Course Updates

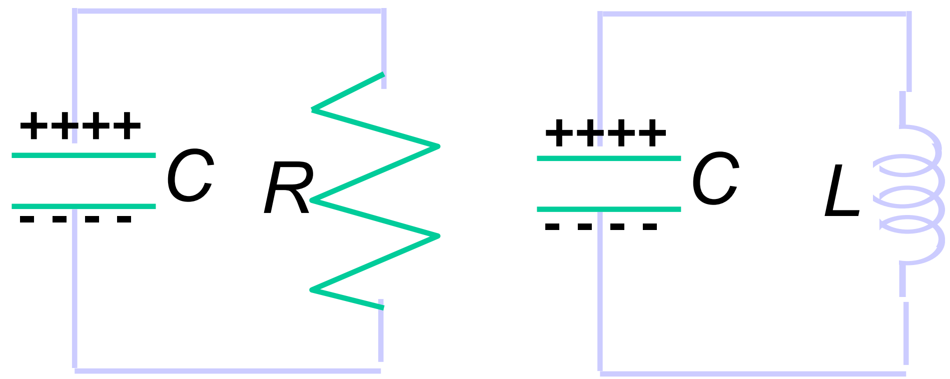
http://www.phys.hawaii.edu/~varner/PHYS272-Spr10/physics272.html

Reminders:

1) Assignment #10 → due Wednesday
2) Quiz # 5 Friday
3) Today finish RLC circuits

LC Circuits

- **Consider the RC and LC series circuits shown:**



- **Suppose that the circuits are formed at t=0 with the capacitor charged to value Q.**

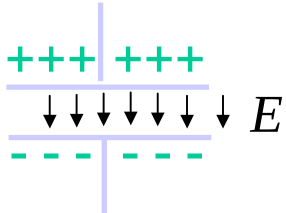
There is a qualitative difference in the time development of the currents produced in these two cases. Why??

- Consider from point of view of energy!
  - In the RC circuit, any current developed will cause energy to be dissipated in the resistor.
  - In the LC circuit, there is NO mechanism for energy dissipation; energy can be stored both in

Energy in the *Electric* and *Magnetic* Fields

Energy stored in a capacitor ...

$$U = \frac{1}{2} C V^2$$

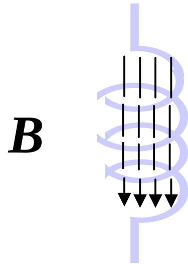


$$u_{\text{electric}} = \frac{1}{2}\varepsilon_0 E^2$$

… energy density …

Energy stored in an inductor ….

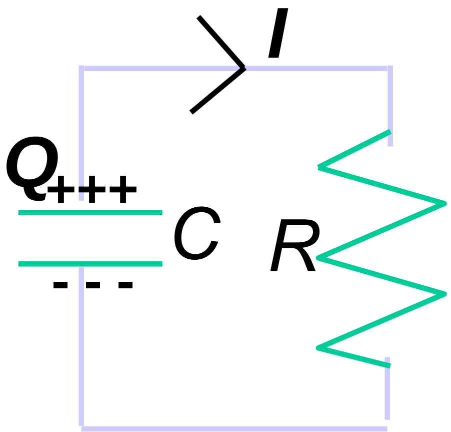
$$U = \frac{1}{2} L I^2$$

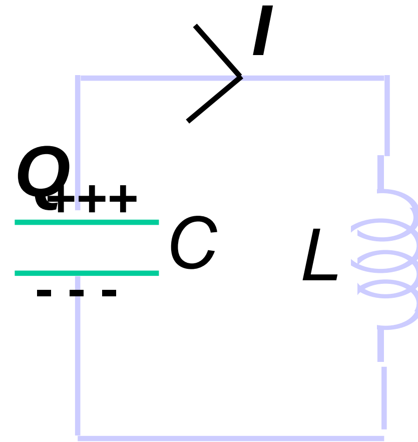


$$u_{\text{magnetic}} = \frac{1}{2}\frac{B^2}{\mu_0}$$

… energy density …

RC/LC Circuits



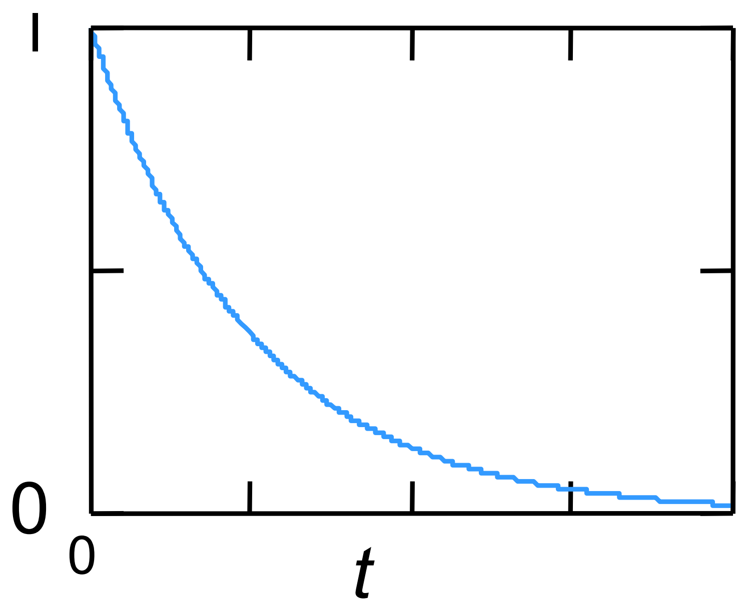


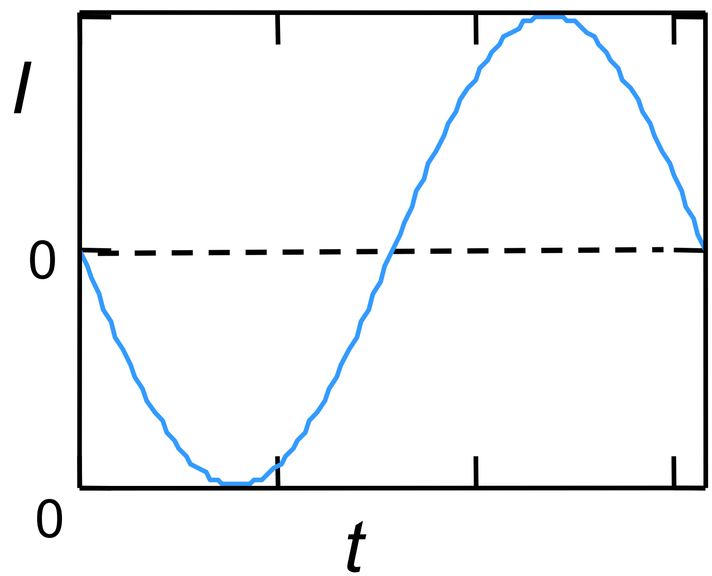
RC:

current decays exponentially

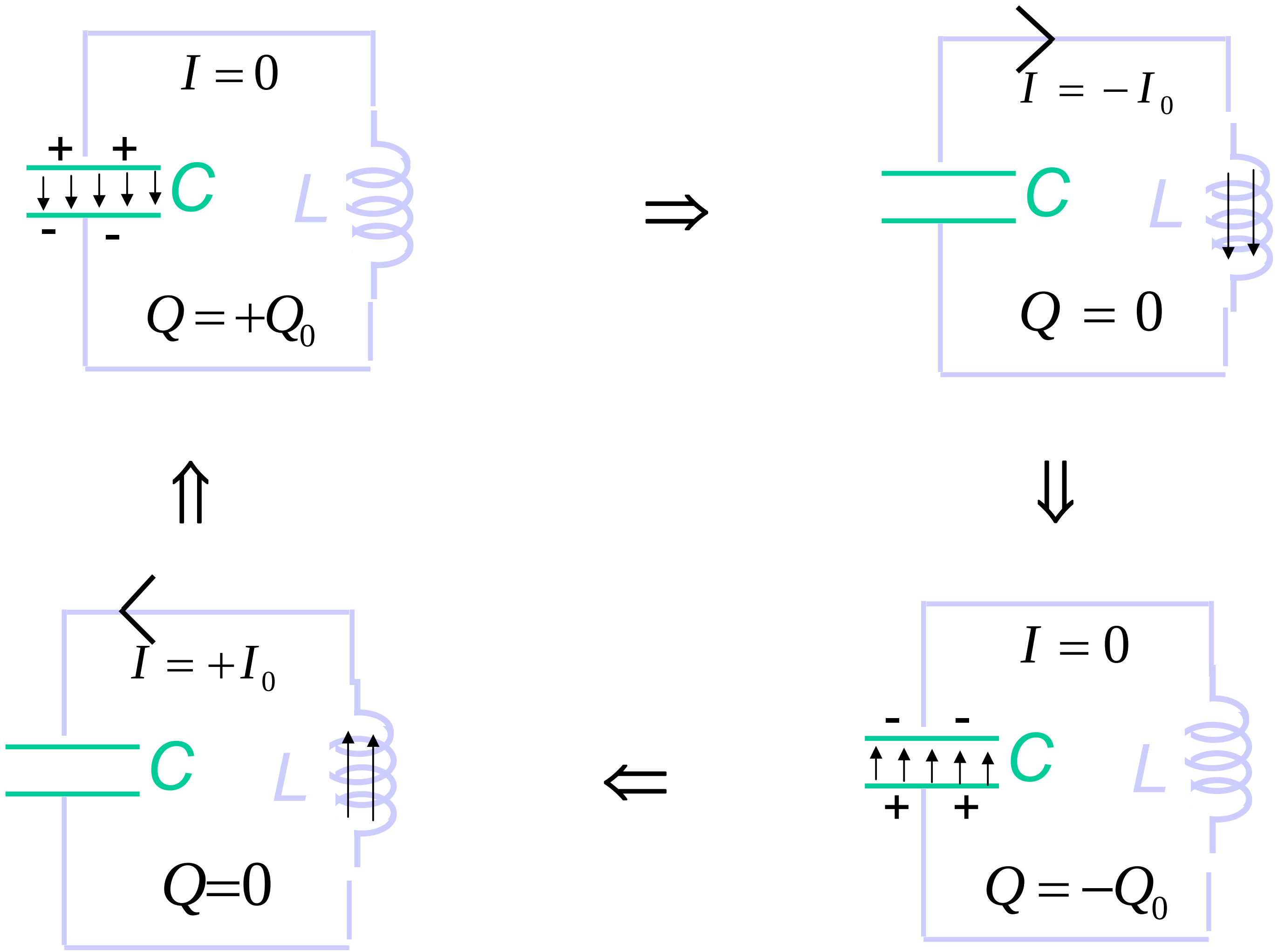
LC:

current oscillates



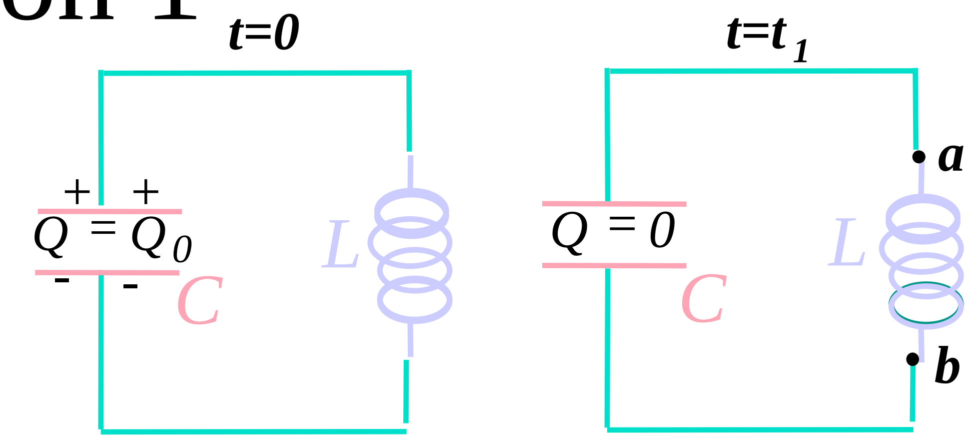


LC Oscillations
(qualitative)

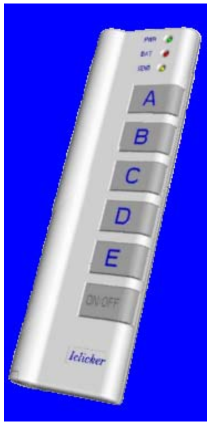


Question 1

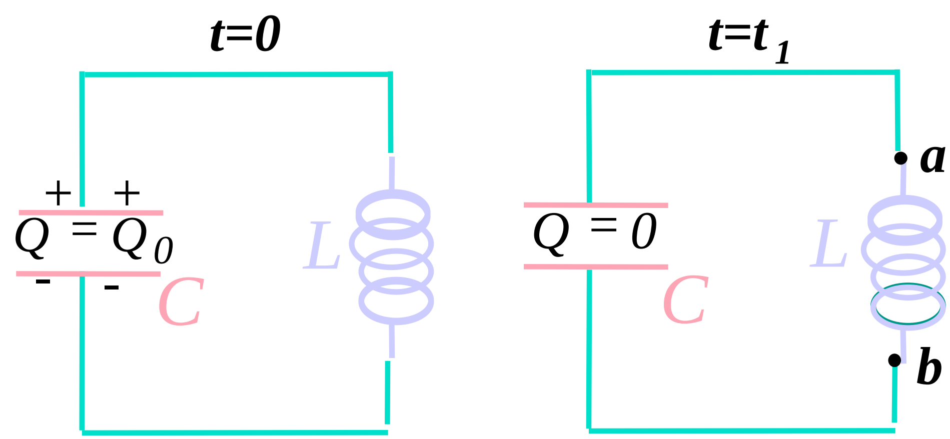
- At t=0, the capacitor in the LC circuit shown has a total charge Q_0. At $t = t_1$, the capacitor is uncharged.
  - What is the value of $V_{ab}=V_b-V_a$, the voltage across the inductor at time t_1



(a) $V_{ab} < 0$ (b) $V_{ab} = 0$ (c) $V_{ab} > 0$



Question 1



- At t=0, the capacitor in the LC circuit shown has a total charge Q_0. At $t = t_1$, the capacitor is uncharged.
  - What is the value of $V_{ab}=V_b-V_a$, the voltage across the inductor at time t_1?

(a) $V_{ab} < 0$

(b) $\boxed{V_{ab} = 0}$

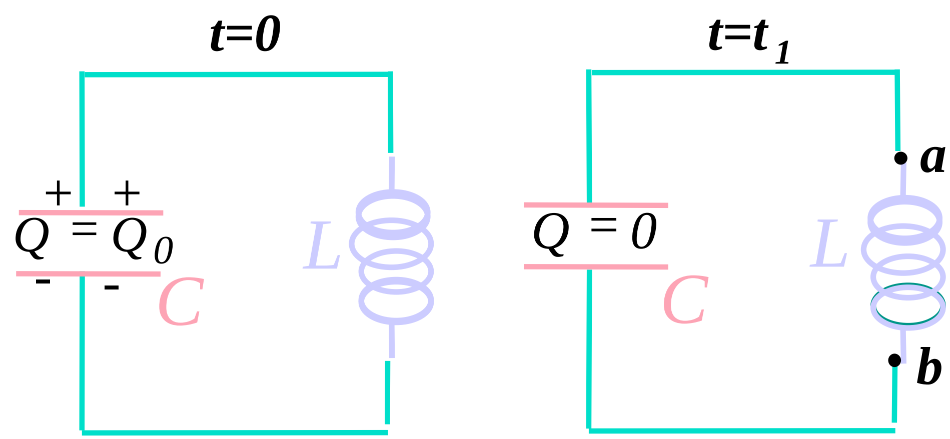
(c) $V_{ab} > 0$

- **V_{ab} is the voltage across the inductor, but it is also (minus) the voltage across the capacitor!**

- **Since the charge on the capacitor is zero, the voltage across the capacitor is zero!**

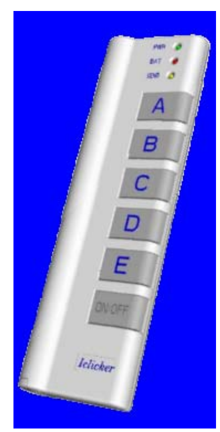
Question 2

- At t=0, the capacitor in the LC circuit shown has a total charge Q_0. At $t = t_1$, the capacitor is uncharged.



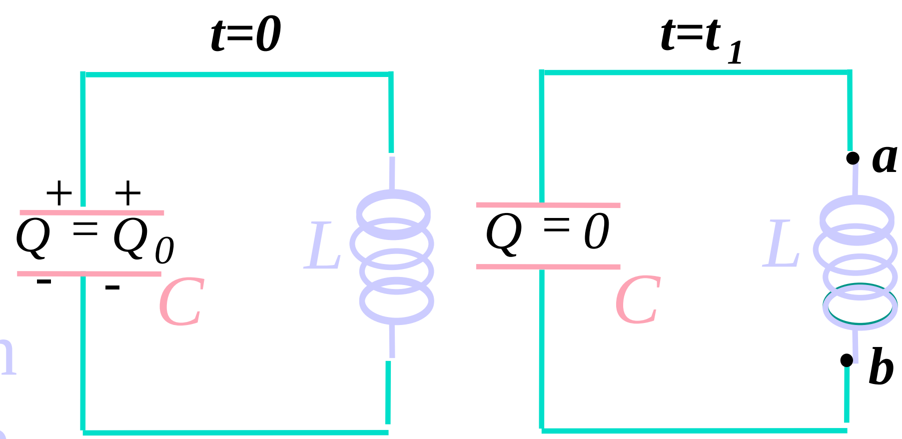
– What is the relation between U_{L1}, the energy stored in the inductor at $t=t_1$, and U_{C1}, the energy stored in the capacitor at $t=t_1$?

(a) $U_{L1} < U_{C1}$ (b) $U_{L1} = U_{C1}$ (c) $U_{L1} > U_{C1}$



Question 2

- At t=0, the capacitor in the LC circuit shown has a total charge Q_0. At $t = t_1$, the capacitor is uncharged.
  - What is the relation between U_{L1}, the energy stored in the inductor at $t=t_1$, and U_{C1}, the energy stored in the capacitor at $t=t_1$?



(a) $U_{L1} < U_{C1}$

(b) $U_{L1} = U_{C1}$

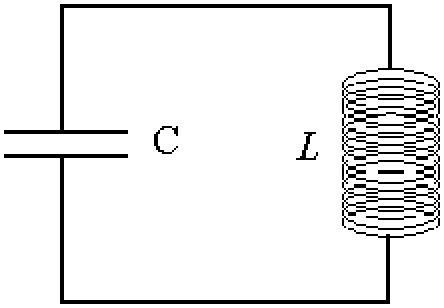
(c) $\boxed{U_{L1} > U_{C1}}$

- At $t=t_1$, the charge on the capacitor is zero.
- At $t=t_1$, the current is a maximum.

$$U_{C1} = \frac{Q_1^2}{2C} = 0$$

$$U_{L1} = \frac{1}{2} L I_1^2 = \frac{Q_0^2}{2C} > 0$$

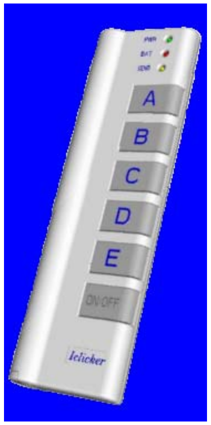
Question 3:



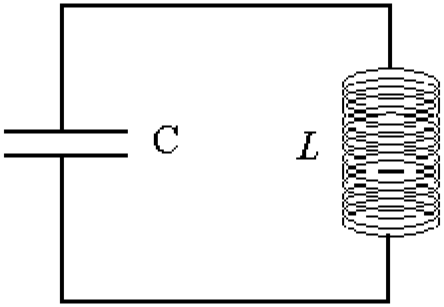
At time $t = 0$ the capacitor is fully charged with Q_{max}, and the current through the circuit is 0.

What is the potential difference across the inductor at $t = 0$?

a) $V_L = 0$ b) $V_L = Q_{max}/C$ c) $V_L = Q_{max}/2C$



Question 3:



At time $t = 0$ the capacitor is fully charged with Q_{max}, and the current through the circuit is 0.

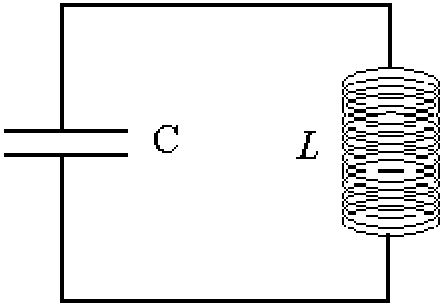
What is the potential difference across the inductor at $t = 0$?

a) $V_L = 0$

b) $V_L = Q_{max}/C$

c) $V_L = Q_{max}/2C$

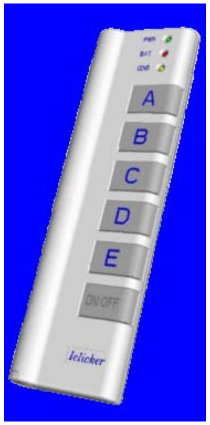
Question 4:



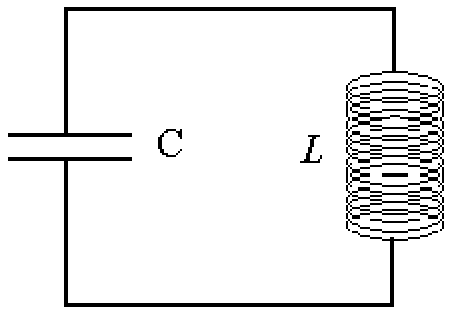
At time $t = 0$ the capacitor is fully charged with Q_{max}, and the current through the circuit is 0.

What is the potential difference across the inductor when the current is maximum?

a) $V_L = 0$ b) $V_L = Q_{max}/C$ c) $V_L = Q_{max}/2C$



Question 4:



At time $t = 0$ the capacitor is fully charged with Q_{max}, and the current through the circuit is 0.

What is the potential difference across the inductor when the current is maximum?

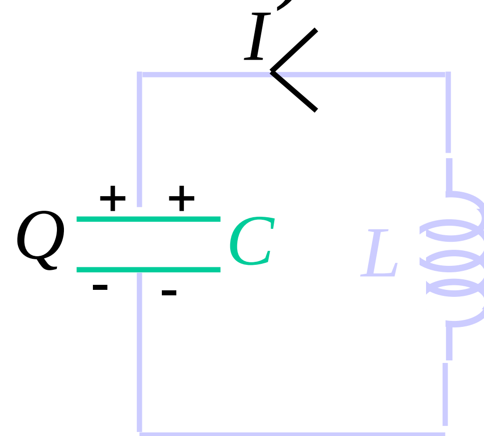
a) $V_L = 0$

b) $V_L = Q_{max}/C$

c) $V_L = Q_{max}/2C$

LC Oscillations
(quantitative, but only for R=0)

- **What is the oscillation frequency ω_0?**
- **Begin with the loop rule:**



$$\boxed{L\frac{d^2Q}{dt^2}+\frac{Q}{C}=0}$$

- **Guess solution: (just harmonic oscillator!)**

$$\boxed{Q=Q_0\cos(\omega t+\phi)}$$

remember:

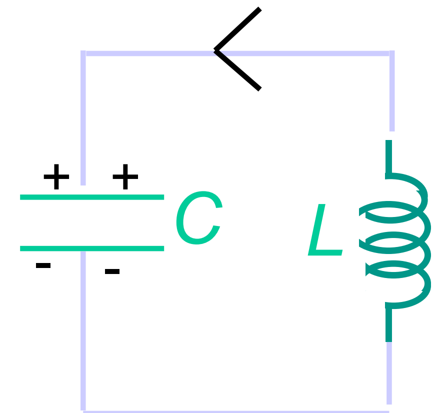
$$\boxed{m\frac{d^2x}{dt^2}+kx=0}$$

where ϕ, $\mathbf{Q_0}$ determined from initial conditions

- **Procedure: differentiate above form for Q and substitute into loop equation to find ω.**
- **Note: Dimensional analysis →**

$$\omega=\frac{1}{\sqrt{LC}}$$

LC Oscillations
(quantitative)



- **General solution:**

$$\boxed{Q = Q_0 \cos(\omega t + \phi)}$$

$$\boxed{L\frac{d^2Q}{dt^2} + \frac{Q}{C} = 0}$$

- **Differentiate:**

$$\frac{dQ}{dt} = -\omega Q_0 \sin(\omega t + \phi)$$

$$\frac{d^2Q}{dt^2} = -\omega^2 Q_0 \cos(\omega t + \phi)$$

- **Substitute into loop eqn:**

$$L\left(-\omega^2 Q_0 \cos(\omega t + \phi)\right) + \frac{1}{C}\left(Q_0 \cos(\omega t + \phi)\right) = 0 \quad \Rightarrow \quad -\omega^2 L + \frac{1}{C} = 0$$

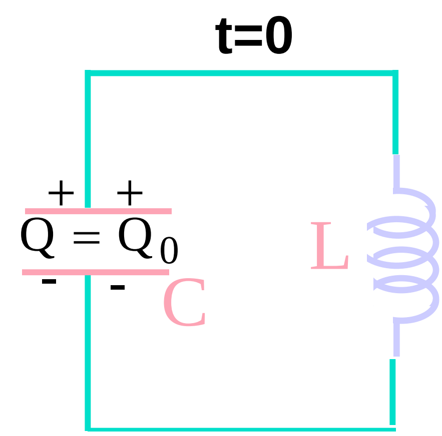
Therefore,

$$\boxed{\omega = \frac{1}{\sqrt{LC}}}$$

which we could have determined from the mass on a spring result:

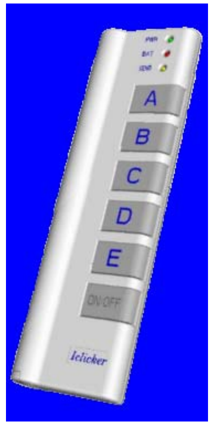
$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1/C}{L}} = \frac{1}{\sqrt{LC}}$$

Question 5

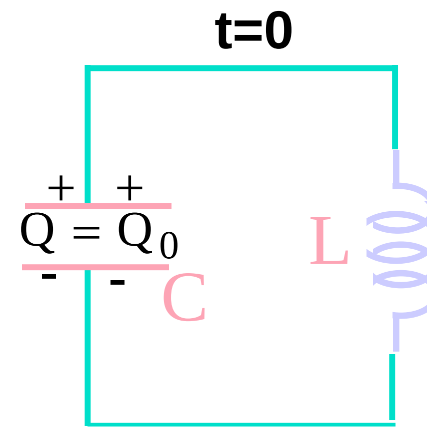


- At t=0 the capacitor has charge Q_0; the resulting oscillations have frequency ω_0. The maximum current in the circuit during these oscillations has value I_0.
  - What is the relation between ω_0 and ω_2, the frequency of oscillations when the initial charge = $2Q_0$?

(a) $\omega_2 = 1/2\ \omega_0$ (b) $\omega_2 = \omega_0$ (c) $\omega_2 = 2\omega_0$



Question 5



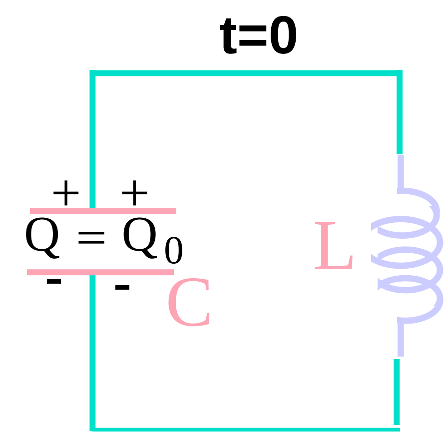
- At t=0 the capacitor has charge Q_0; the resulting oscillations have frequency ω_0. The maximum current in the circuit during these oscillations has value I_0.
  - What is the relation between ω_0 and ω_2, the frequency of oscillations when the initial charge = $2Q_0$?

(a) $\omega_2 = 1/2\ \omega_0$ (b) $\boxed{\omega_2 = \omega_0}$ (c) $\omega_2 = 2\omega_0$

- **Q_0 determines the amplitude of the oscillations (initial condition)**
- **The frequency of the oscillations is determined by the circuit parameters (L, C), just as the frequency of oscillations of a mass on a spring was determined by the physical parameters (k, m)!**

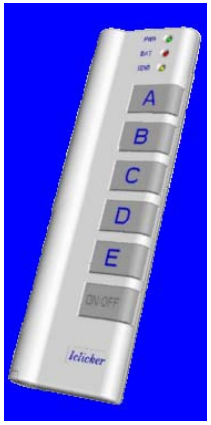
Question 6

- At t=0 the capacitor has charge Q_0; the resulting oscillations have frequency ω_0. The maximum current in the circuit during these oscillations has value I_0.

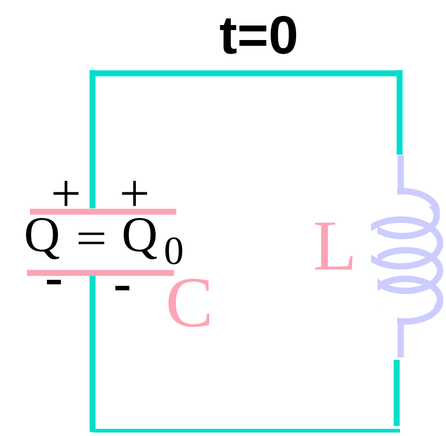


– What is the relation between I_0 and I_2, the maximum current in the circuit when the initial charge = $2Q_0$?

(a) $I_2 = I_0$ (b) $I_2 = 2I_0$ (c) $I_2 = 4I_0$



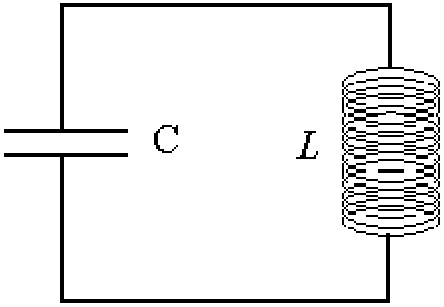
Question 6



- At t=0 the capacitor has charge Q_0; the resulting oscillations have frequency ω_0. The maximum current in the circuit during these oscillations has value I_0.
  - What is the relation between I_0 and I_2, the maximum current in the circuit when the initial charge = $2Q_0$?

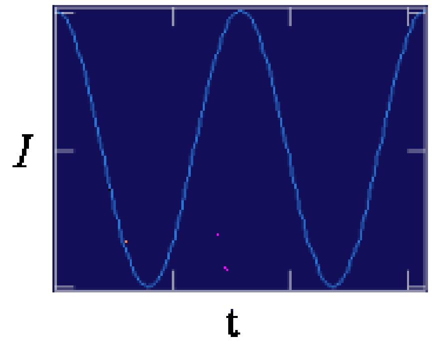
(a) $I_2 = I_0$ (b) $\boxed{I_2 = 2I_0}$ (c) $I_2 = 4I_0$

- **The initial charge determines the total energy in the circuit:** $U_0 = Q_0^2/2C$
- **The maximum current occurs when Q=0!**
- **At this time, all the energy is in the inductor: $U = 1/2\, LI_o^2$**
- **Therefore, doubling the initial charge quadruples the total energy.**
- **To quadruple the total energy, the max current must double!**



Confirmation 1:

The current in a LC circuit is a sinusoidal oscillation, with frequency ω.

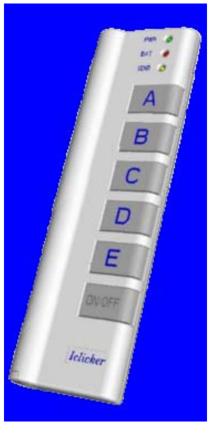


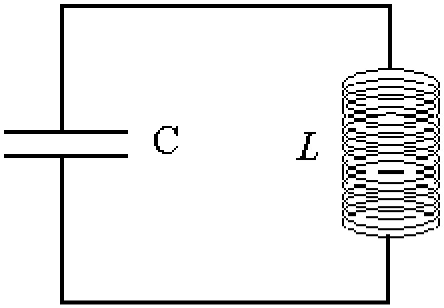
If the inductance of the circuit is increased, what will happen to the frequency ω?

a) increase

b) decrease

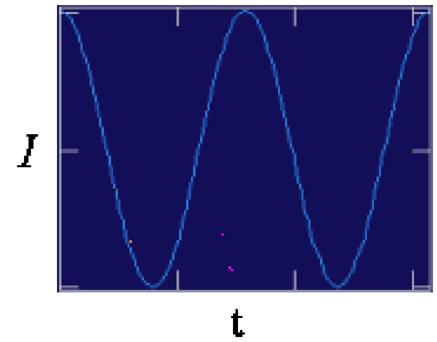
c) doesn't change





Confirmation 1:

The current in a LC circuit is a sinusoidal oscillation, with frequency ω.

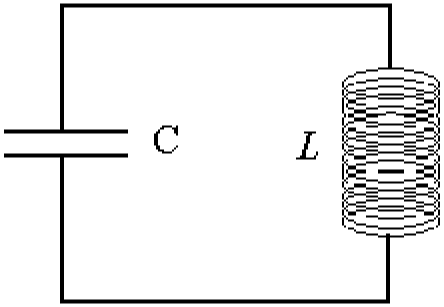


5) If the inductance of the circuit is increased, what will happen to the frequency ω?

a) increase

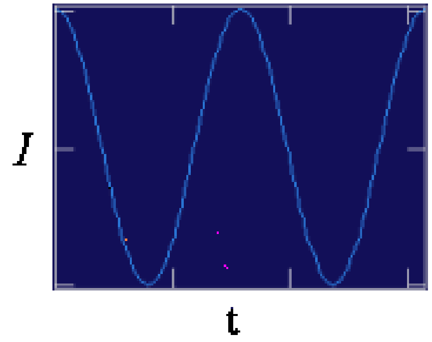
b) decrease

c) doesn’t change



Confirmation 2:

The current in a LC circuit is a sinusoidal oscillation, with frequency ω.

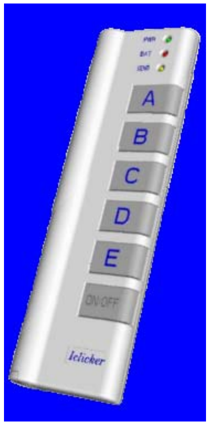


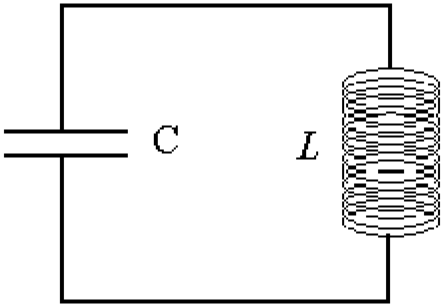
If the capacitance of the circuit is increased, what will happen to the frequency?

a) increase

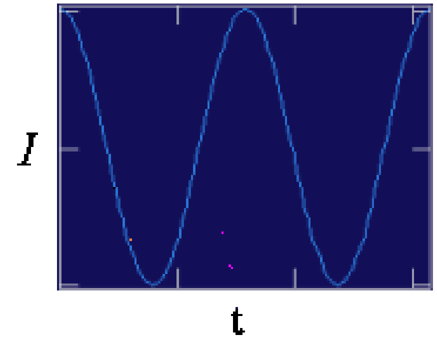
b) decrease

c) doesn't change





Confirmation 2:



The current in a LC circuit is a sinusoidal oscillation, with frequency ω.

If the capacitance of the circuit is increased, what will happen to the frequency?

a) increase

b) decrease

c) doesn’t change

LC Oscillations
Energy Check

- Oscillation frequency $\omega = \frac{1}{\sqrt{LC}}$ has been found from the loop equation.
- The other unknowns (Q_0, ϕ) are found from the initial conditions. E.g., in our original example we assumed initial values for the charge (***Q_i***) and current (**0**). For these values: $Q_0 = Q_i$, $\phi = 0$.
- **Question: Does this solution conserve energy?**

$$U_E(t) = \frac{1}{2}\frac{Q^2(t)}{C} = \frac{1}{2C}Q_0^2\cos^2(\omega t + \phi)$$

$$U_B(t) = \frac{1}{2}Li^2(t) = \frac{1}{2}L\omega^2 Q_0^2 \sin^2(\omega t + \phi)$$

Energy Check

Energy in Capacitor

$$\boxed{U_E(t) = \frac{1}{2C} Q_0^2 \cos^2(\omega t + \phi)}$$

Energy in Inductor

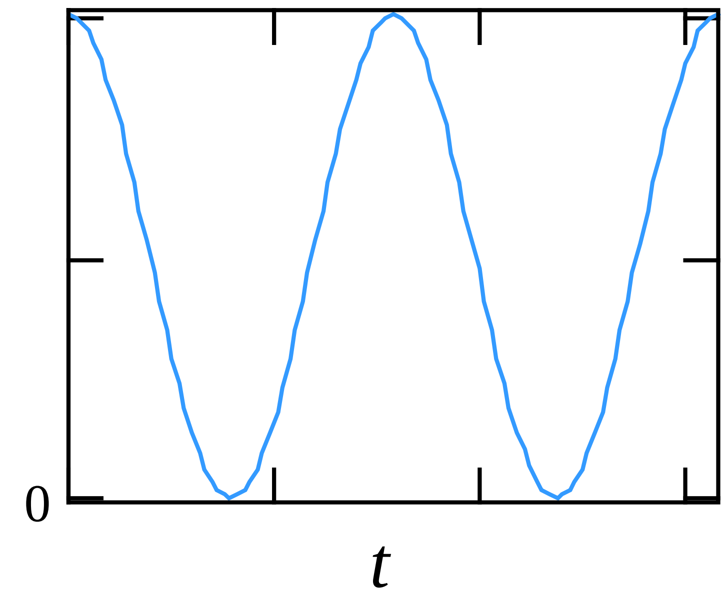
$$U_B(t) = \frac{1}{2} L\omega^2 Q_0^2 \sin^2(\omega t + \phi)$$

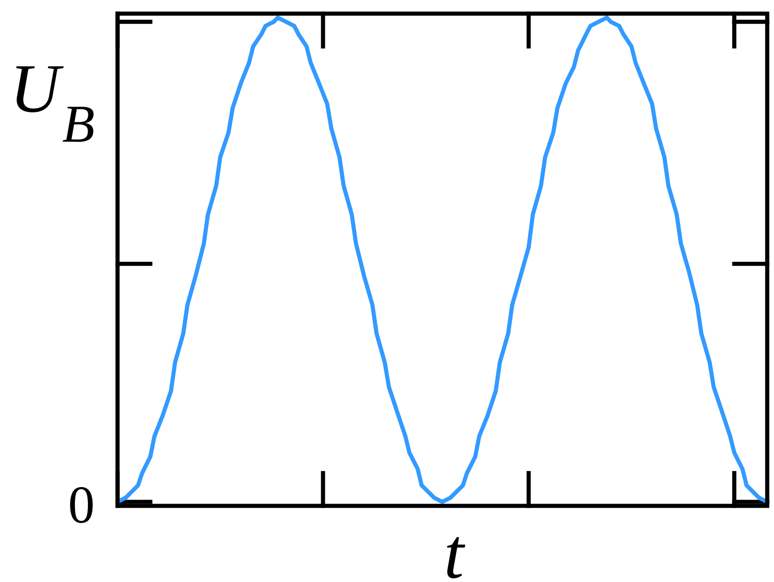
$$\omega = \frac{1}{\sqrt{LC}} \qquad \Downarrow$$

$$\boxed{U_B(t) = \frac{1}{2C} Q_0^2 \sin^2(\omega t + \phi)}$$

Therefore,

$$\boxed{U_E(t) + U_B(t) = \frac{Q_0^2}{2C}}$$





Inductor-Capacitor Circuits

Solving a LC circuit problem; Suppose ω=1/sqrt(LC)=3 and given the initial conditions,

$$Q(t=0)=5C$$

$$I(t=0)=15A$$

Solve find Q_0 and ϕ_0, to get complete solution using,

$$Q(t=0)=5=Q\cos(0+\phi_0)$$

$$I(t=0)=15=-Q\omega\sin(0+\phi_0)=-3Q\sin(0+\phi_0)$$

and we find,

$$(5)^2+\left(-\frac{15}{3}\right)^2=Q^2\left[\sin^2(\phi_0)+\cos^2(\phi_0)\right]=Q^2,\quad Q=5\sqrt{2}$$

$$\phi_0=inv.\tan\left(-\frac{15}{5\cdot 3}\right),\quad \phi_0=-45^\circ$$

Mathematical Insert

The following are all equally valid solutions

$$Q(t) = Q_0 \quad \cos(\omega t + \phi_0)$$

$$Q(t) = Q_0 \quad \sin(\omega t + \phi_1)$$

$$Q(t) = Q_0 \quad (\cos(\omega t)\cos(\phi_0) - \sin(\omega t)\sin(\phi_0))$$

$$Q(t) = A\cos(\omega t) + B\sin(\omega t)$$

The LC circuit eqn is the analog of the spring force eqn

$$\boxed{\frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0} \qquad \boxed{m\frac{d^2x}{dt^2} = -Kx}$$

$$\omega = \frac{1}{\sqrt{LC}} \qquad \omega = \sqrt{\frac{k}{m}}$$

Inductor-Capacitor-Resistor Circuit

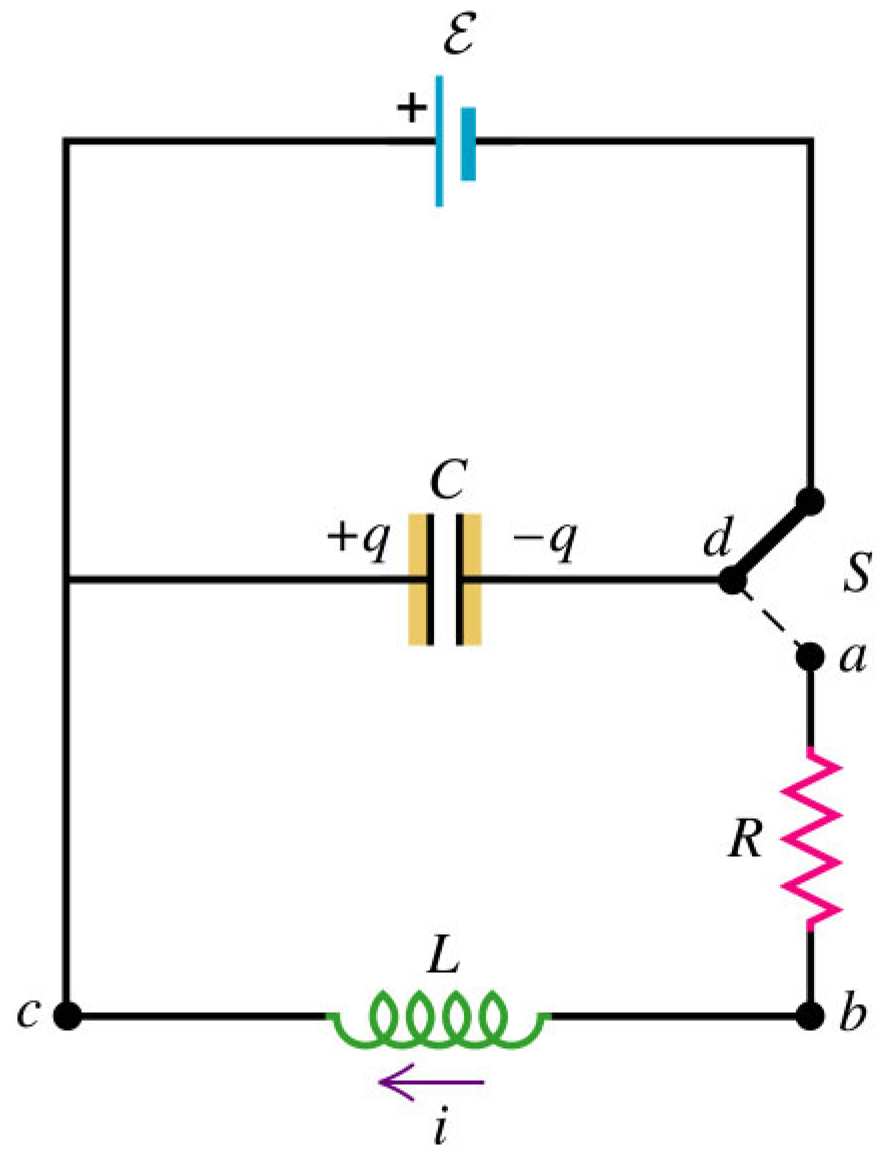
$$0 = \frac{Q}{C} + RI + L\frac{d^2Q}{dt^2}$$

$$0 = L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C}$$

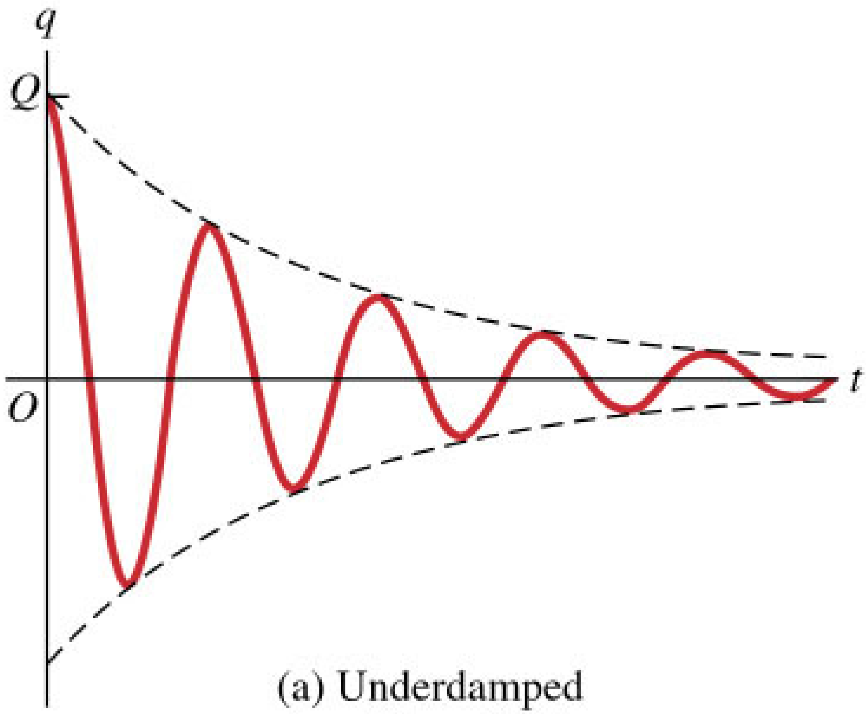
Solution will have form of

$$Q(t) = Ae^{-\alpha t}\cos(\omega' t + \phi)$$

If, $$\frac{1}{LC} > \frac{R^2}{4L^2}$$

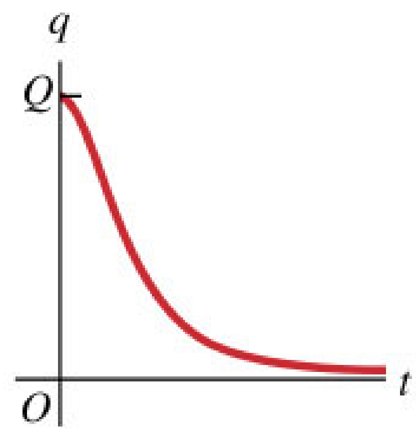


Inductor-Capacitor-Resistor Circuit
3 solutions, depending on L,R,C values



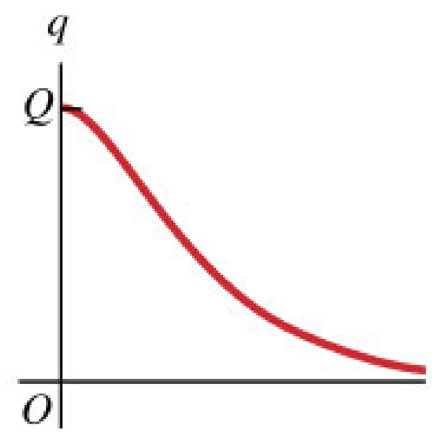
(a) Underdamped circuit (small R)

$$\frac{1}{LC} > \frac{R^2}{4L^2}$$



(b) Critically damped circuit (larger R)

$$\frac{1}{LC} = \frac{R^2}{4L^2}$$



(c) Overdamped circuit (very large R)

$$\frac{1}{LC} < \frac{R^2}{4L^2}$$

Inductor-Capacitor-Resistor Circuit Solving for all the terms

$$Q(t) = Ae^{-\alpha t}\cos(\omega' t + \phi)$$

$$= Ae^{-\left(\frac{R}{2L}\right)t}\cos\left(\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\,t + \phi\right)$$

$$\alpha = \frac{R}{2L} \text{ and } \omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

Solution for underdamped circuit; $\dfrac{1}{LC} > \dfrac{R^2}{4L^2}$

For other solutions, use starting form, solve for λ and λ′,

$$Q(t) = Ae^{-\lambda t} + Be^{-\lambda' t}$$

For next time

- Homework #10 [due Wednesday]

- Quiz on Friday: Faraday's Law, Inductance and Inductors