Course Updates

http://www.phys.hawaii.edu/~varner/PHYS272-Spr10/physics272.html

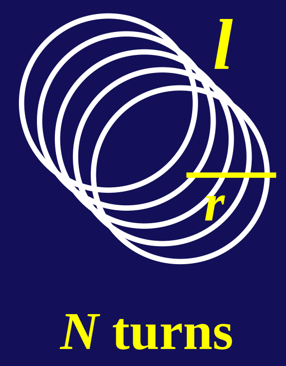
Reminders:

1) Assignment #10 → due next Wednesday
2) Midterm #2 take-home due today
3) Quiz # 5 next week
4) Inductance, Inductors, RLC

Self inductance of long solenoid

- Long Solenoid:

  N turns total, radius *r*, Length *l*



$$r << l \Rightarrow B = \mu_0 \frac{N}{l} I$$

For a single turn, $A = \pi r^2 \Rightarrow \phi = BA = \mu_0 \frac{N}{l} I \pi r^2$

The flux through a turn is given by:

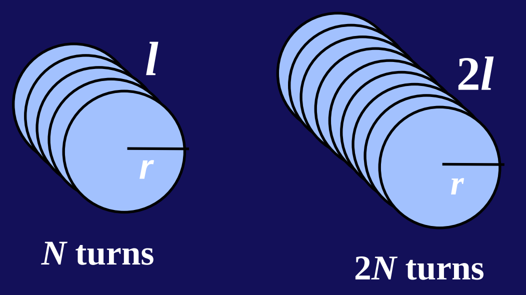
$$\Phi_B = \mu_0 \frac{N}{l} I \pi r^2$$

Inductance of solenoid can then be calculated as:

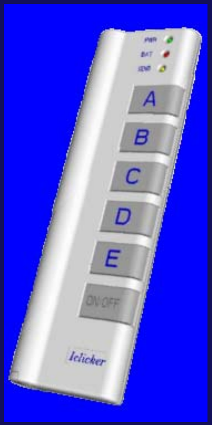
$$\boxed{L \equiv \frac{N\Phi_B}{I} = \mu_0 \frac{N^2}{l} \pi r^2 = \mu_0 \left(\frac{N}{l}\right)^2 l \pi r^2}$$

Question 1a

- Consider the two inductors shown:
  - Inductor 1 has length l, N total turns and has inductance L_1.
  - Inductor 2 has length $2l$, $2N$ total turns and has inductance L_2.
  - What is the relation between L_1 and L_2?



(a) $B_2 < B_1$ **(b)** $B_2 = B_1$ **(c)** $B_2 > B_1$



Question 1a

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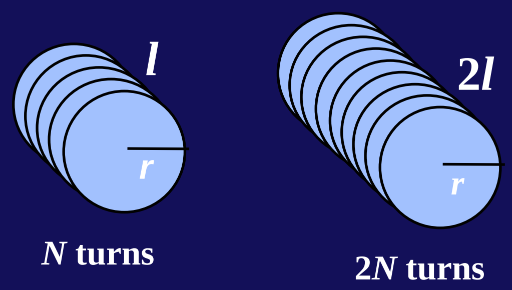
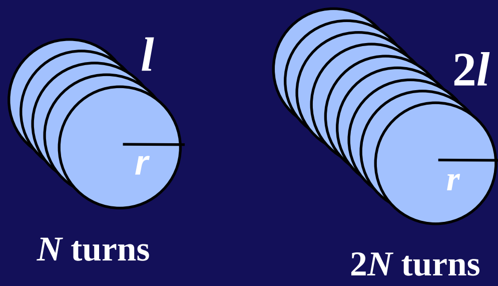


(a) $B_2 < B_1$ **(b)** $\boxed{B_2 = B_1}$ **(c)** $B_2 > B_1$

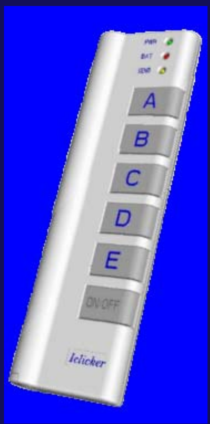
- To calculate the flux, we first need to calculate the magnetic field B produced by the current: $B = \mu_0(N/l)I$
  - i.e., the B field is proportional to the number of turns per unit length.
  - Therefore, $B_1 = B_2$. But does that mean $L_1 = L_2$?

Question 1b

- Consider the two inductors shown:
  - Inductor 1 has length l, N total turns and has inductance L_1.
  - Inductor 2 has length $2l$, $2N$ total turns and has inductance L_2.
  - What is the relation between L_1 and L_2?



(a) $L_2 < L_1$ **(b)** $L_2 = L_1$ **(c)** $L_2 > L_1$



Question 1b

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  - Inductor 1 has length l, N total turns and has inductance L_1.
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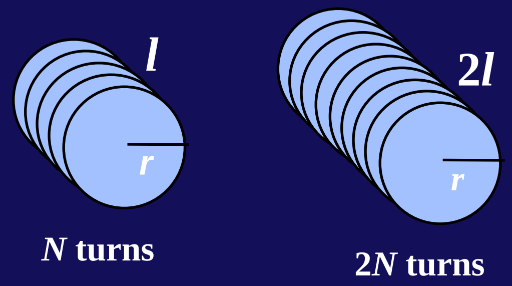
  - What is the relation between L_1 and L_2?



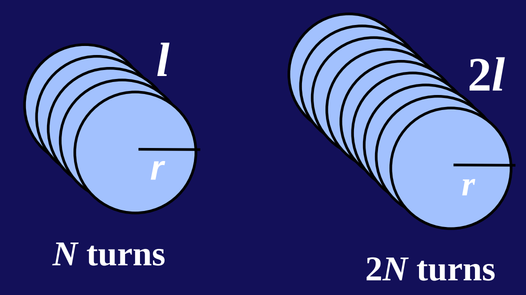
(a) $L_2 < L_1$ **(b)** $L_2 = L_1$ $\boxed{\textbf{(c)}\ L_2 > L_1}$

- To determine the self-inductance L, we need to determine the flux Φ_B which passes through the coils when a current I flows: $L \equiv N\Phi_B / I$.

So larger by the additional factor of N

Inductor addition rules

- To calculate L, we need to calculate the flux.
  - Since $B_1=B_2$, the flux through any given turn is the same in each inductor
  - There are twice as many turns in inductor 2; therefore the net flux through inductor 2 is twice the flux through inductor 1! Therefore, $L_2 = 2L_1$.



Inductors in series *add* (like resistors):

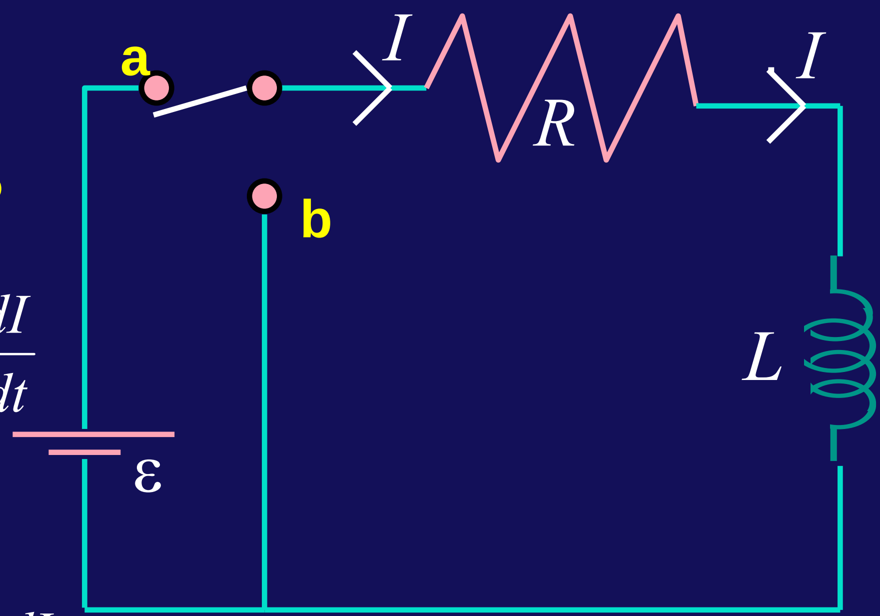
$$\boxed{L_{\text{eff}} = L_1 + L_2}$$

And inductors in parallel add like resistors in parallel:

$$\boxed{\frac{1}{L_{\text{eff}}} = \frac{1}{L_1} + \frac{1}{L_2}}$$

Energy of an Inductor

- How much energy is stored in an inductor when a current is flowing through it?



- Start with loop rule: $\varepsilon = IR + L\dfrac{dI}{dt}$
- Multiply this equation by I:

$$\varepsilon I = I^2 R + LI\frac{dI}{dt}$$

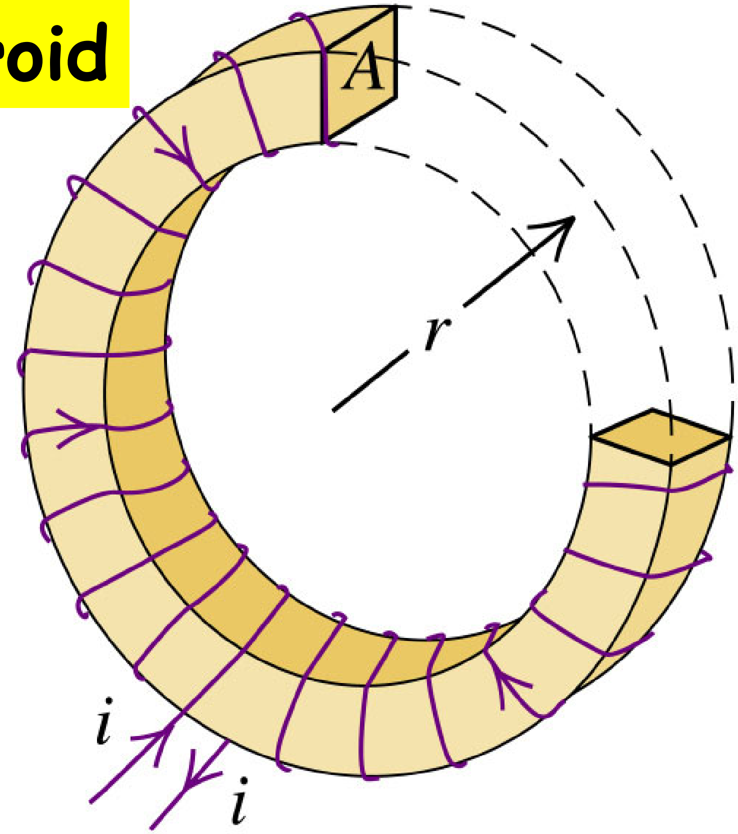
- From this equation, we can identify P_L, the rate at which energy is being stored in the inductor:

$$P_L = \frac{dU}{dt} = LI\frac{dI}{dt}$$

- We can integrate this equation to find an expression for U, the energy stored in the inductor when the current = I:

$$U = \int_0^U dU = \int_0^I LI\,dI \quad \Rightarrow \quad \boxed{U = \frac{1}{2}LI^2}$$

Magnetic field Energy in a toroid



Consider a toroid magnet, the B field is , B = $\mu_0 NI/2\pi r$ (ex.28.11). The energy is,

$$U = \frac{1}{2}LI^2 = \frac{1}{2}\left(\frac{\mu_0 N^2 A}{2\pi r}\right)I^2$$

Substituting the B field into the Eqn., we have,

$$\frac{U}{A2\pi r} = \frac{1}{2}\left(\frac{\mu_0 N^2 I^2}{(2\pi r)^2}\right) = \frac{1}{2\mu_0}\left(\frac{\mu_0{}^2 N^2 I^2}{(2\pi r)^2}\right) = \frac{B^2}{2\mu_0}$$

$$\frac{U}{A2\pi r} = \frac{U}{volume} = \frac{B^2}{2\mu_0} = Energy\ density$$

Energy in Electric Fields and Magnetic Fields

In chapter 24.3, we discussed energy in a parallel plate with area A and separation d, The electric field energy in the capacitor was

$$U = \frac{1}{2}CV^2 = \frac{1}{2}C(Ed)^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2 = \frac{1}{2}\varepsilon_0 (Ad) E^2$$

$$\frac{U}{Ad} = \frac{energy}{volume} = \frac{\varepsilon_0}{2}E^2$$

Now we find the magnetic field energy in the toroid magnet is

$$\frac{U}{A2\pi r} = \frac{energy}{volume} = \frac{1}{2\mu_0}B^2$$

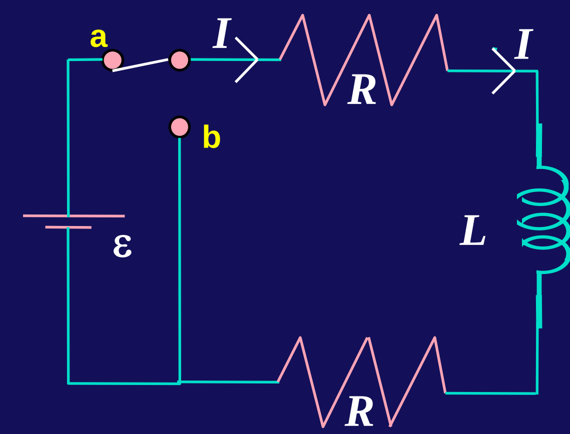
The $\left|\vec{B}\right|^2, \left|\vec{E}\right|^2$ fields are proportional to the energy density

Inductors in Circuits

General rule: inductors resist change in current

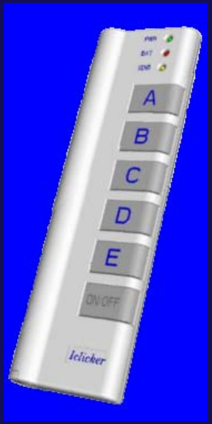
- **Hooked to current source**
  - **Initially, the inductor behaves like an open switch.**
  - **After a long time, the inductor behaves like an ideal wire.**
- **Disconnected from current source**
  - **Initially, the inductor behaves like a current source.**
  - **After a long time, the inductor behaves like an open switch.**

Question 2

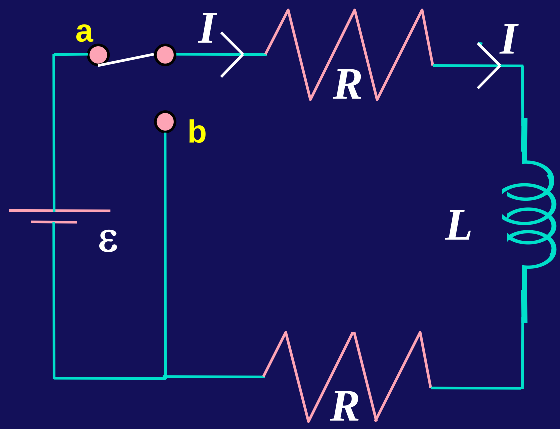


- At t=0 the switch is thrown from position b to position a in the circuit shown:
  - What is the value of the current I_∞ a long time after the switch is thrown?

(a) $I_\infty = 0$ **(b)** $I_\infty = \varepsilon/2R$ **(c)** $I_\infty = 2\varepsilon/R$



Question 2



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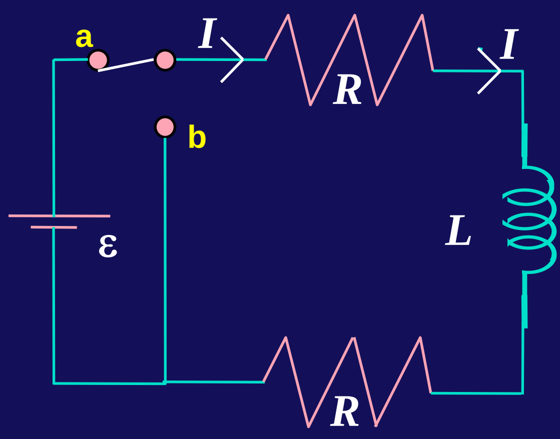
- A long time after the switch is thrown, the current approaches an asymptotic value: as $t \to \infty$, $dI/dt \to 0$.

- As $dI/dt \to 0$, the voltage across the inductor $\to 0$. Therefore, $I_\infty = \varepsilon/2R$.

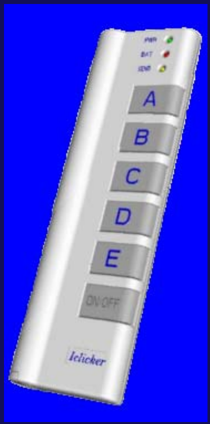
Question 3

- At t=0 the switch is thrown from position b to position a in the circuit shown:

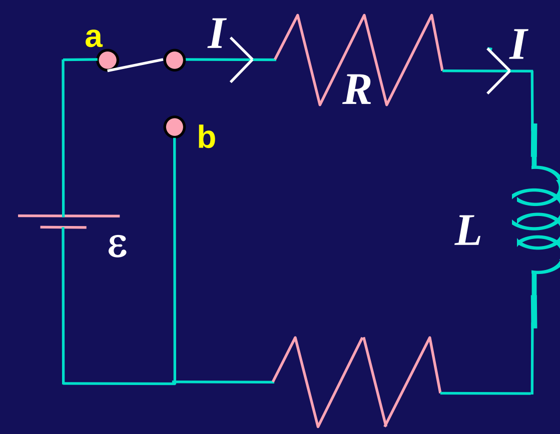


  – What is the value of the current I_0 immediately after the switch is thrown?

(a) $I_0 = 0$ **(b)** $I_0 = \varepsilon/2R$ **(c)** $I_0 = 2\varepsilon/R$



Question 3



- At t=0 the switch is thrown from position b to position a in the circuit shown:
  - What is the value of the current I_0 immediately after the switch is thrown?

$\boxed{\textbf{(a)}\ I_0 = 0}$ **(b)** $I_0 = \varepsilon/2R$ **(c)** $I_0 = 2\varepsilon/R$

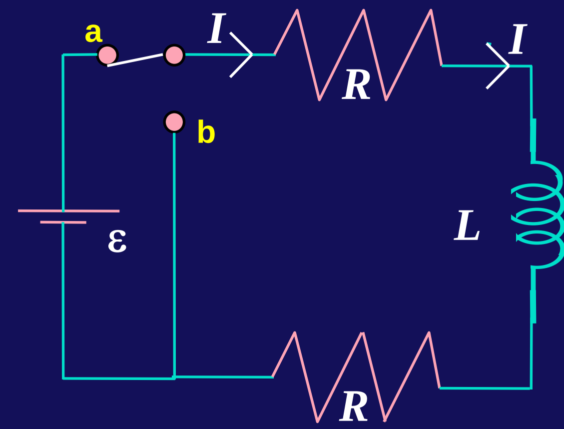
• Just after the switch is thrown, the rate of change of current is as large as it can be (we had been assuming it was ∞!)

• The inductor limits dI/dt to be initially equal to ε/L. The voltage across the inductor = ε; the current, then, must be 0!

• Another way: the moment the switch is thrown, the current tries to generate a huge B-field. There is a huge change in flux through coil—an emf is generated to oppose this. Initially, then, *no current* flows through→ no voltage drop across the resistors.

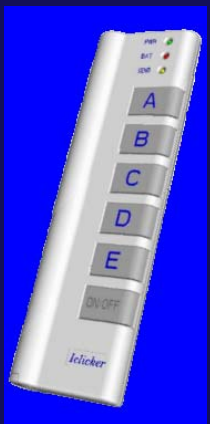
Question 4

- At t=0 the switch is thrown from position b to position a in the circuit shown:

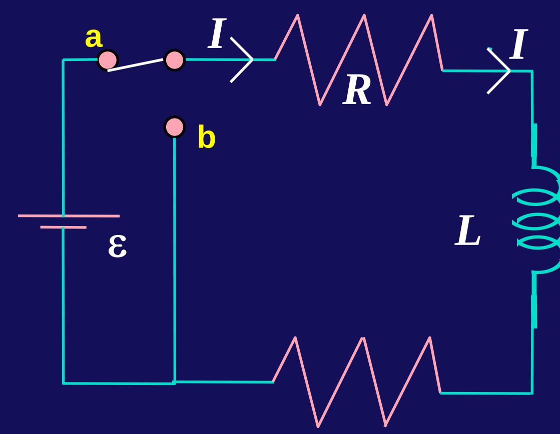


- After a long time the switch is opened.
  - What is the value of the current I_0 just after the switch is opened?

(a) $I_0 = 0$ **(b)** $I_0 = \varepsilon/2R$ **(c)** $I_0 = 2\varepsilon/R$



Question 4



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  - –What is the value of the current I_0 just after the switch is opened?

(a) $I_0 = 0$ **(b)** $\boxed{I_0 = \varepsilon/2R}$ **(c)** $I_0 = 2\varepsilon/R$

• Just after the switch is thrown, the inductor induces an emf to keep current flowing: emf = L dI/dt (can be *much* larger than ε)

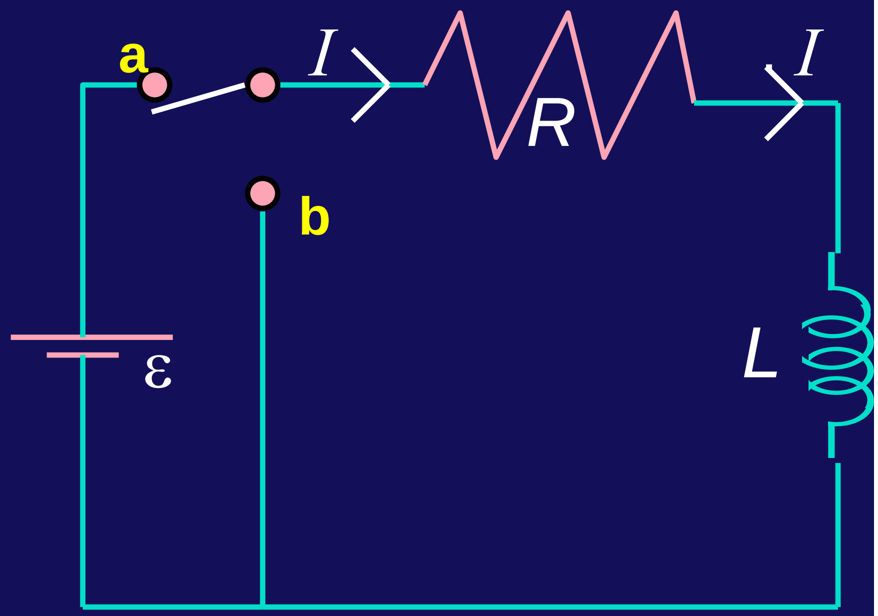
• However, now there's no place for the current to go → charges build up on switch contacts → high voltage across switch gap

•If the electric field exceeds the "dielectric strength" (~30 kV/cm in air) → breakdown → SPARK!

This phenomenon is used in "flyback generators" to create high voltages; it also destroys lots of electronic equipment!

RL Circuits, Quantitative

- At $t=0$, the switch is closed and the current I starts to flow.



- Loop rule:

$$\boxed{\varepsilon - IR - L\frac{dI}{dt} = 0}$$

Note that this equation is identical in form to that for the RC circuit with the following substitutions:

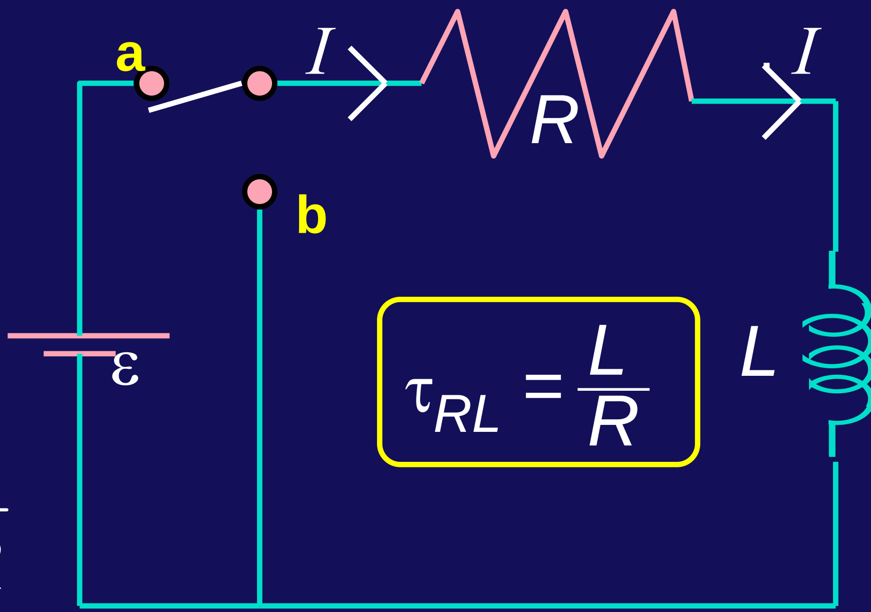
RC: $\boxed{\varepsilon - \frac{Q}{C} - R\frac{dQ}{dt} = 0}$ $\Rightarrow$ **RC→RL:** $R \to L$ $\quad Q \to I$ $\quad \frac{1}{C} \to R$

Therefore, $\tau_{RC} = RC$ $\Rightarrow$ $\boxed{\tau_{RL} = \frac{L}{R}}$

RL Circuits

- To find the current *I* as a function of time *t*, we need to choose an exponential solution which satisfies the boundary condition:

$$\frac{dI}{dt}(t=\infty)=0 \Rightarrow \quad I(t=\infty)=\frac{\varepsilon}{R}$$



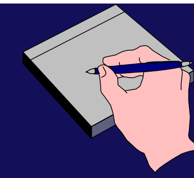
$$\tau_{RL} = \frac{L}{R}$$

  - We therefore write: $\boxed{I = \frac{\varepsilon}{R}\left(1 - e^{-Rt/L}\right)}$

- **The voltage drop across the inductor is given by:**

$$\boxed{V_L = L\frac{dI}{dt} = \varepsilon e^{-Rt/L}}$$

RL Circuit (ε on)



Sketch curves !

Current

$$I = \frac{\varepsilon}{R}\left(1 - e^{-Rt/L}\right)$$

Max = ε/*R*

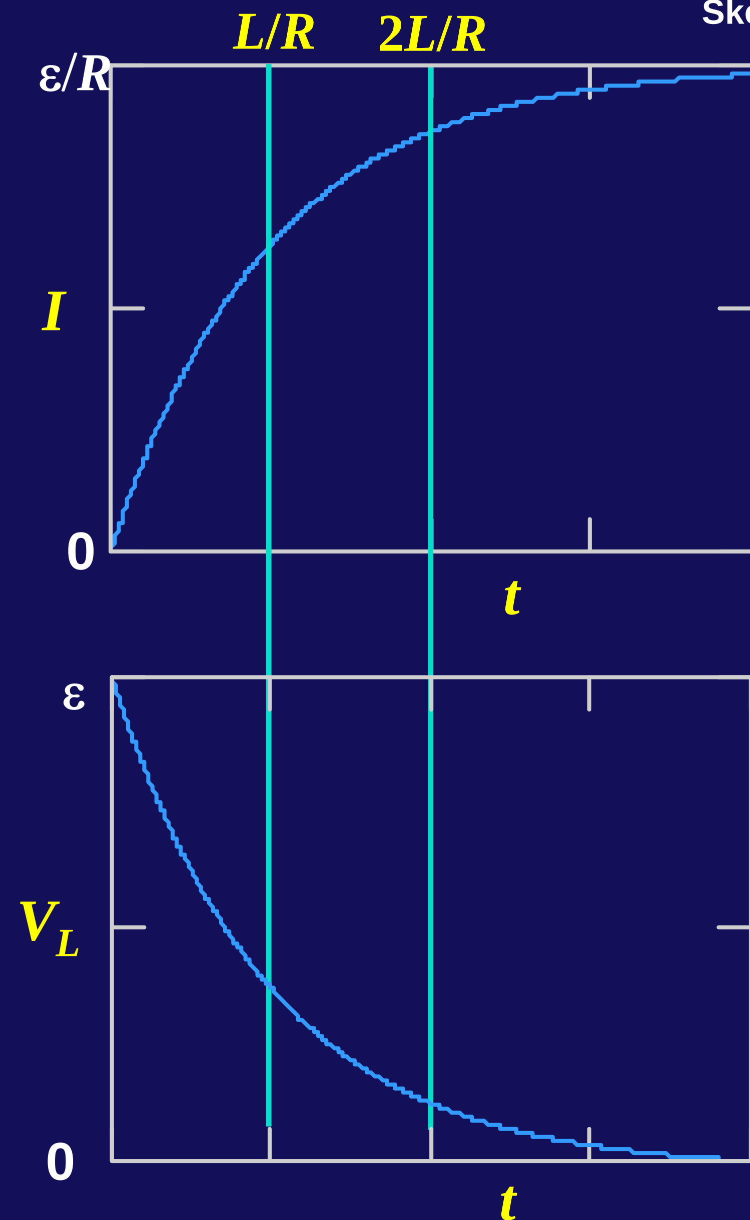
63% Max at *t*=*L*/*R*

Voltage on *L*

$$V_L = L\frac{dI}{dt} = \varepsilon e^{-Rt/L}$$

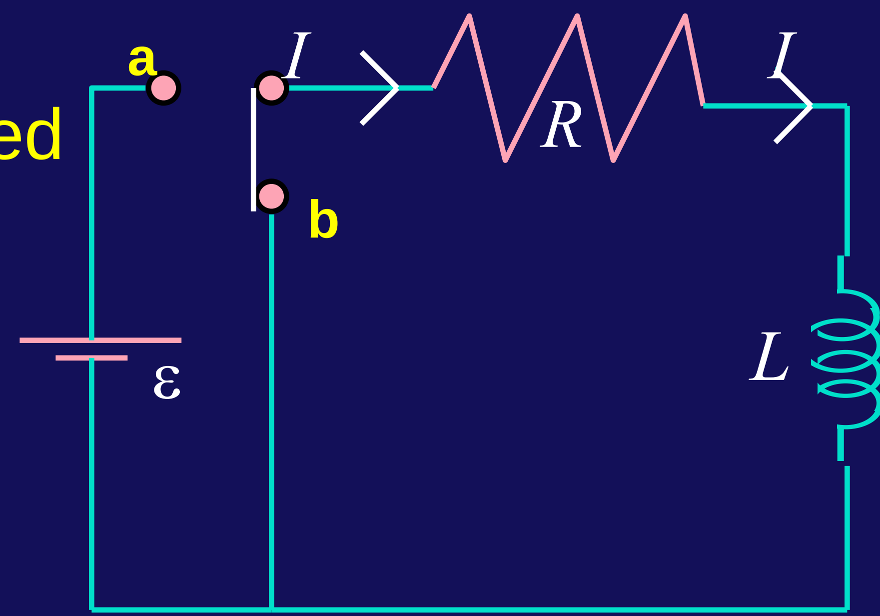
Max = ε/*R*

37% Max at *t*=*L*/*R*



RL Circuits

- After the switch has been in position a for a long time, redefined to be t=0, it is moved to position b.



- Loop rule:

$$\boxed{IR + L\frac{dI}{dt} = 0}$$

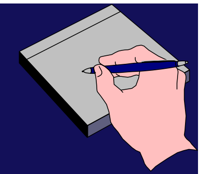
- The appropriate initial condition is: $I(t=0) = \dfrac{\varepsilon}{R}$

- **The solution then must have the form:**

$$\boxed{I = \frac{\varepsilon}{R} e^{-Rt/L}}$$

$$\boxed{V_L = L\frac{dI}{dt} = -\varepsilon e^{-Rt/L}}$$

RL Circuit (ε off)



Sketch curves !

Current

$$I = \frac{\varepsilon}{R} e^{-Rt/L}$$

Max = ε/*R*

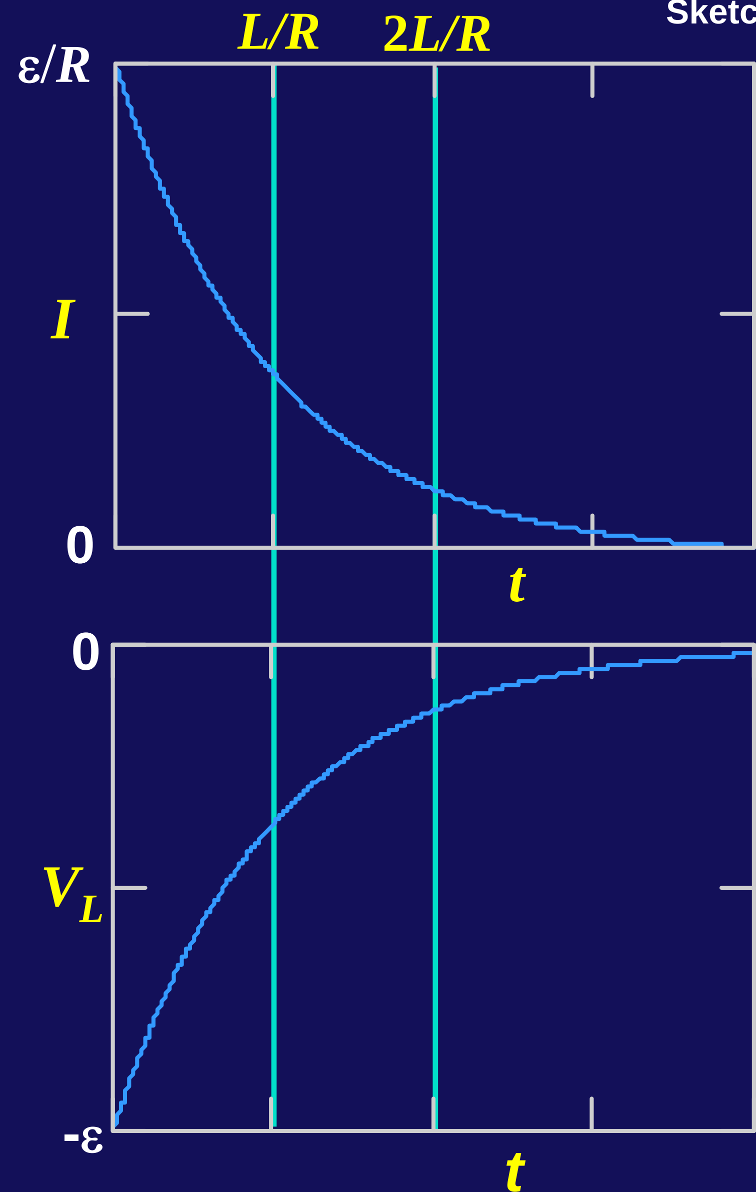
37% Max at *t*=*L*/*R*

Voltage on *L*

$$V_L = L\frac{dI}{dt} = -\varepsilon e^{-Rt/L}$$

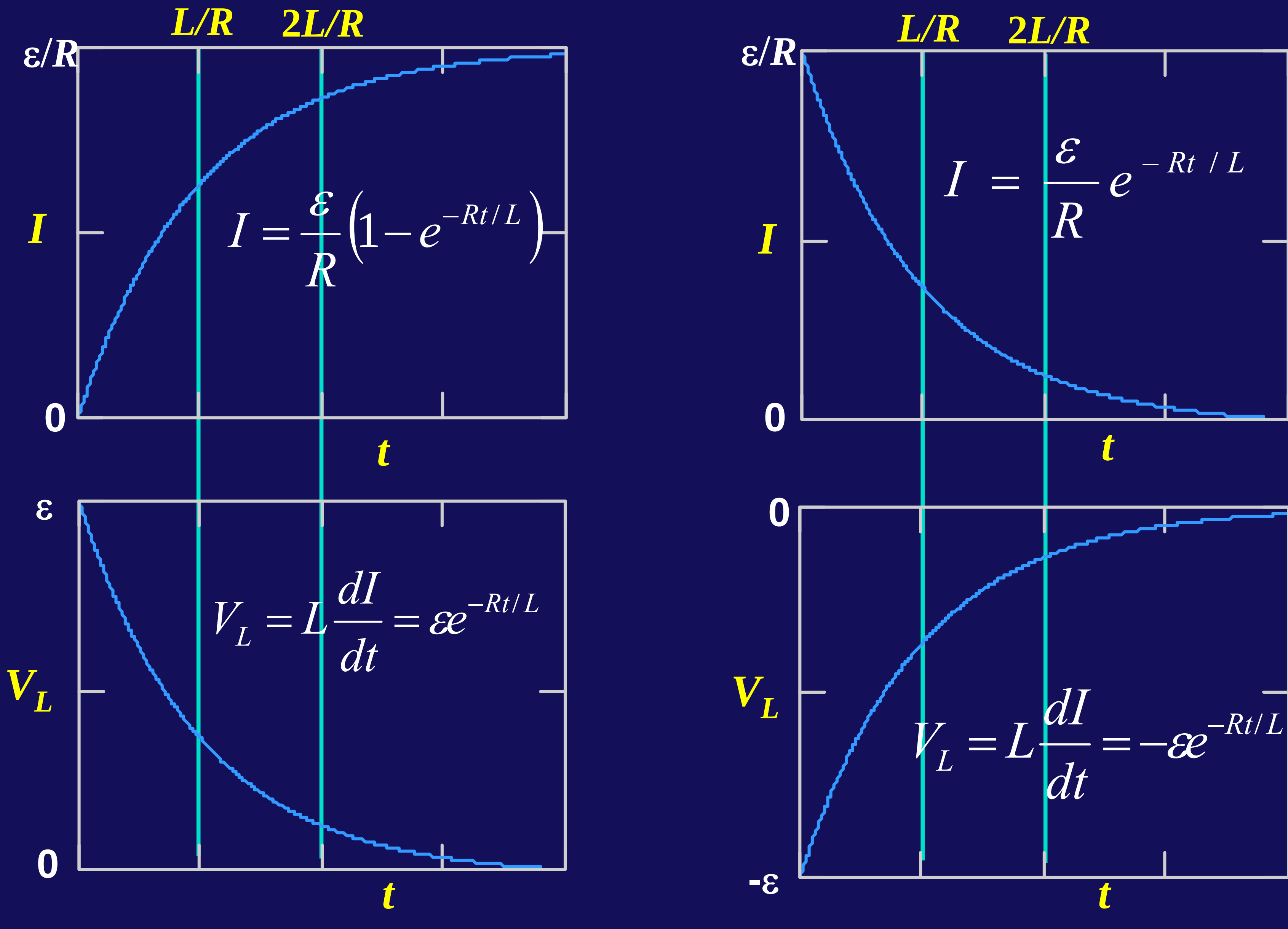
Max = -ε

37% Max at *t*=*L*/*R*



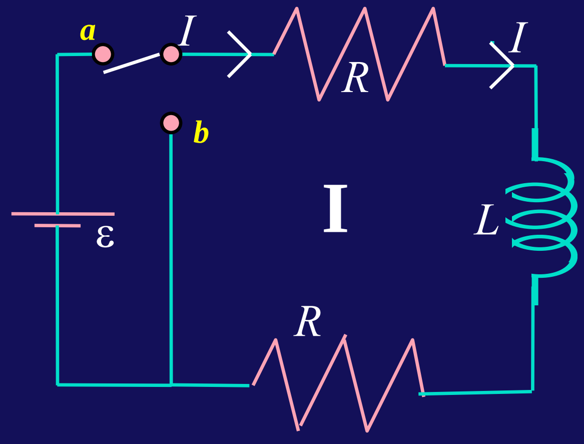
ε on

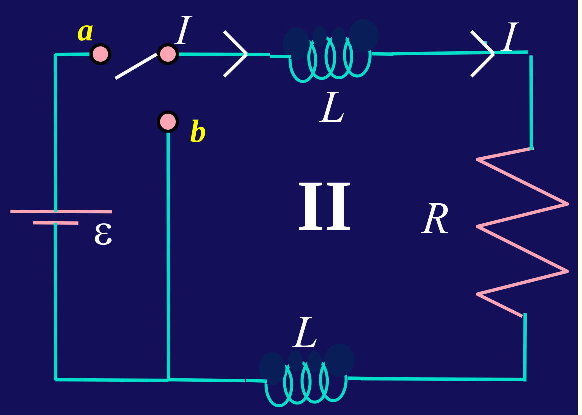
ε off


L/R
2L/R
ε/R
I
0
t
$I = \frac{\varepsilon}{R}\left(1 - e^{-Rt/L}\right)$
ε
V_L
0
t
$V_L = L\frac{dI}{dt} = \varepsilon e^{-Rt/L}$
L/R
2L/R
ε/R
I
0
t
$I = \frac{\varepsilon}{R} e^{-Rt/L}$
0
V_L
-ε
t
$V_L = L\frac{dI}{dt} = -\varepsilon e^{-Rt/L}$

Question 5

- At t=0, the switch is thrown from position b to position a as shown:
  - Let t_I be the time for circuit I to reach 1/2 of its asymptotic current.
  - Let t_{II} be the time for circuit II to reach 1/2 of its asymptotic current.
  - What is the relation between t_I and t_{II}?





(a) $t_{II} < t_I$ **(b)** $t_{II} = t_I$ **(c)** $t_{II} > t_I$

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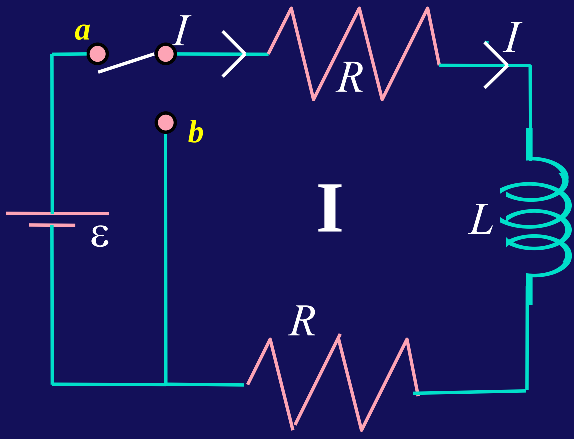
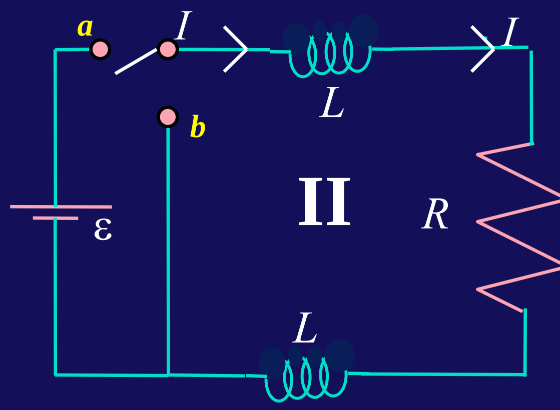




(a) $t_{II} < t_I$ **(b)** $t_{II} = t_I$ $\boxed{\textbf{(c)}\ t_{II} > t_I}$

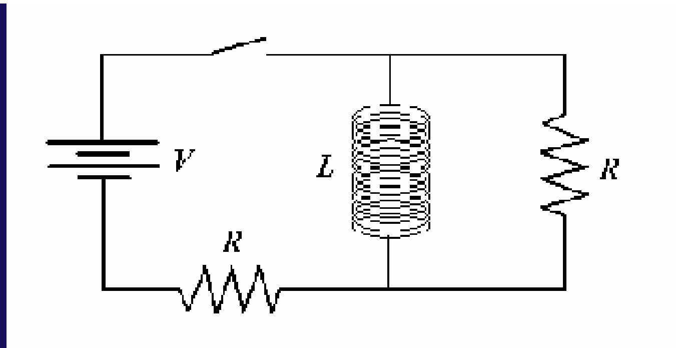
- We must determine the time constants of the two circuits by writing down the loop equations.

I: $\varepsilon - IR - L\dfrac{dI}{dt} - IR = 0 \Rightarrow \tau_I = \dfrac{L}{2R}$

II: $\varepsilon - L\dfrac{dI}{dt} - IR - L\dfrac{dI}{dt} = 0 \Rightarrow \tau_{II} = \dfrac{2L}{R}$

This confirms that inductors in series *add*!

Example 1:



At t = 10 hrs the switch is opened, abruptly disconnecting the battery from the circuit. What will happen to all the energy stored in the solenoid?

Energy stored in the inductor: $U = 1/2\, L\, I^2$

When the switch is opened, this energy is dissipated in the resistor.

An inductor doesn't like change!!!

When the switch is opened, the inductor will try to maintain the current that was flowing through it before the switch is opened. Since the battery is disconnected from the circuit, the energy which is necessary to keep current flowing through the resistor is provided by the inductor.

For next time

- Midterm 2 – take home for problems didn't get on exam [to get Scaled grade] due NOW (some did not pick up)

- Homework #10 assigned [due next Wednesday]

- Quiz next week