitude depends on (1/ω). So HIGH frequencies are reduced or FILTERED out.

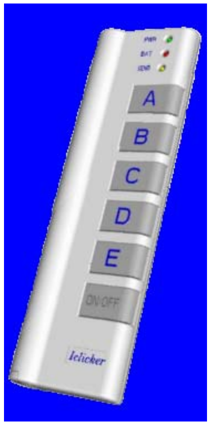
# Question 1




An RL circuit is driven by an AC generator as shown in the figure.

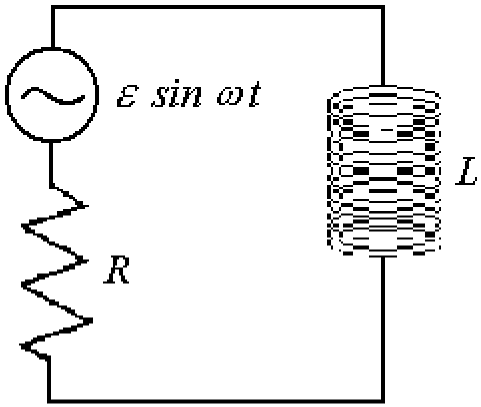
For what driving frequency ω of the generator, will the current through the resistor be largest

a) ω large  b) ω small  c) ω doesn't matter



# Question 1

An RL circuit is driven by an AC generator as shown in the figure.

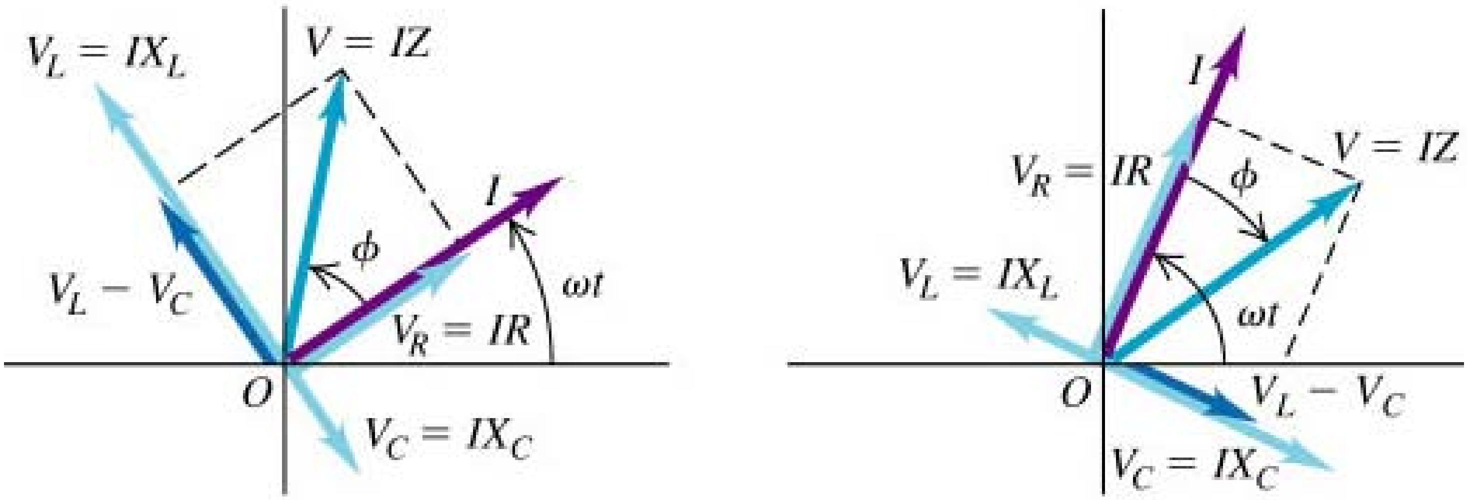



For what driving frequency ω of the generator, will the current through the resistor be largest

a) ω large b) ω small c) ω doesn't matter

The current amplitude is inversely proportional to the frequency of the generator.

# Alternating Currents: LRC circuit




(b) (c)

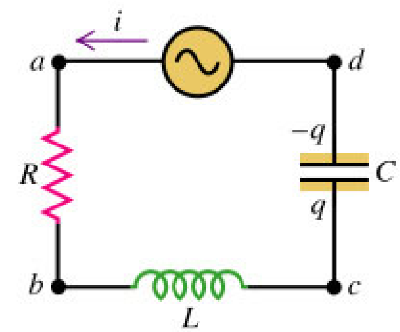



Figure (b) has $X_L > X_C$ and (c) has $X_L < X_C$ .

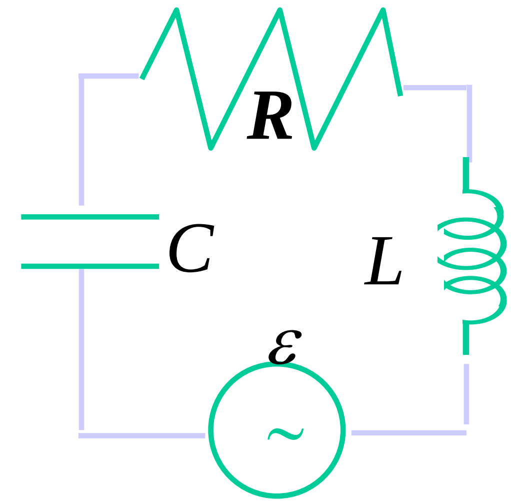
Using Phasors, we can construct the phasor diagram for an LRC Circuit. This is similar to 2-D vector addition. We add the phasors of the resistor, the inductor, and the capacitor. The inductor phasor is +90 and the capacitor phasor is -90 relative to the resistor phasor.

Adding the three phasors vectorially, yields the voltage sum of the resistor, inductor, and capacitor, which must be the same as the voltage of the AC source. Kirchoff's voltage law holds for AC circuits.

Also $V_R$ and I are in phase.

# Phasors

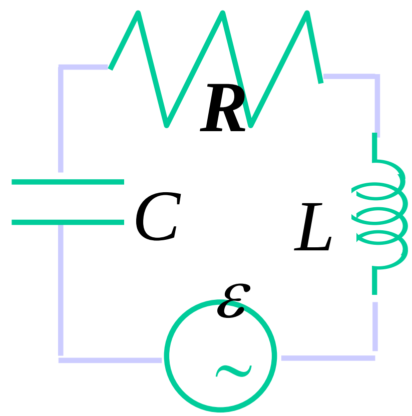
**Problem**: Given $V_{\text{drive}} = \varepsilon_m \sin(\omega t)$,
find $V_R$, $V_L$, $V_C$, $I_R$, $I_L$, $I_C$




**Strategy**:

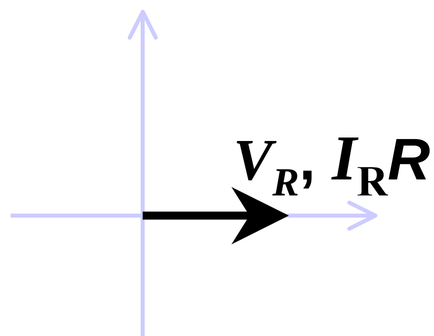
**We will use Kirchhoff's voltage law that the (phasor) sum of the voltages $V_R$, $V_C$, and $V_L$ must equal $V_{drive}$.**

# Phasors, cont.

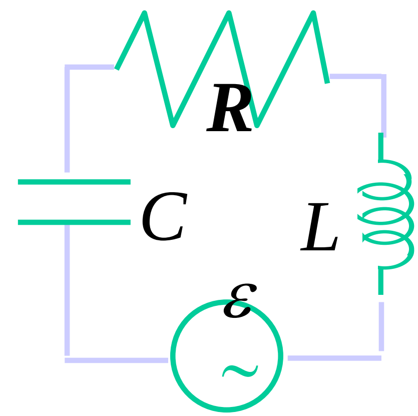



**Problem**: Given $V_{\text{drive}} = \varepsilon_m \sin(\omega t)$,
find $V_R$, $V_L$, $V_C$, $I_R$, $I_L$, $I_C$

1. Draw $V_R$ phasor along $x$-axis (this direction is chosen for convenience). Note that since $V_R = I_R R$, this is also the direction of the current phasor $i_R$. Because of Kirchhoff's current law, $I_L = I_C = I_R \equiv I$ (i.e., the same current flows through each element).

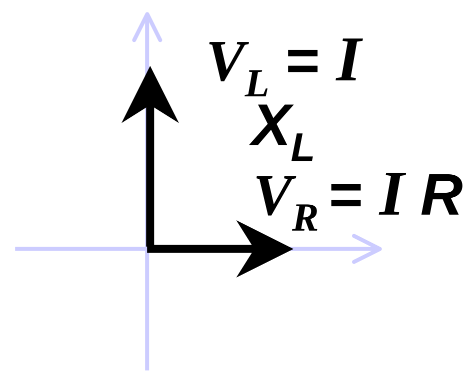


# Phasors, cont.

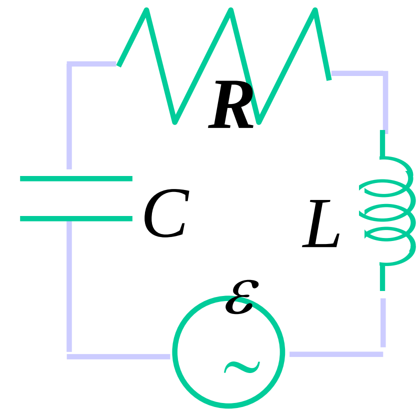



**<u>Problem</u>: Given** $V_{\text{drive}} = \varepsilon_m \sin(\omega t)$,
**find** $V_R$, $V_L$, $V_C$, $I_R$, $I_L$, $I_C$

2. **Next draw the phasor for $V_L$. Since the inductor current $I_L$ always lags $V_L$ → draw $V_L$ 90° further counterclockwise. The length of the $V_L$ phasor is $I_L\, X_L = I\, \omega L$**

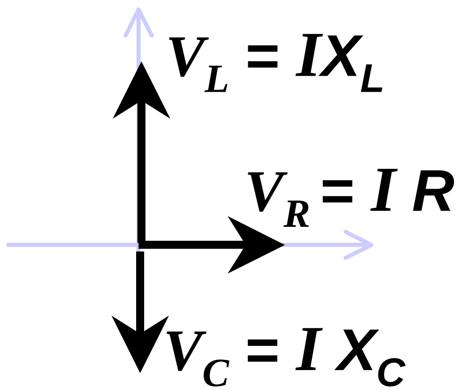


# Phasors, cont.



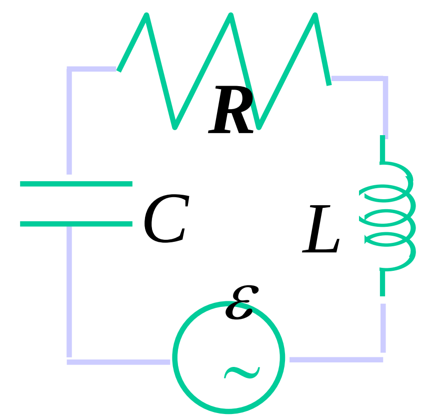

**<u>Problem</u>: Given $V_{\text{drive}} = \varepsilon_m \sin(\omega t)$, find $V_R$, $V_L$, $V_C$, $I_R$, $I_L$, $I_C$**

**3. The capacitor voltage $V_C$ always lags $I_C$ → draw $V_C$ 90° further clockwise. The length of the $V_C$ phasor is $I_C X_C = I/\omega C$**

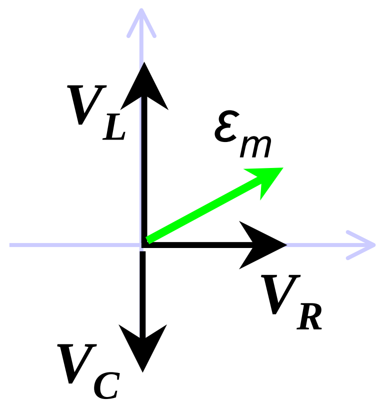



**The lengths of the phasors depend on $R$, $L$, $C$, and $\omega$. The relative orientation of the $V_R$, $V_L$, and $V_C$ phasors is *<u>always</u>* the way we have drawn it. Memorize it!**

# Phasors, cont.




**<u>Problem</u>: Given $V_{\text{drive}} = \varepsilon_m \sin(\omega t)$,**
**find $V_R$, $V_L$, $V_C$, $I_R$, $I_L$, $I_C$**

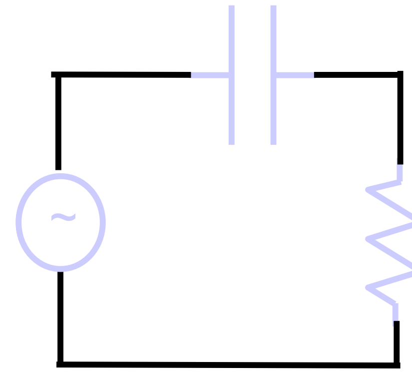


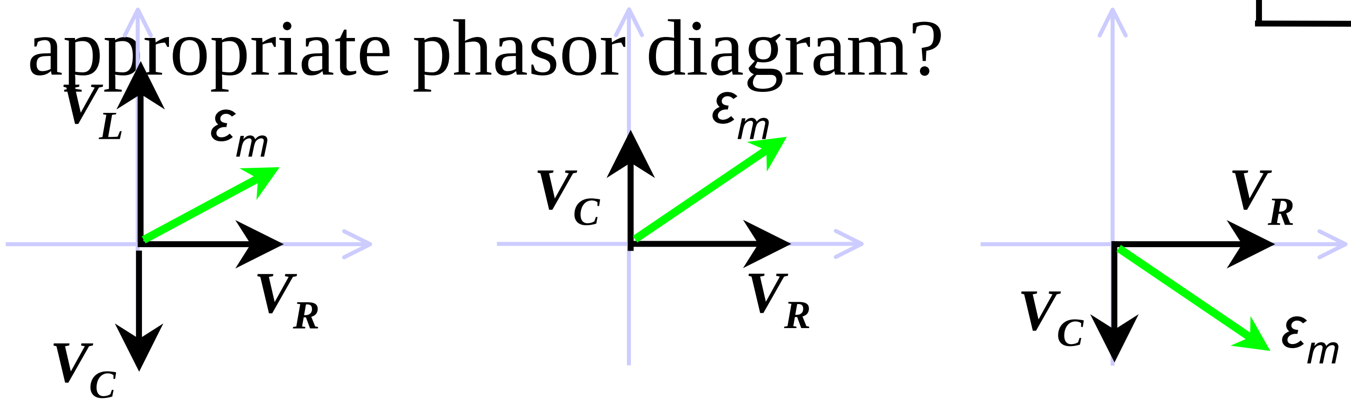

- **The phasors for $V_R$, $V_L$, and $V_C$ are added like *vectors* to give the drive voltage $V_R + V_L + V_C = \varepsilon_m$ :**

- From this diagram we can now easily calculate quantities of interest, like the net current $I$, the maximum voltage across any of the elements, and the phase between the current the drive voltage (and thus the power).

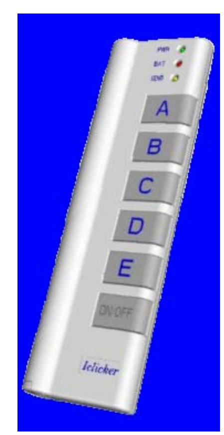
# Question 2

A series RC circuit is driven by emf $\varepsilon$. Which of the following could be an appropriate phasor diagram?






**(a)** **(b)** **(c)**



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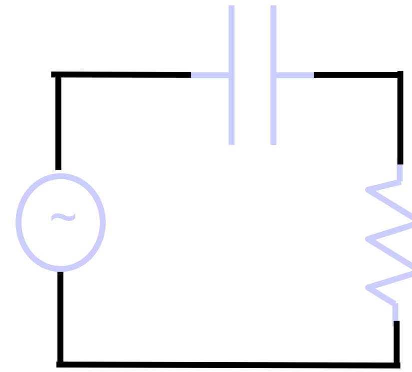
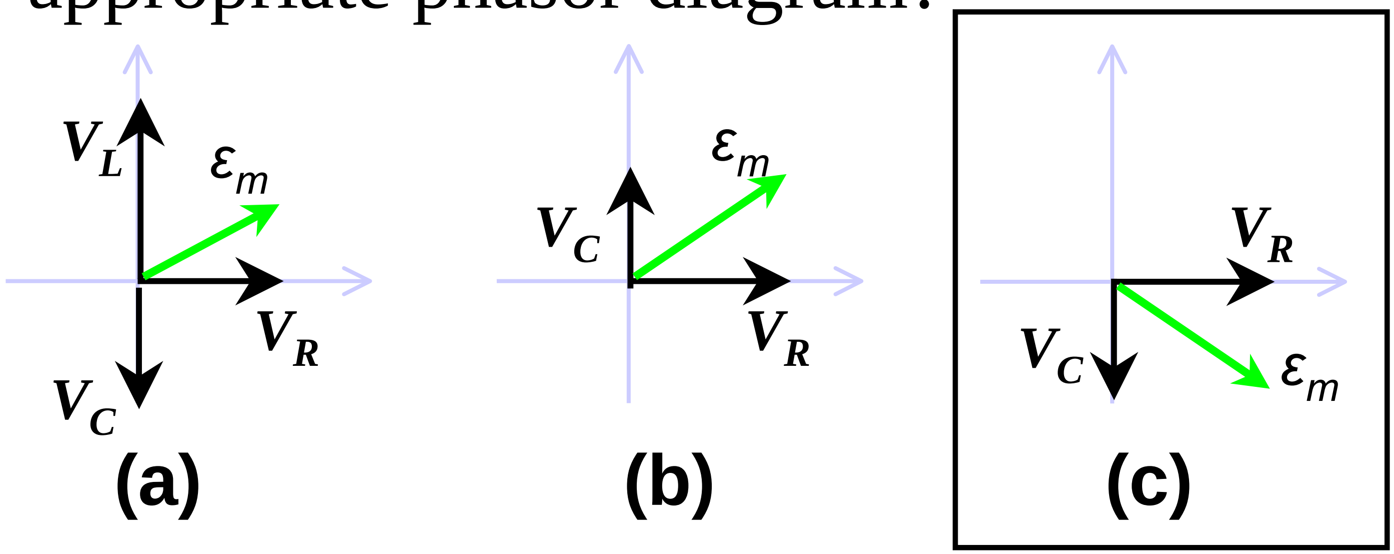
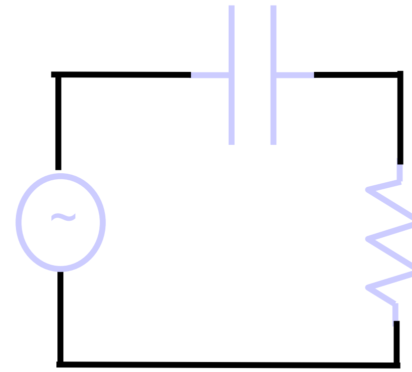


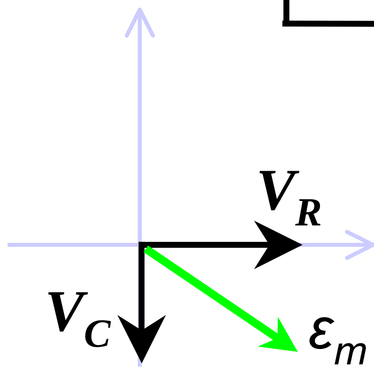



- The phasor diagram for the driven series RLC circuit always has the voltage across the capacitor lagging the current by 90°. The vector sum of the $V_C$ and $V_R$ phasors must equal the generator emf phasor $\varepsilon_m$.

# Question 3

A series RC circuit is driven by emf $\varepsilon$.



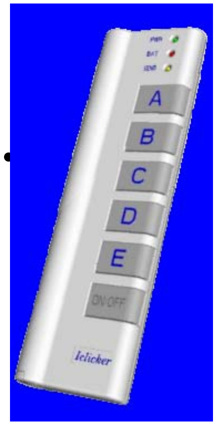



For this circuit which of the following is true?

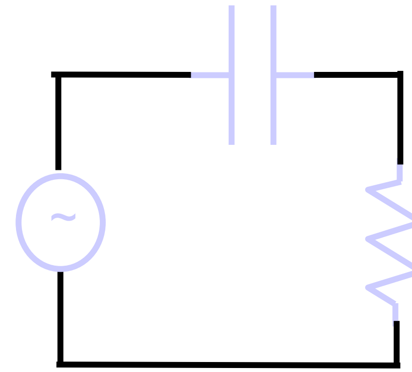
(a) The drive voltage is in phase with the current.

(b) The drive voltage lags the current.

(c) The drive voltage leads the current.



# Question 3



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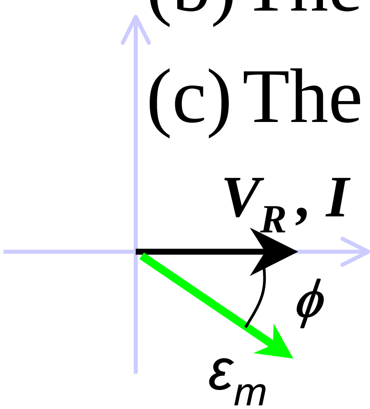



First, remember that the current phasor $I$ is always in the same orientation as the resistor voltage phasor $V_R$ (since the current and voltage are always in phase). From the diagram, we see that the drive phasor $\varepsilon_m$ is *lagging* (clockwise) $I$. Just as $V_C$ lags $I$ by 90°, in an AC driven RC circuit, the drive voltage will also lag $I$ by some angle less than 90°. The precise phase lag $\phi$ depends on the values of $R$, $C$ and $\omega$.

## Voltage *V(t)* across AC source

$$v(t) = \sqrt{(V_R)^2 + (V_L - V_C)^2}\cos(\omega t + \phi)$$

$$= \sqrt{(IR)^2 + (IX_L - IX_C)^2}\cos(\omega t + \phi)$$

$$= I\sqrt{(R)^2 + (X_L - X_C)^2}\cos(\omega t + \phi) = IZ\cos(\omega t + \phi)$$

$$Z = \sqrt{(R)^2 + (X_L - X_C)^2}$$

Z is called "impedance"

$$V_R = IR$$
$$V_L = IX_L$$
$$V_C = IX_C$$

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{\omega L - 1/\omega C}{R}$$

Also:

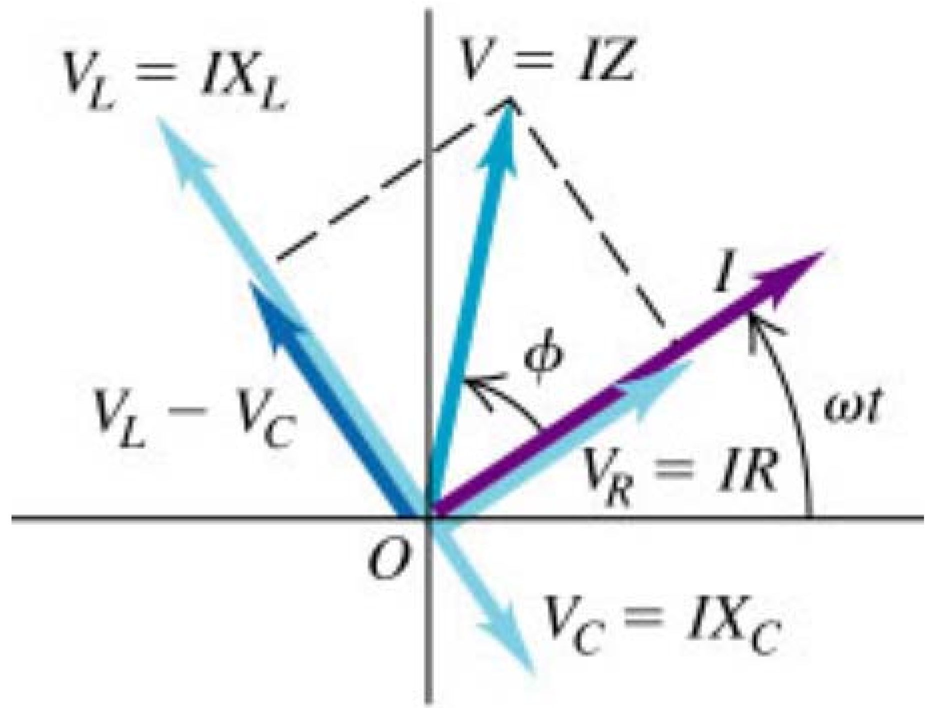
$$V = V^{MAX} = I\ Z$$

Like: $V_R = IR$

$$V_{rms} = I_{rms} Z$$

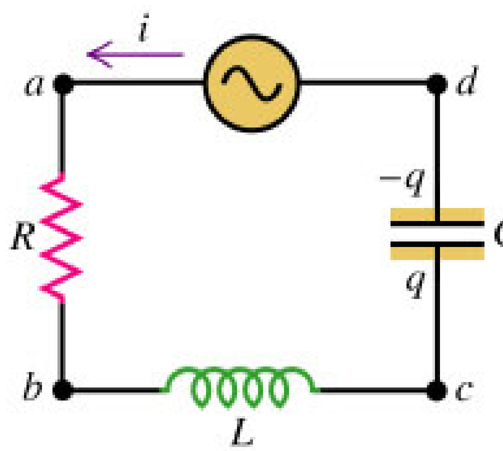
$$i = I\cos\omega t$$

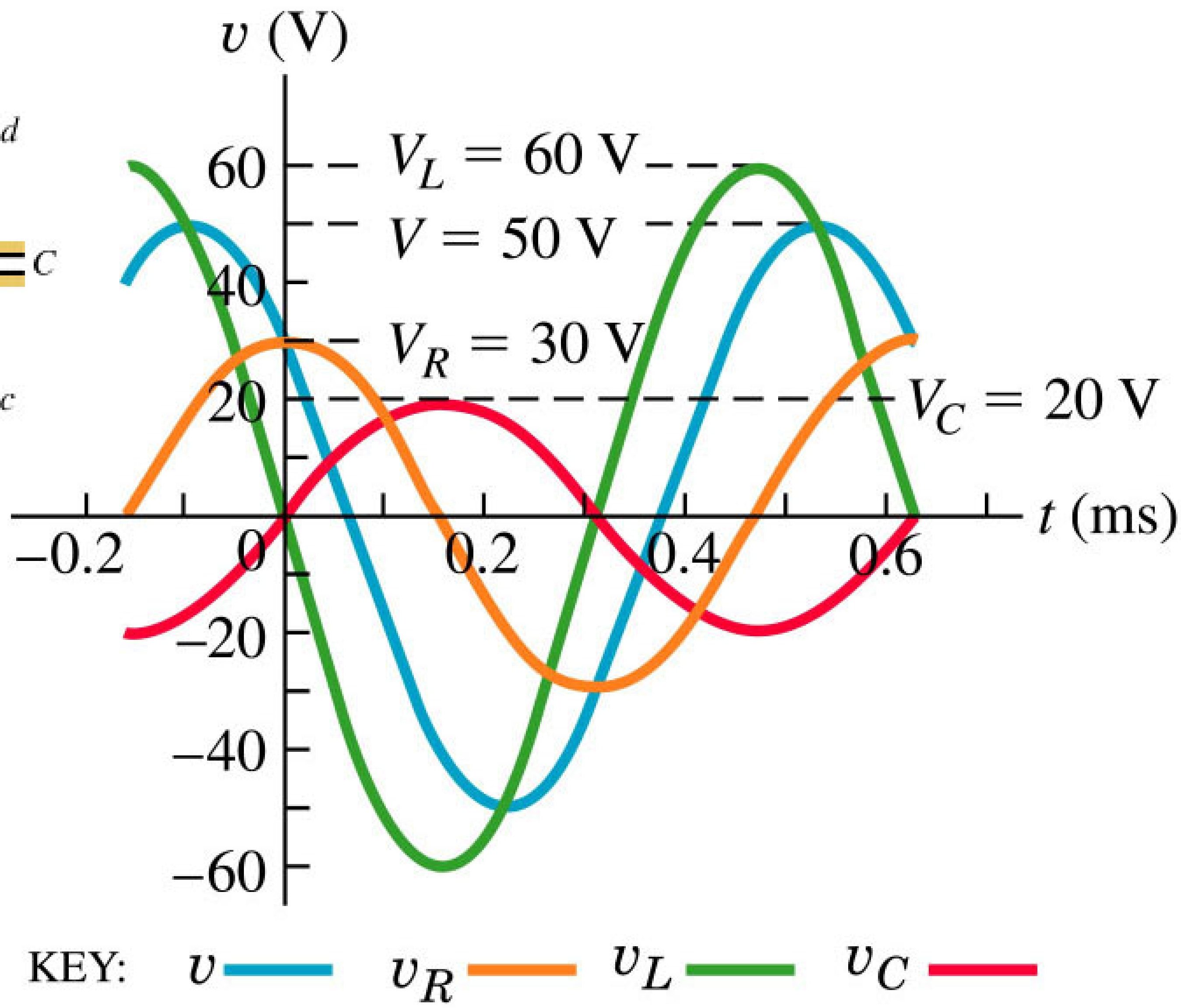
$$v = V\cos(\omega t + \phi)$$



# Alternating Currents: LRC circuit, Fig. 31.11

**Y&F Example 31.4**

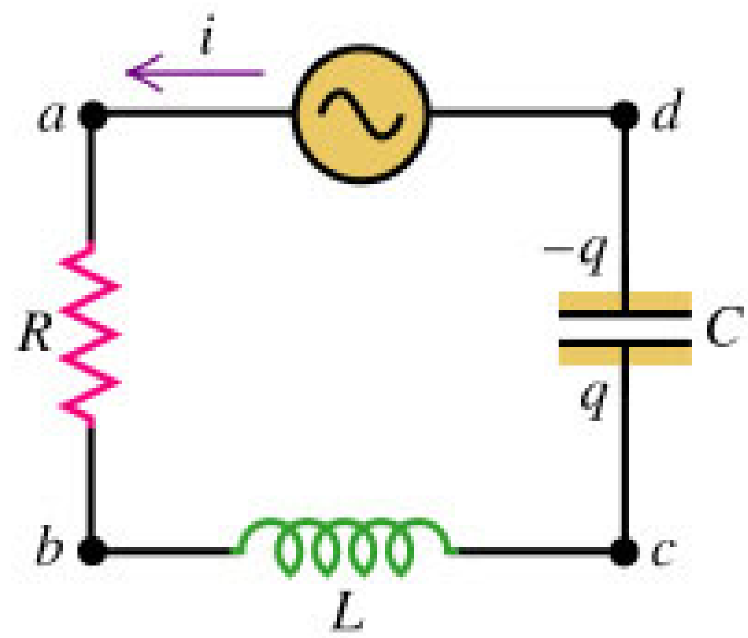



**V=50v**
**ω=10000rad/s**
**R=300ohm**
**L=60mH**
**C=0.5μC**

# LRC series circuit; Summary of instantaneous Current and voltages

$$V_R = IR$$
$$V_L = IX_L$$
$$V_C = IX_C$$




$$i(t) = I\cos(\omega t)$$
$$v_R(t) = IR\cos(\omega t)$$
$$v_C(t) = IX_C\cos(\omega t - 90) = I\frac{1}{\omega C}\cos(\omega t - 90)$$
$$v_L(t) = IX_L\cos(\omega t + 90) = I\omega L\cos(\omega t + 90)$$
$$v_{ad}(t) = I\sqrt{(X_R)^2 + (X_L - X_C)^2}\cos(\omega t + \phi)$$
$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{\omega L - 1/\omega C}{R}$$

# For next time

- Homework #10 [due now]
- Homework #11 posted this afternoon, due next Wed.
- Quiz on Friday:  Faraday’s Law, Inductance and Inductors, RLC circuits