Course Updates

http://www.phys.hawaii.edu/~varner/PHYS272-Spr10/physics272.html

Reminders:

1) Assignment #8 available

2) Chapter 28 this week
Magnetism

Biot-Savart’s Law (Continued)
Magnetic Fields from a long wire

An infinitely long wire along the y-axis with the current moving +y. What is the magnetic field at position x on the x-axis? By symmetry we argue the field must be in the –z direction only.

\[
\vec{B} = \frac{I \mu_0}{4\pi} \int \frac{d\vec{l} \times \hat{r}}{r^2} = \frac{I \mu_0}{4\pi} \int \frac{\sin \phi}{r^2} dl (-\hat{z})
\]

Since, \(\sin(\phi) = \sin(\pi - \phi)\), we have

\[
\int \frac{\sin \phi}{r^2} dy = \int \frac{\sin(\pi - \phi)}{r^2} dy = \int_{-\infty}^{\infty} \frac{x}{r^3} dy = x \int_{-\infty}^{\infty} \frac{1}{(y^2 + x^2)^{3/2}} dy
\]

\[
= x \left[ \frac{1}{x^2} \frac{y}{\sqrt{y^2 + x^2}} \right]_{-\infty}^{+\infty} = \frac{2}{x}
\]

\[
\vec{B} = \frac{\mu_0 I}{2\pi x} (-\hat{z})
\]
Magnetic Fields from a long wire

Magnetic fields at points a, b, and c loop around

$$\left| \vec{B} \right| = \frac{\mu_0 I}{2\pi r}$$
Magnetic Fields from a wire segment

Here the problem is the same except wire not infinitely long!

\[
\vec{B} = I \frac{\mu_0}{4\pi} \int \frac{d\vec{l} \times \hat{r}}{r^2} = -\hat{z}I\mu_0 \int \frac{\sin \phi}{r^2} dy;
\]

\[
\int \frac{\sin \phi}{r^2} dy = \int \frac{\sin(\pi - \phi)}{r^2} dy = x \int_{-b}^{+a} \frac{1}{(y^2 + x^2)^{3/2}} dy
\]

\[
= x \left[ \frac{1}{x^2} \frac{y}{\sqrt{y^2 + x^2}} \right]_{-b}^{+a} = \frac{1}{x} \left\{ \frac{a}{\sqrt{a^2 + x^2}} + \frac{b}{\sqrt{b^2 + x^2}} \right\}
\]

\[
\vec{B} = \frac{\mu_0 I}{4\pi x} \left\{ \frac{a}{\sqrt{a^2 + x^2}} + \frac{b}{\sqrt{b^2 + x^2}} \right\} (-\hat{z})
\]

\[
= \frac{\mu_0 I}{4\pi x} \{\sin \theta_a + \sin \theta_b\} (-\hat{z})
\]

But must be careful of signs.
Putting it all together

- We know that a current-carrying wire can experience force from a $B$-field.
- We know that a current-carrying wire produces a $B$-field.
- Therefore: We expect one current-carrying wire to exert a force on another current-carrying wire:

  - Current goes together $\rightarrow$ wires come together
  - Current goes opposite $\rightarrow$ wires go opposite
Suppose we have two parallel wires with currents $I'$ and $I$ which are distance $r$ apart. The bottom wire will produce a magnetic field of

$$|\vec{B}| = \frac{\mu_0 I}{2\pi r}$$

on the top wire. The force on a length $L$ on the top wire will be

$$|F| = I'LB = I'LLBIF = I'LLBIF = \frac{\mu_0 I}{2\pi r}$$

The force is downward in the direction of the other wire. Force/unit length is

$$|F| = \frac{\mu_0 I I'}{L 2\pi r}$$

(1st derived by Mr. Ampere)
Question 1:

- Two slack wires are carrying current in opposite directions. What will happen to the wires? They will:
  
a) attract
  
b) repel
  
c) twist due to torque
Two slack wires are carrying current in opposite directions. What will happen to the wires? They will:

a) attract

b) repel

c) twist due to torque
Now, two slack wires are carrying current in the same direction. What will happen to the wires? They will:

a) attract
b) repel
c) twist due to torque
Now, two slack wires are carrying current in the same direction. What will happen to the wires? They will:

- a) attract
- b) repel
- c) twist due to torque
a) Find $B$ due to one wire at the position of the other wire
b) Use $\vec{F} = \vec{I} \times \vec{B}$ to find the direction of $F$ in each case

a: Point your thumb down, your fingers wrap in the direction of $B$ around the wire: The direction of $B$ due to the left wire at the position of the right wire is out of the screen.

b: $I$ is up, $B$ is out of the screen, so $F$ is to the right. $\Rightarrow$ the force is repulsive.
Question 3

- A current $I$ flows in the $+y$ direction in an infinite wire; a current $I$ also flows in the loop as shown in the diagram.

What is $F_x$, the net force on the loop in the $x$-direction?

(a) $F_x < 0$  (b) $F_x = 0$  (c) $F_x > 0$
Question 3

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- Forces cancel on the top and bottom of the loop.
- Forces do not cancel on the left and right sides of the loop.

- The left segment is in a larger magnetic field than the right

- Therefore, $F_{\text{left}} > F_{\text{right}}$
Magnetic Field of a wire loop

Suppose a wire loop is centered at the origin in the y-z plane. What is the B field along the center line axis (x-axis)? By symmetry, the net B field must be only along the x-axis as the y and z components will cancel.

\[ d\vec{B} = \frac{\mu_0 I \, dl \times \hat{r}}{4\pi r^2} \]

\[ dB_x = dB \cos \theta = dB \frac{a}{R} \]

\[ dB = \frac{\mu_0 I \, dl}{4\pi r^2} \]

\[ B_x = \frac{\mu_0 I}{4\pi} \int \frac{1}{r^2} \left(\frac{a}{r}\right) dl = \frac{\mu_0 I}{4\pi} \frac{a}{r^3} \int dl \]

\[ = \frac{\mu_0 I \, a}{4\pi \, r^3} \frac{2\pi a}{2} = \frac{\mu_0 I a^2}{2\sqrt{x^2 + a^2}} \]

For \( x \gg a \): \[ B_x = \frac{\mu_0 I a^2}{2x^3} \]
Circular Loop, anywhere on axis

\[ B_z = \frac{\mu_0 I R^2}{2 \left( z^2 + R^2 \right)^{3/2}} \]

Expressed in terms of the magnetic moment \( \mu = I \pi R^2 \)

\[ B_z \quad (z \gg R) \approx \frac{\mu_0 I R^2}{2z^3} \]

Note the typical \( 1/z^3 \) dipole field behavior!
Question 4

- Equal currents $I$ flow in identical circular loops as shown in the diagram. The loop on the right (left) carries current in the ccw (cw) direction as seen looking along the $+z$ direction.

- What is the magnetic field $B_z(A)$ at point $A$, the midpoint between the two loops?

(a) $B_z(A) < 0$  (b) $B_z(A) = 0$  (c) $B_z(A) > 0$
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- The right current loop gives rise to $B_z < 0$ at point $A$.
- The left current loop gives rise to $B_z > 0$ at point $A$.
- From symmetry, the magnitudes of the fields must be equal.
- Therefore, $B(A) = 0$
Question 5

- Equal currents $I$ flow in identical circular loops as shown in the diagram. The loop on the right (left) carries current in the ccw (cw) direction as seen looking along the $+z$ direction.

- What is the magnetic field $B_z(B)$ at point B, just to the right of the right loop?

(a) $B_z(B) < 0$  (b) $B_z(B) = 0$  (c) $B_z(B) > 0$
Question 5

- Equal currents $I$ flow in identical circular loops as shown in the diagram. The loop on the right (left) carries current in the ccw (cw) direction as seen looking along the $+z$ direction.

\[ (a) \quad B_z(B) < 0 \quad (b) \quad B_z(B) = 0 \quad (c) \quad B_z(B) > 0 \]

- The signs of the fields from each loop are the same at B as they are at A
- However, point B is closer to the right loop, so its field wins!
4 more lectures until Spring Break

- HW #8 → need magnetic equivalent of Gauss’ Law, next time

- Office Hours immediately after this class (9:30 - 10:00) in WAT214 (1:30-2/1-1:30 M/WF)

- Last day to drop soon - grade feedback on webpage