Course Updates

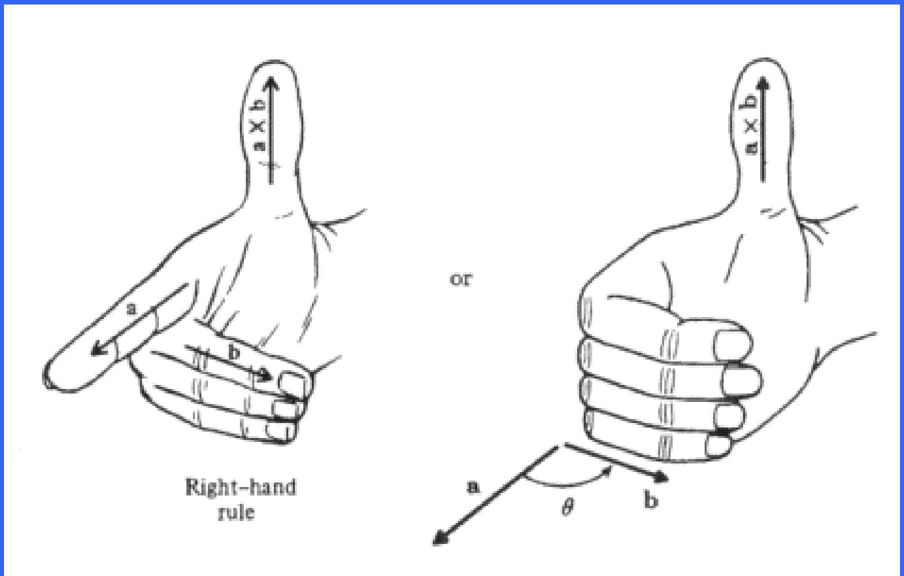
http://www.phys.hawaii.edu/~varner/PHYS272-Spr10/physics272.html

Reminders:

1) Assignment #8 → will be able to do after today
2) Finish Chapter 28 today
3) Quiz next Friday
4) Review of 3 right-hand rules

B Force (single charge):

$$\vec{F} = q\vec{v} \times \vec{B}$$

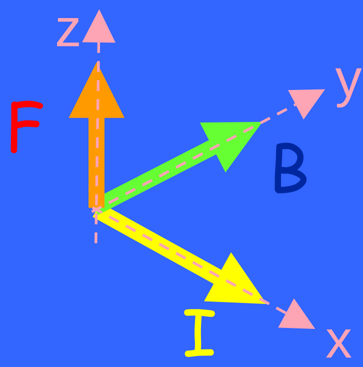


Lots of q:

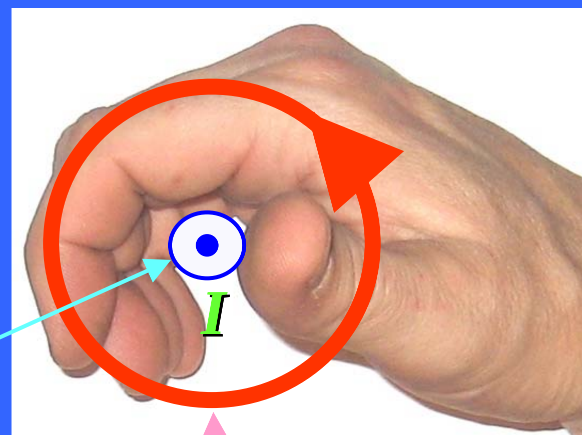
$$\vec{F} = q\sum_i \vec{v}_i \times \vec{B}$$

$$\vec{F} = qN\vec{v}_{avg} \times \vec{B}$$

$$\vec{F} = I\vec{L} \times \vec{B}$$



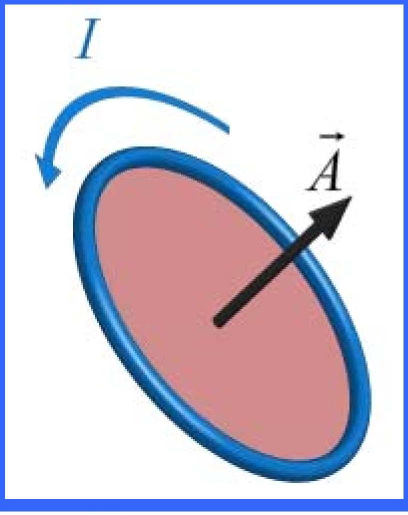
Straight wire

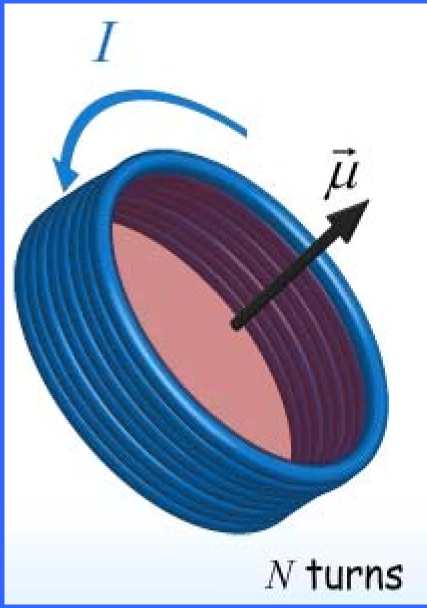


Current in Wire

Resulting B field

Magnetic Dipole Moment





Area vector

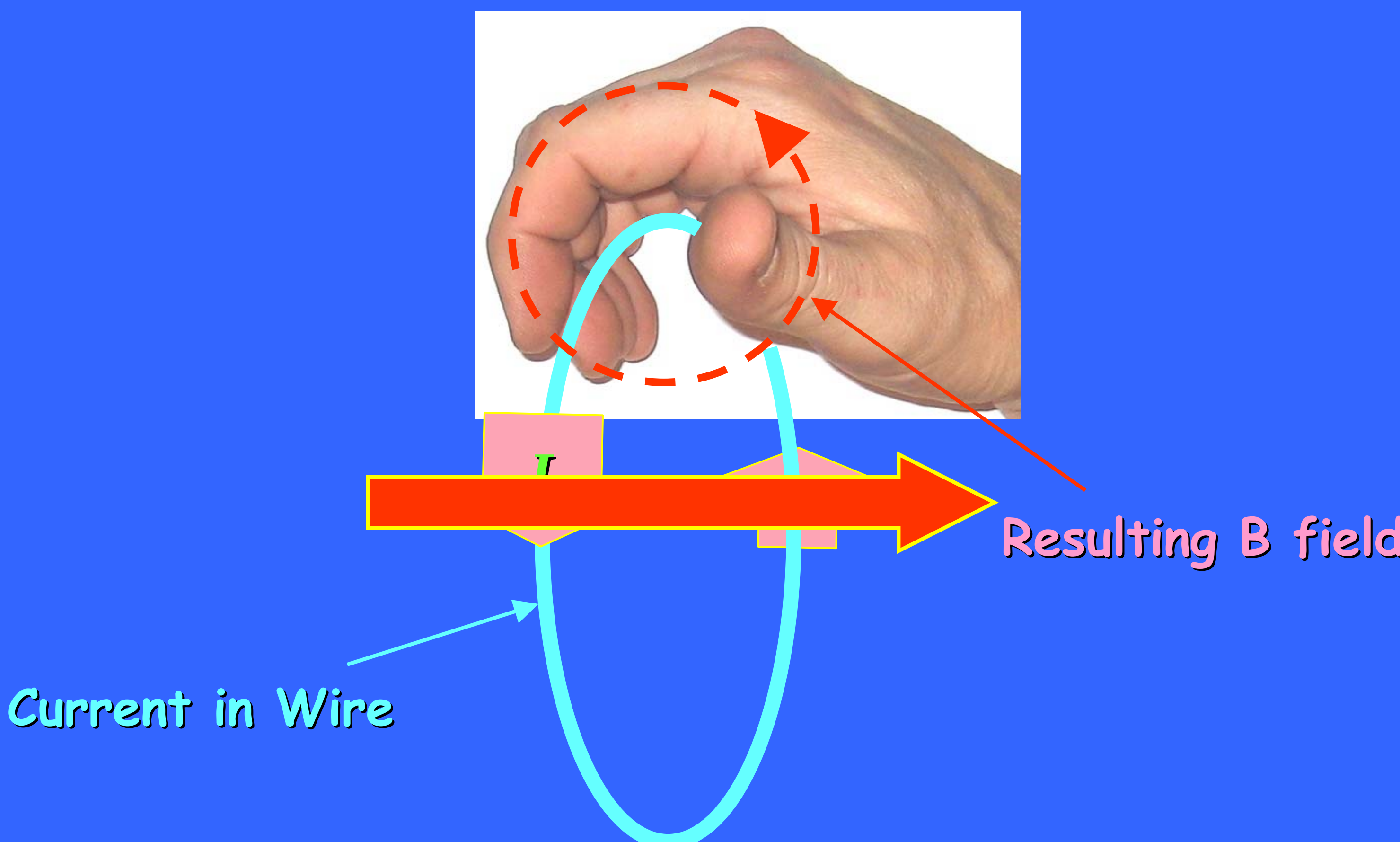
Magnitude = Area

Direction uses R.H.R.

Magnetic Dipole moment

$$\vec{\mu} \equiv NI\vec{A}$$

Consistent? Yes!



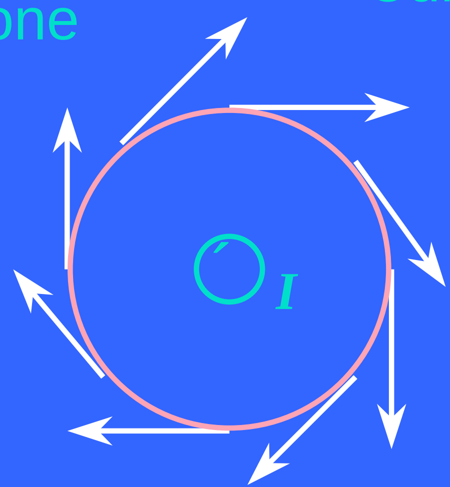
Today is Ampere’s Law Day

"High symmetry"

$$\boxed{\oint \vec{B} \bullet d\vec{l} = \mu_0 I}$$

Integral around a path …
hopefully a simple one

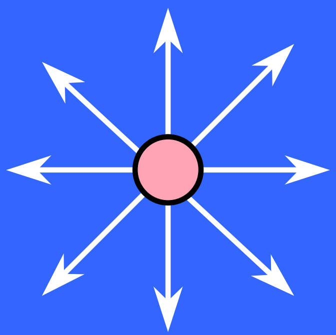
Current “enclosed” by that path



Calculation of Electric Field

- **Two Ways to calculate**
  - **Coulomb's Law**

$$d\vec{E} = k\frac{dq}{r^2}\hat{r}$$



"Brute force"

  - **Gauss' Law**

$$\varepsilon_0 \oint \vec{E} \bullet d\vec{S} = q$$

"High symmetry"

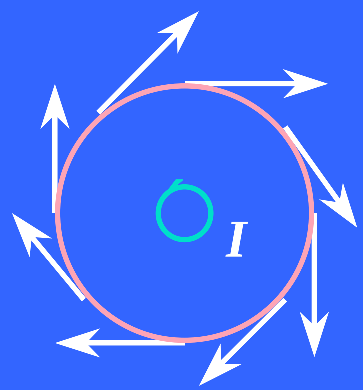
What are the analogous equations for the Magnetic Field?

Calculation of Magnetic Field

- **Two Ways to calculate**
  - **Biot-Savart Law ("Brute force")**
  - **Ampere's Law ("High symmetry")**

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2}$$

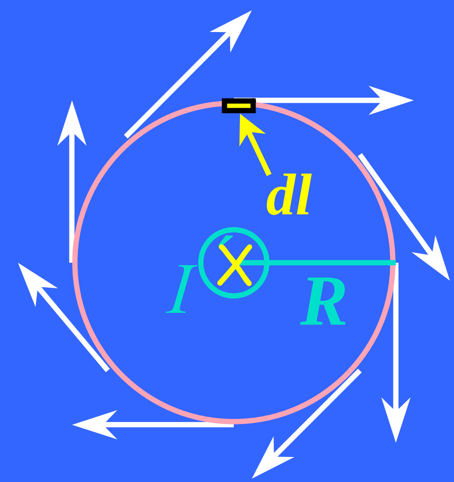
$$\oint \vec{B} \bullet d\vec{l} = \mu_0 I$$



–AMPERIAN SURFACE/LOOP

These are the analogous equations

B-field of ∞ Straight Wire, revisited



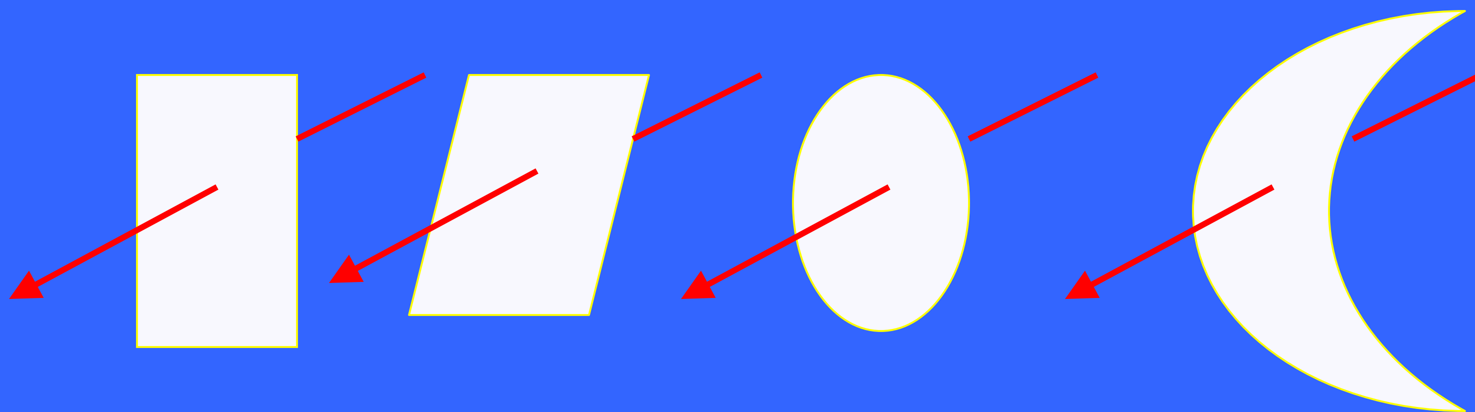
- Calculate field at distance R from wire using Ampere's Law:
- **Choose loop to be circle of radius R centered on the wire in a plane ⊥ to wire.**
  - **Why?**
    - **Magnitude of B is constant (function of R only)**
    - **Direction of B is parallel to the path.**
  - **Evaluate line integral in Ampere's Law:** $\oint \vec{B} \bullet d\vec{l} = B(2\pi R)$
  - **Current enclosed by path = I**
  - **Apply Ampere's Law:**

$$2\pi RB = \mu_0 I \quad \Rightarrow \quad \boxed{B = \frac{\mu_0 I}{2\pi R}}$$

Ampere's Law simplifies the calculation thanks to symmetry of the current! (axial/cylindrical)

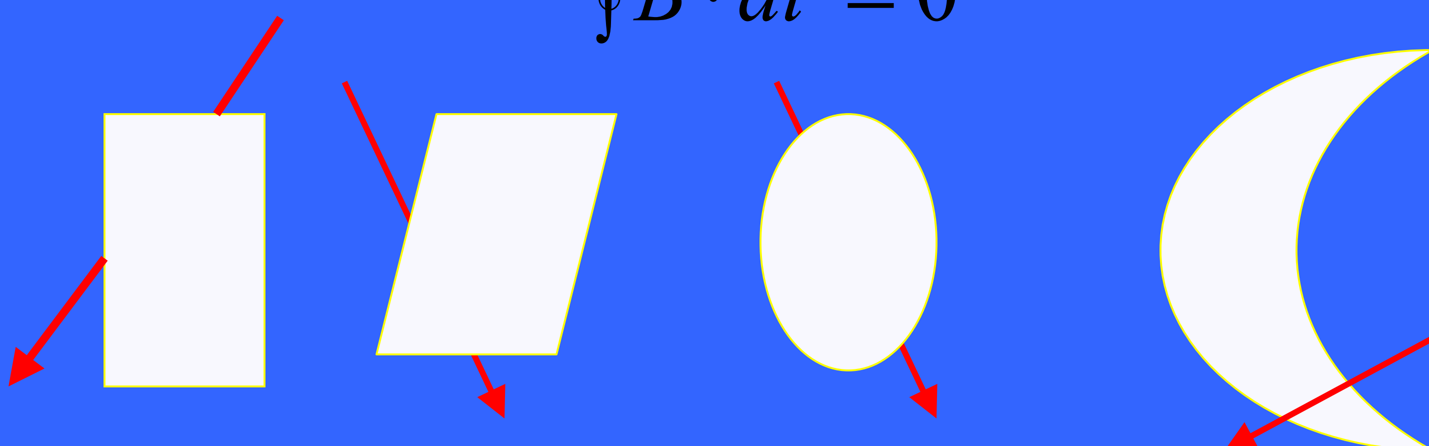
Arbitrary closed path about a straight current wire will satisfy

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$$



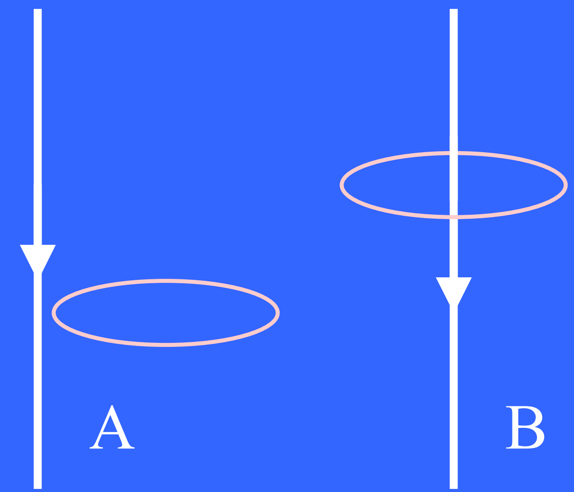
Arbitrary closed path outside a current wire will satisfy

$$\oint \vec{B} \cdot d\vec{l} = 0$$



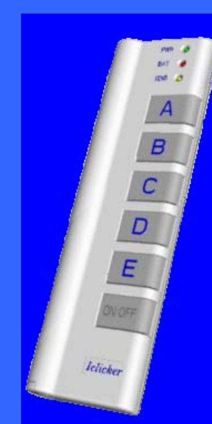
Question 1:

Two identical loops are placed in proximity to two identical current carrying wires.



➔For which loop is $\int \vec{B} \bullet \vec{dl}$ the greatest?

A)	B)	C)
A	B	Same



Question 1:

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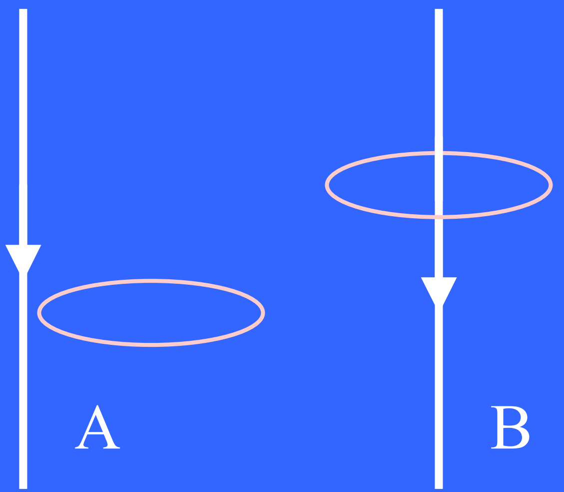
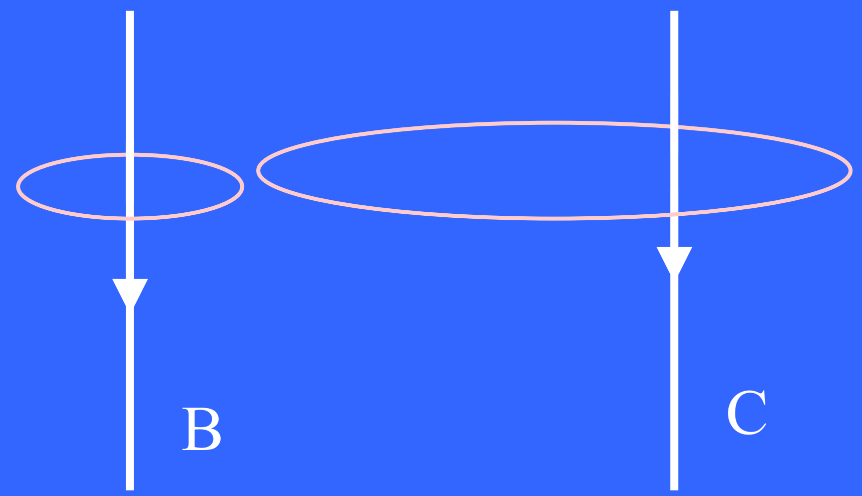


➔For which loop is $\int \vec{B} \bullet \vec{dl}$ the greatest?

A)	B)	C)
A	B	Same

Question 2:

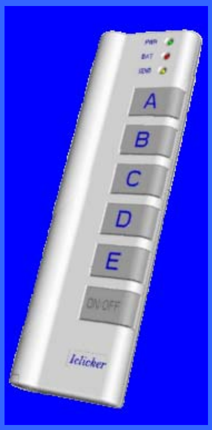


Now compare loops B and C. For which loop is $\int \vec{B} \bullet \vec{dl}$ the greatest?

A) B

B) C

C) Same



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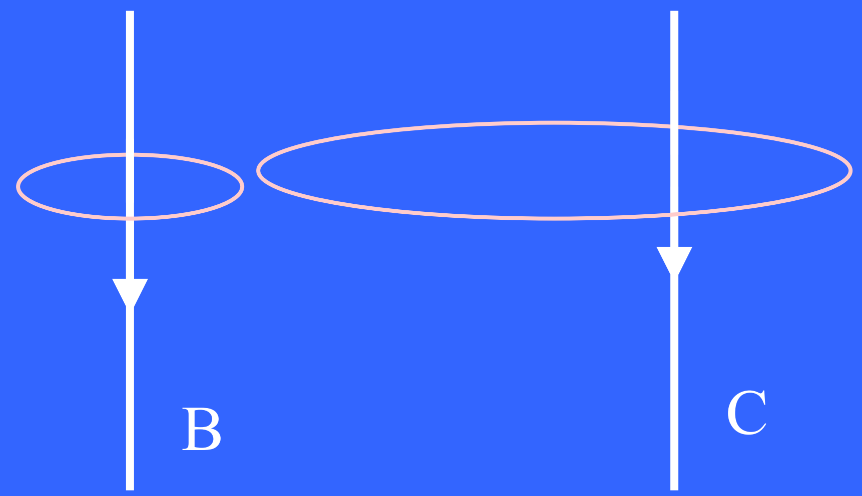

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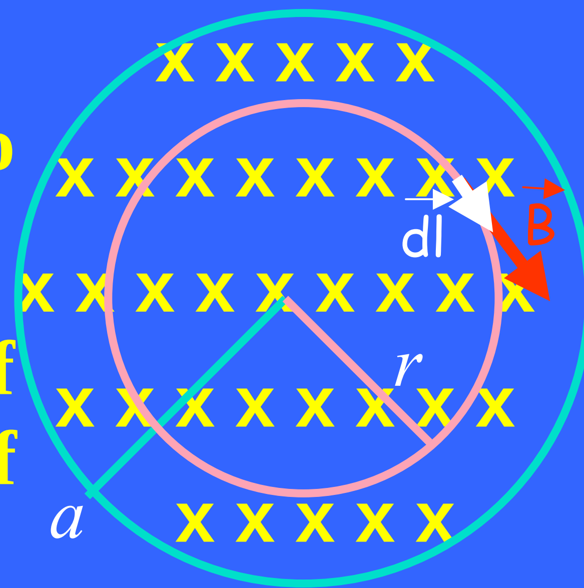
A) B

B) C

C) Same

B Field Inside a Long Wire

- **Suppose a total current *I* flows through the wire of radius *a* into the screen as shown.**
- **Calculate B field as a function of *r*, the distance from the center of the wire.**



- **B** field direction tangent to circles.
- **B** field is only a function of $r \Rightarrow$ take path to be circle of radius r:

$$\Rightarrow \oint \vec{B} \bullet d\vec{l} = B(2\pi r)$$

- Current passing through circle:

$$I_{\text{enclosed}} = \frac{r^2}{a^2} I$$

- Ampere's Law:

$$\oint \vec{B} \bullet d\vec{l} = \mu_o I_{\text{enclosed}} \quad \Rightarrow \quad \boxed{B = \frac{\mu_0 I}{2\pi} \frac{r}{a^2}}$$

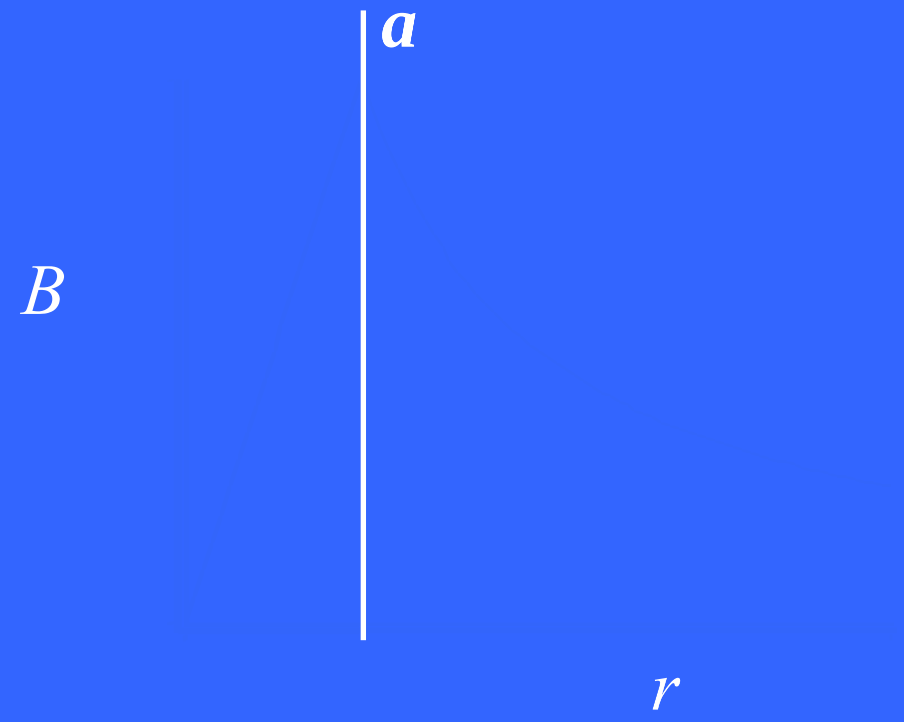
B Field of a Long Wire

- **Inside the wire: (*r* < *a*)**

$$\boxed{B = \frac{\mu_0 I}{2\pi}\frac{r}{a^2}}$$

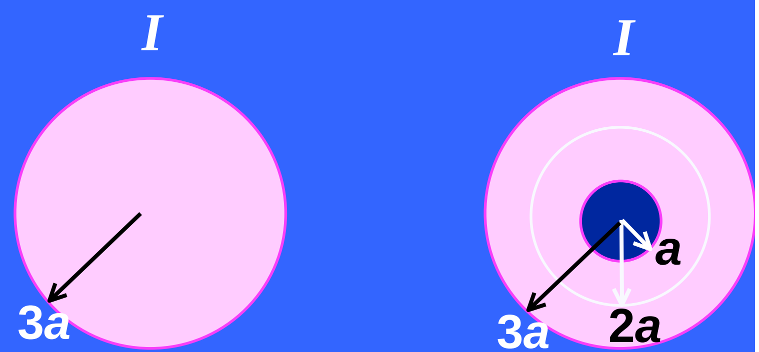
- **Outside the wire:**
(*r* > *a*)

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

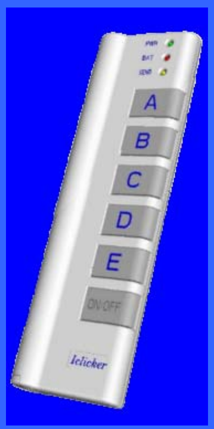


Question 3

- **Two cylindrical conductors each carry current *I* into the screen as shown. The conductor on the left is solid and has radius $R = 3a$. The conductor on the right has a hole in the middle and carries current only between $R = a$ and $R = 3a$.**
  - **What is the relation between the magnetic field at $R = 6a$ for the two cases (L = left, R = right)?**



(a) $B_L(6a) < B_R(6a)$ (b) $B_L(6a) = B_R(6a)$ (c) $B_L(6a) > B_R(6a)$



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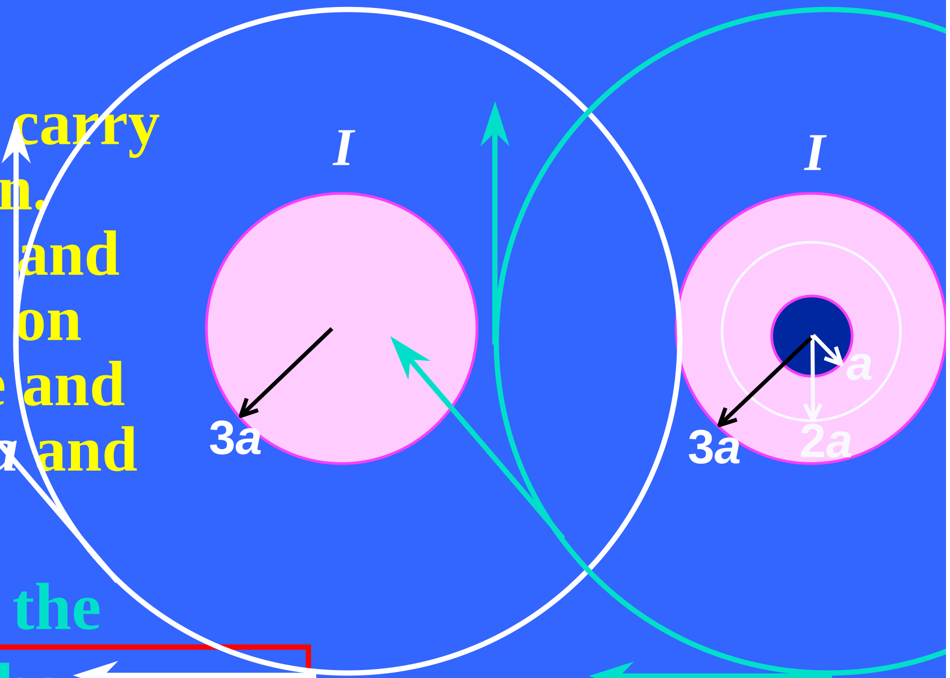
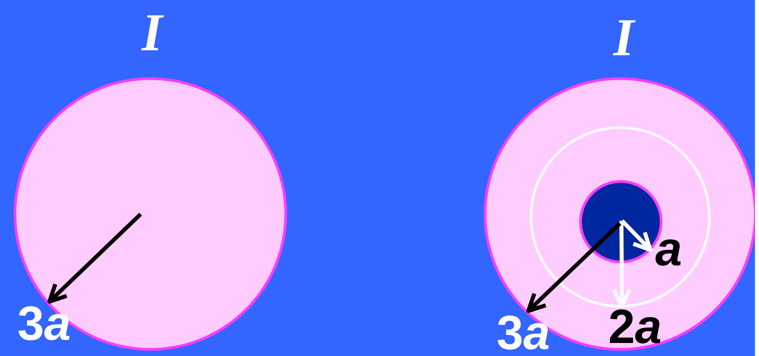


(a) $B_L(6a) < B_R(6a)$ (b) $B_L(6a) = B_R(6a)$ (c) $B_L(6a) > B_R(6a)$

- Use Ampere's Law in both cases by drawing a loop in the plane of the screen at R=6a
- Both fields have cylindrical symmetry, so they are tangent to the loop at all points, thus the field at R=6a only depends on **current enclosed**
- $I_{enclosed} = I$ in both cases

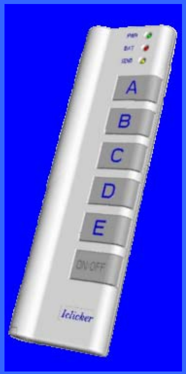
Question 4

- **Two cylindrical conductors each carry current *I* into the screen as shown. The conductor on the left is solid and has radius $R = 3a$. The conductor on the right has a hole in the middle and carries current only between $R = a$ and $R = 3a$.**



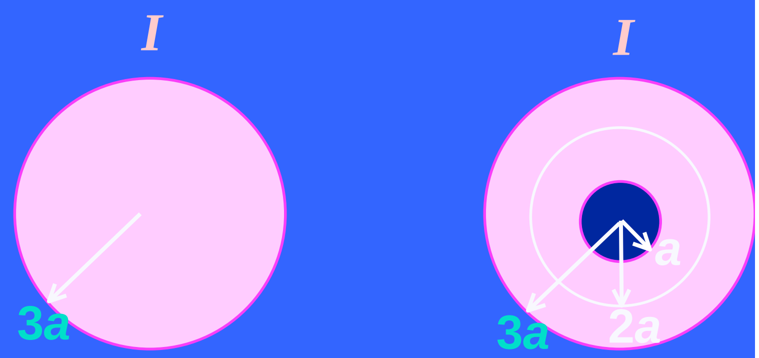
- What is the relation between the magnetic field at $R = 2a$ for the two cases (L = left, R = right)?

(a) $B_L(2a) < B_R(2a)$ (b) $B_L(2a) = B_R(2a)$ (c) $B_L(2a) > B_R(2a)$



Question 4

- **Two cylindrical conductors each carry current *I* into the screen as shown. The conductor on the left is solid and has radius $R=3a$. The conductor on the right has a hole in the middle and carries current only between $R=a$ and $R=3a$.**
  - **What is the relation between the magnetic field at $R = 2a$ for the two cases (L=left, R=right)?**



(a) $B_L(2a) < B_R(2a)$ (b) $B_L(2a) = B_R(2a)$ (c) $\boxed{B_L(2a) > B_R(2a)}$

Again, field only depends upon current enclosed...

LEFT cylinder:

$$I_L = \frac{\pi(2a)^2}{\pi(3a)^2} I = \frac{4}{9} I$$

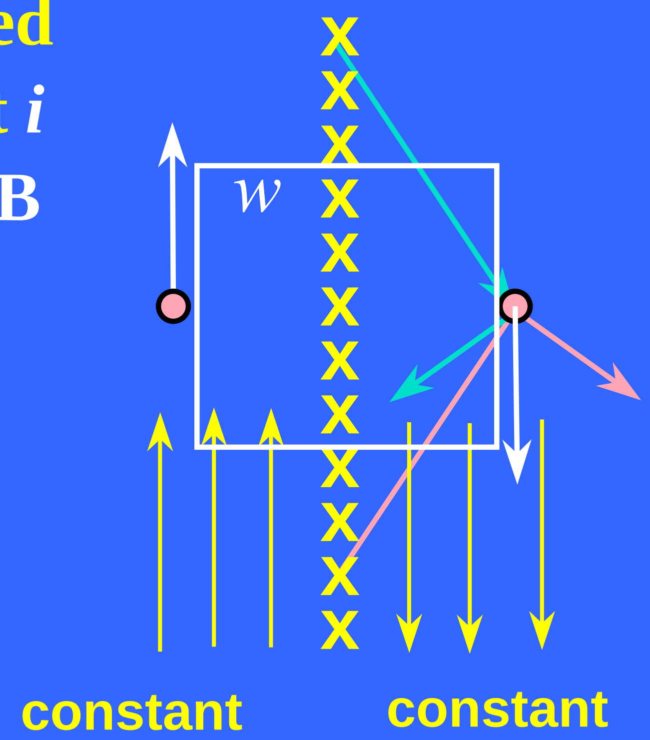
RIGHT cylinder:

$$I_R = \frac{\pi\left((2a)^2 - a^2\right)}{\pi\left((3a)^2 - a^2\right)} I = \frac{3}{8} I$$

B Field of ∞ Current Sheet

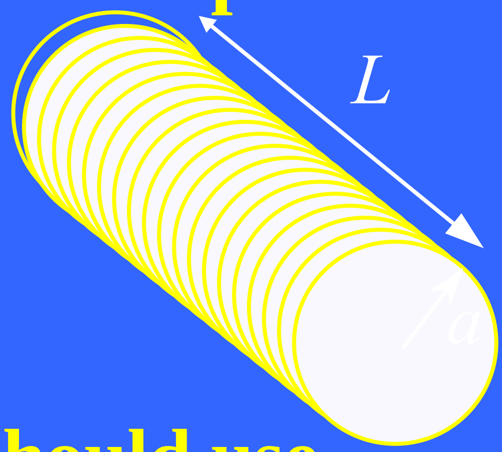
- **Consider an ∞ sheet of current described by *n* wires/length each carrying current *i* into the screen as shown. Calculate the B field.**
- What is the direction of the field?
  - Symmetry ⇒ vertical direction
- Calculate using Ampere's law for a square of side w:
- $$\oint \vec{B} \bullet d\vec{l} = Bw + 0 + Bw + 0 = 2Bw$$
- $$I = nwi$$

therefore, $\oint \vec{B} \bullet d\vec{l} = \mu_0 I \Rightarrow$ $\boxed{B = \frac{\mu_0 n i}{2}}$



B Field of an ideal Solenoid

- **A constant magnetic field can (in principle) be produced by an ∞ sheet of current. In practice, however, a constant magnetic field is often produced by a solenoid.**
- A solenoid is defined by a current i flowing through a wire that is wrapped n *turns per unit length* on a cylinder of radius a and length L.

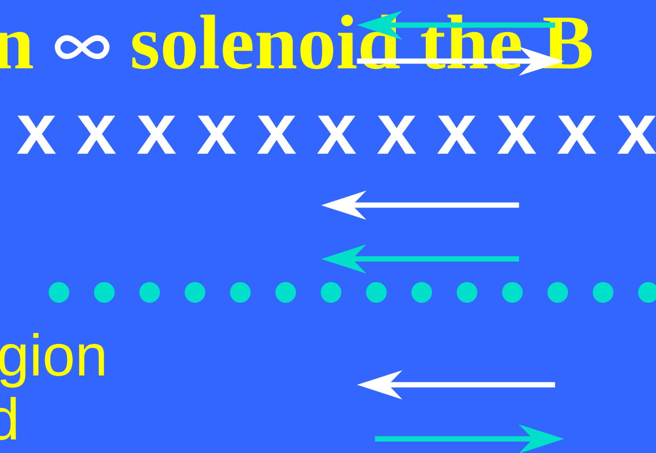


- **To correctly calculate the B-field, we should use Biot-Savart, and add up the field from the different loops.**
- If $a \ll L$, the B field is to first order contained within the solenoid, in the axial direction, and of constant magnitude. In this limit, we can calculate the field using Ampere's Law.

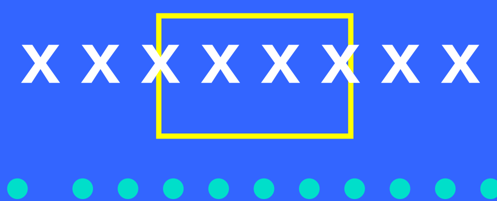
Ideal Solenoid

B Field of an ∞ Solenoid

- **To calculate the B field of the solenoid using Ampere's Law, we need to justify the claim that the B field is nearly 0 outside the solenoid (for an ∞ solenoid the B field is exactly 0 outside).**
- To do this, view the ∞ solenoid from the side as 2 ∞ current sheets.



- The fields are in the same direction in the region between the sheets (inside the solenoid) and cancel outside the sheets (outside the solenoid).
- B is uniform inside solenoid and zero outside.
- **Draw square path of side w:**



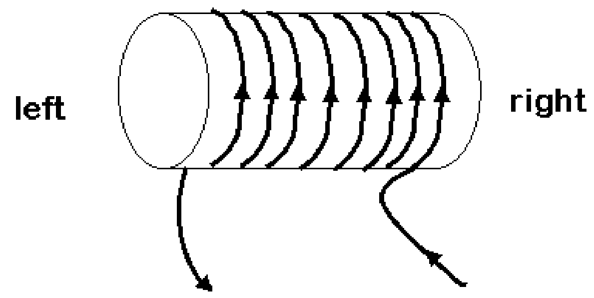
$$\oint \vec{B} \bullet d\vec{l} = Bw$$

$$I = nwi \quad \Rightarrow \quad \boxed{B = \mu_0 n i}$$

Note: $B \propto \dfrac{\text{Amp}}{\text{Length}}$ ☑

Question 5:

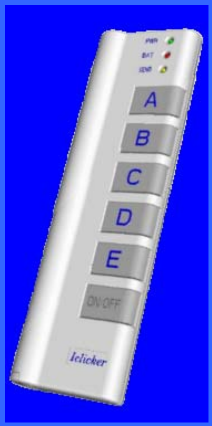
A current carrying wire is wrapped around an iron core, forming an electro-magnet.



Which direction does the magnetic field point inside the iron core?

a) left   b) right   c) up   d) down

e) out of the screen



Question 5:

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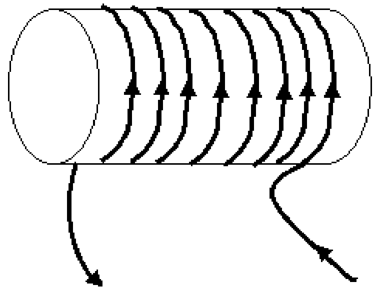


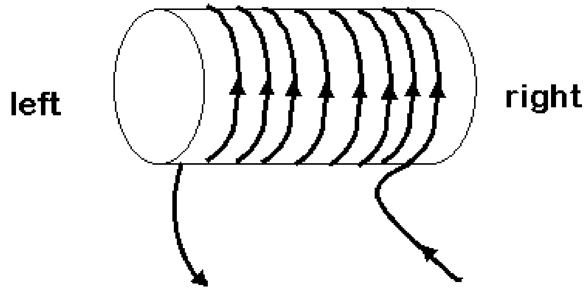
Which direction does the magnetic field point inside the iron core?

a) left　　b) right　　c) up　　d) down

e) out of the screen

Question 6:

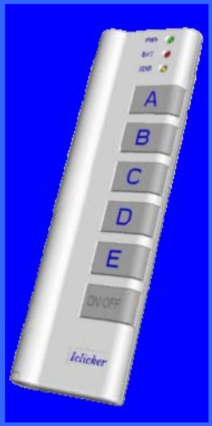
A current carrying wire is wrapped around an iron core, forming an electro-magnet.



Which side of the solenoid should be labeled as the magnetic north pole?

a) right

b) left



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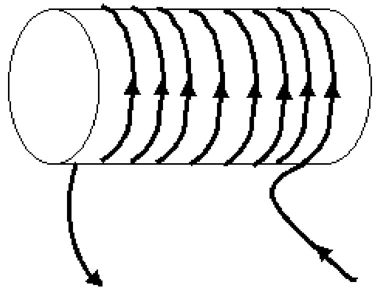

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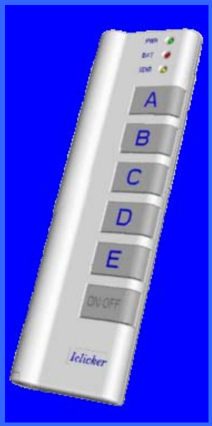
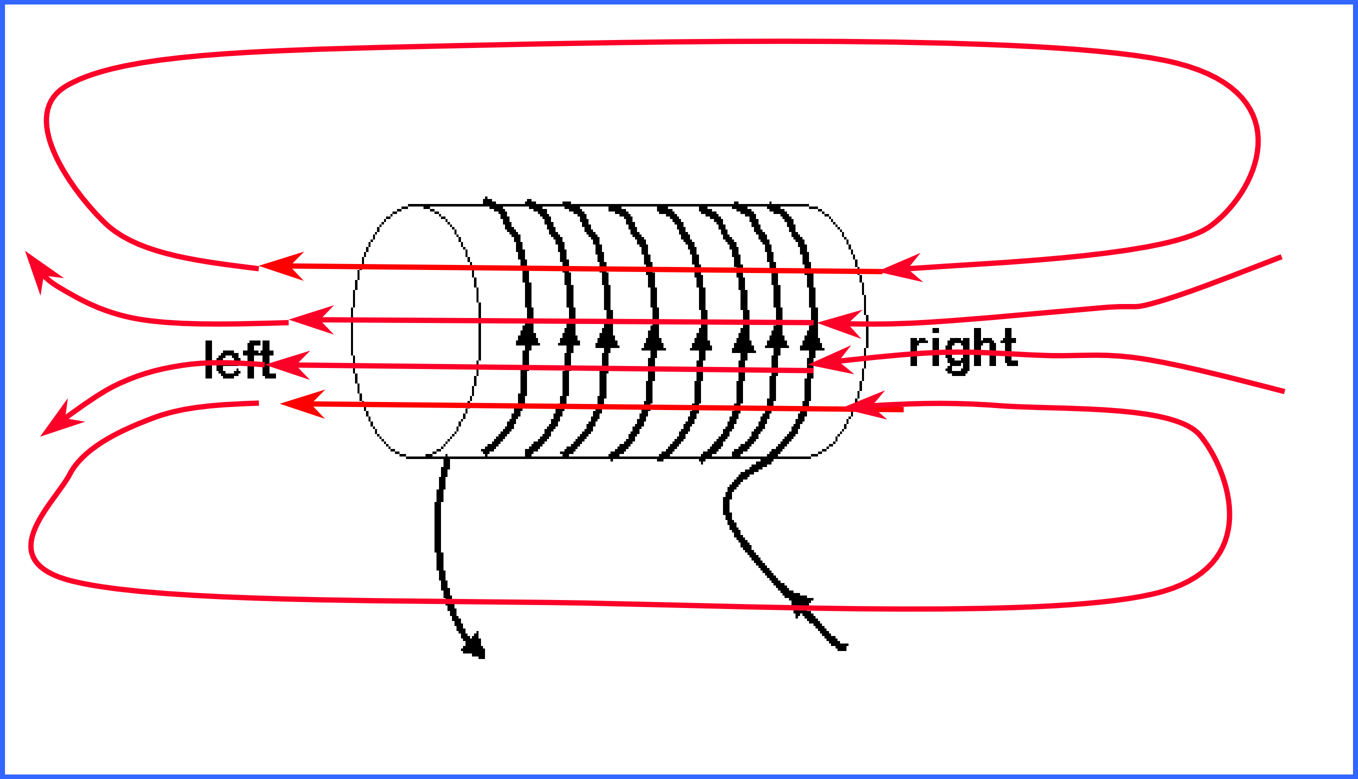
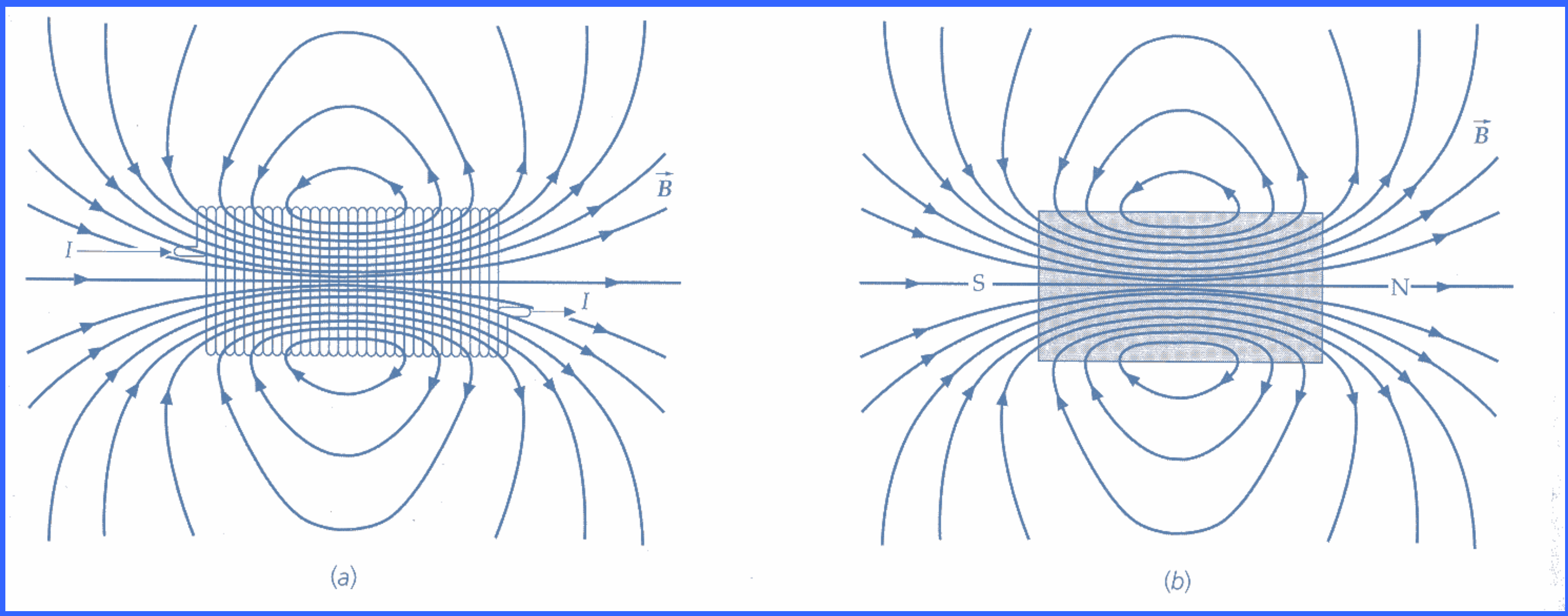


Use the wrap rule to find the B-field: wrap your fingers in the direction of the current, the B field points in the direction of the thumb (to the left). Since the field lines leave the left end of solenoid, the left end is the north pole.



Solenoids

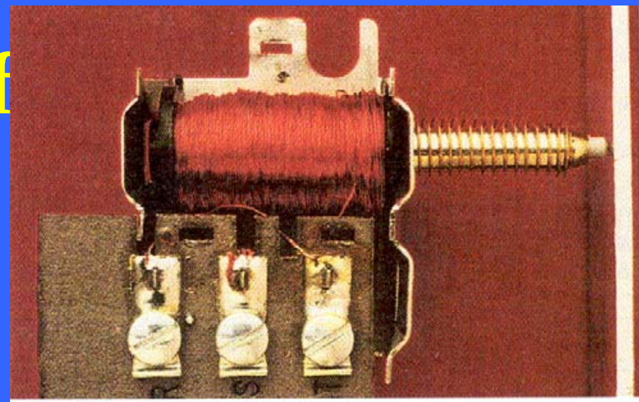
The magnetic field of a solenoid is essentially identical to that of a bar magnet.



The big difference is that we can turn the solenoid *on* and *off* ! It attracts/repels other permanent magnets; it attracts ferromagnets, etc.

Solenoid Applications

Digital [on/of



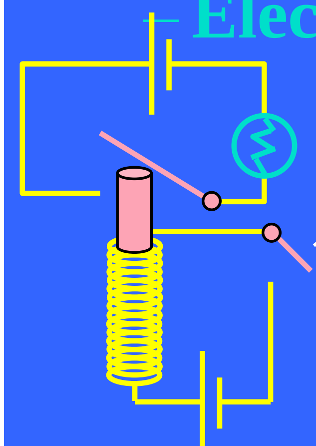
– **Doorbells**

Magnet off → plunger held in place by spring
Magnet on → plunger expelled → strikes bell

– **Power door locks**

– **Magnetic cranes**

– **Electronic Switch “relay”**



Close switch
→ current
→ magnetic field pulls in plunger
→ closes larger circuit

Advantage:

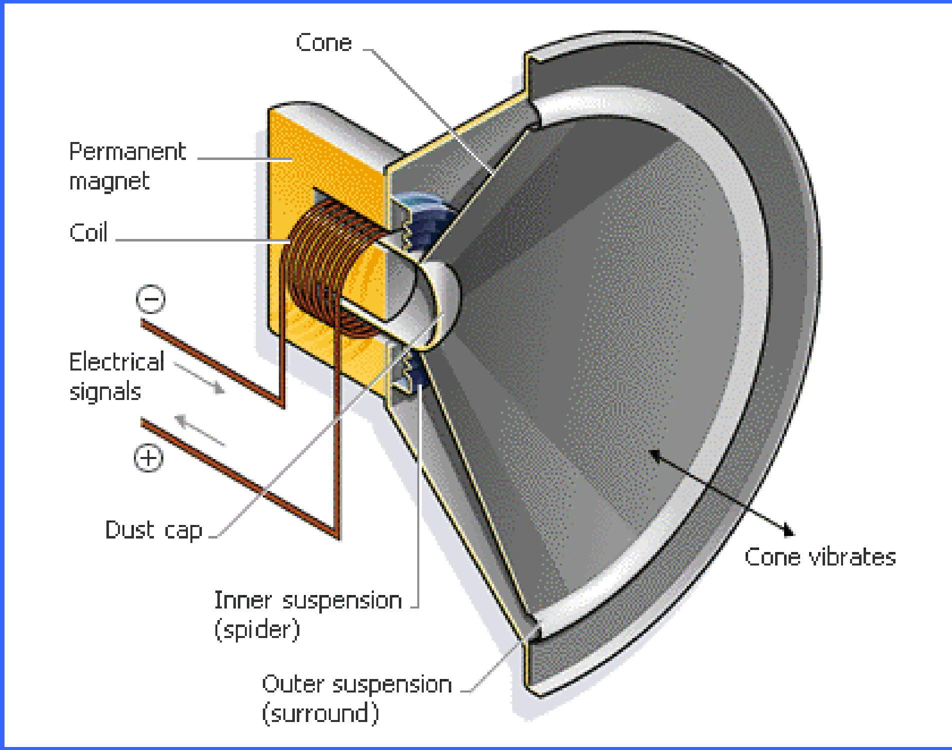
A small current can be used to switch a much larger one

– Starter in washer/dryer, car ignition, …

Solenoid Applications

Analog (deflection ∝ *I*):

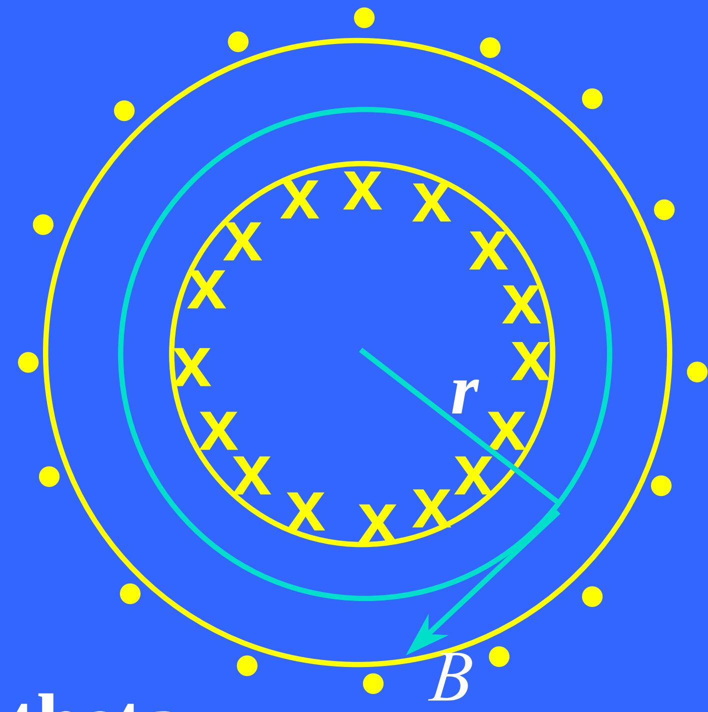
- **Variable A/C valves**
- **Speakers**



Solenoids are everywhere!

In fact, a typical car has over 20 solenoids!

Toroid



- Toroid defined by N *total* turns with current i.
- B=0 outside toroid! (Consider integrating B on circle outside toroid)

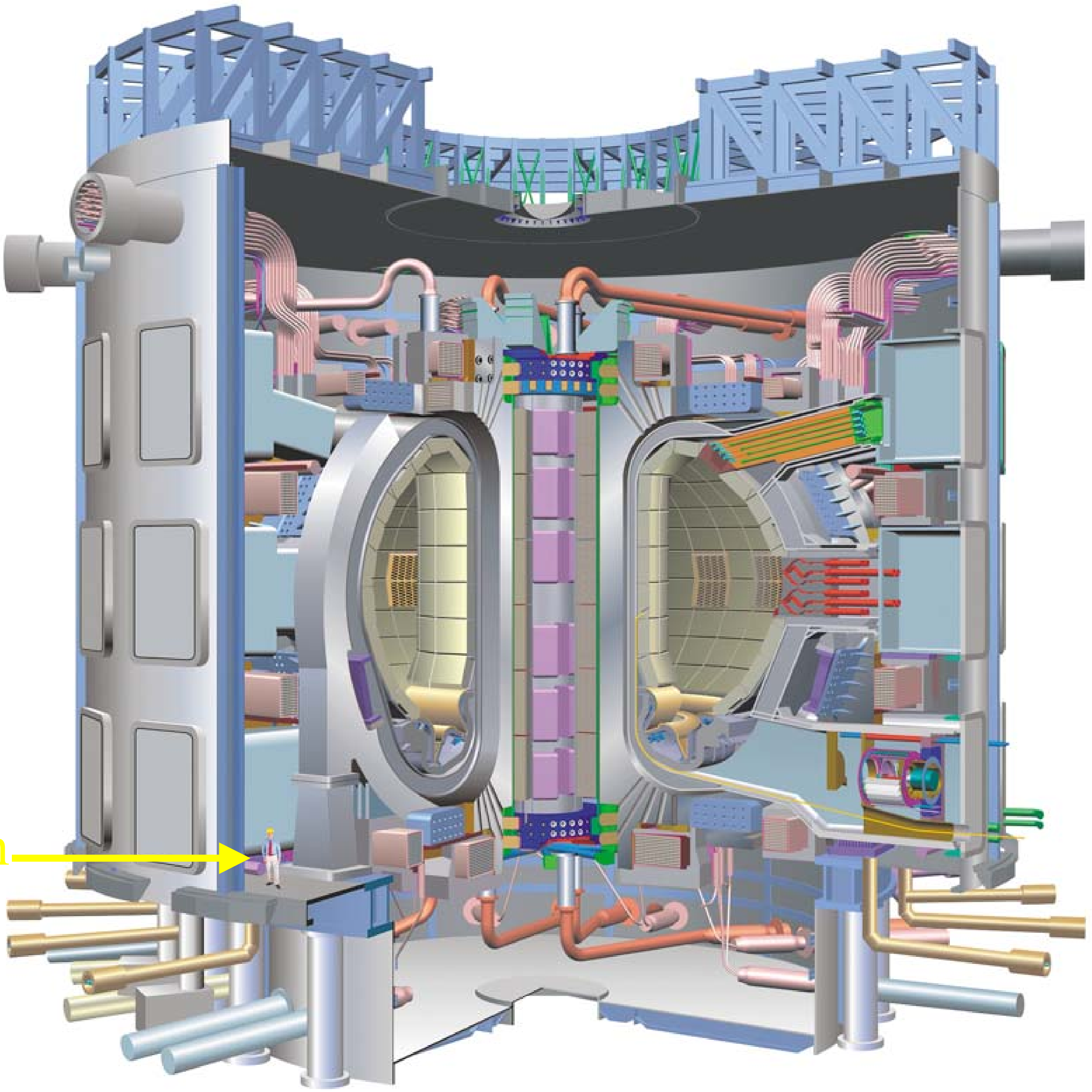
- **Direction? tangent to circle.**
- **Magnitude depends on? r (not theta or z).**
- **To find *B* inside, consider circle of radius *r*, centered at the center of the toroid.**

$$\oint B \bullet dl = B(2\pi r) \qquad I = Ni$$

Apply Ampere's Law: $\oint \vec{B} \bullet d\vec{l} = \mu_0 I \Rightarrow$ $\boxed{B = \frac{\mu_0 N i}{2\pi r}}$

ITER Tokamak; giant toroid for fusion

A joint US-Europe-Japan project to be built in southern France by 2016. A toroidal field contains the hot plasma. Fusion should provide clean power. ITER is prototype for future machines and is suppose to produce 500MW of power.

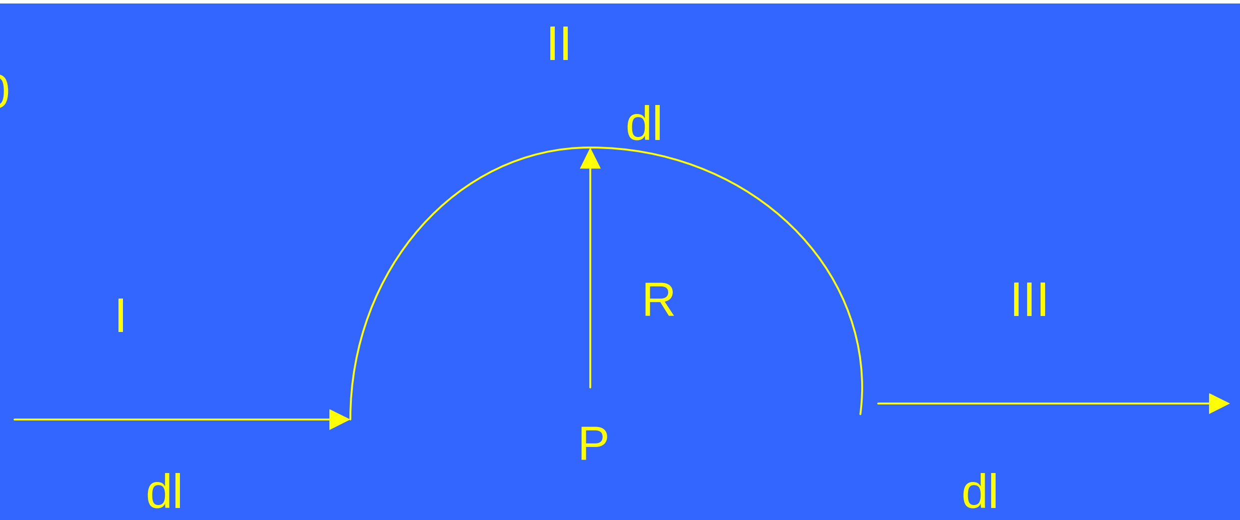


Summary

Example B-field Calculations:

- Inside a Long Straight Wire $$B = \frac{\mu_0 I}{2\pi}\frac{r}{a^2}$$
- Infinite Current Sheet $$B = \frac{\mu_0 n i}{2}$$
- Solenoid $$B = \mu_0 n i$$
- Toroid $$B = \frac{\mu_0 N i}{2\pi r}$$
- Circular Loop $$B_z \approx \frac{\mu_0 i R^2}{2z^3}$$

28.30



$$d\vec{B} = \frac{\mu_o I d\vec{l} \times \hat{r}}{4\pi r^2}$$

On segments I and III, dl x r^ is zero

Make sure you understand why

Is the field from a straight wire always zero ? No !!! See 28.76

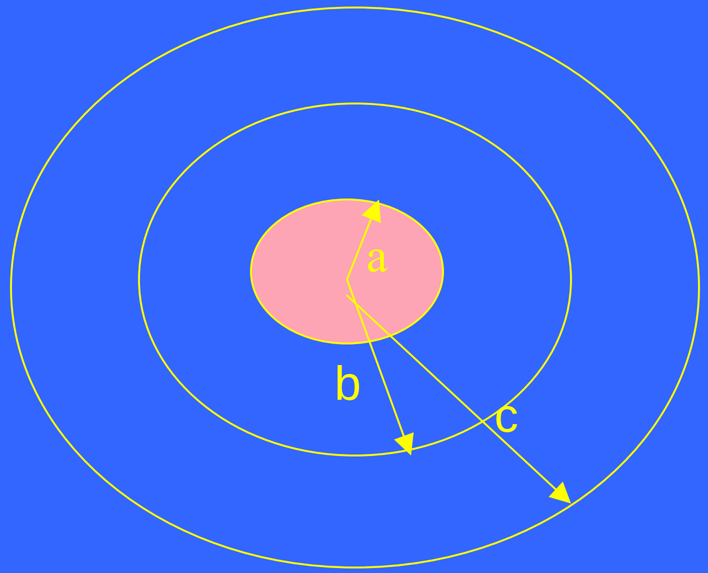
$$dB_{II} = \frac{\mu_o \int dl}{4\pi R^2} = \frac{\mu_o \pi R}{4\pi R^2} = \frac{\mu_o}{4R}$$

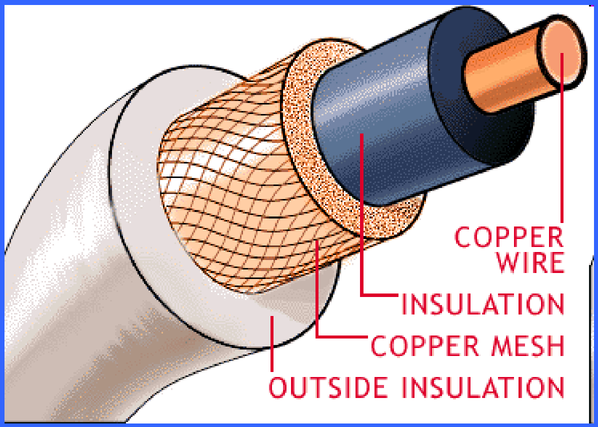
What is the direction of the B field ?

By the right hand rule, it points into the paper

28.37,
28.38

Simplified coaxial cable





Current I flowing out of the page through inner conductor

Current I flowing into the page through outer conductor

$$\oint \vec{B} \bullet d\vec{l} = \mu_o I_C$$

$$B 2\pi r = \mu_o I_c$$

For a<r<b

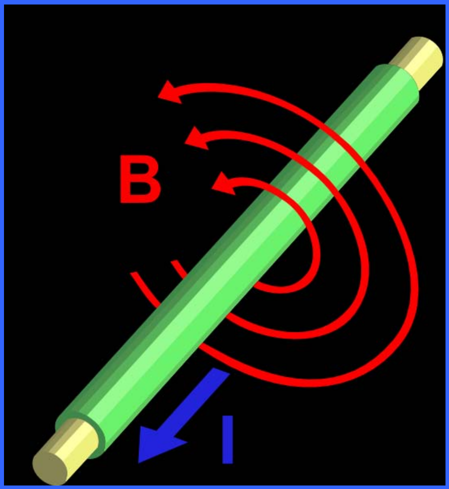
$$B = \frac{\mu_o I}{2\pi r}$$

For a<r<b

For r>c, I_C=+I-I =0
hence B=0 for r>c

28.77

$$\vec{J} = \frac{2I_0}{\pi a^2}\left[1-(\frac{r}{a})^2\right]\hat{k}$$



Find B for r>a

$$\int \vec{B} \bullet d\vec{l} = \mu_o I_c$$

J is current density, so integrate dA to obtain current passing through

$$I = \int_o^a J(r)dA = \int_o^a J(r)2\pi r dr$$

$$I = \int_o^a \frac{2I_0}{\pi a^2}\left[1-(\frac{r}{a})^2\right]2\pi r dr$$

$$I = (4I_o / a^2)\left[\frac{1}{2}r^2 - \frac{1}{4}r^4 / a^2\right]_o^a = I_o$$

$$B(2\pi r) = \mu_o I_C = \mu_o I_0$$

$$B = \mu_o I / 2\pi r$$

More weekend fun?

- HW #8 → get cracking – due Monday
- Office Hours immediately after this class (9:30 – 10:00) in WAT214 (1:30-2/1-1:30 M/WF)
- Next week: Chap 29 [Midterm 2 is Chap 25 – 29]
- Quiz next Friday