Course Updates

http://www.phys.hawaii.edu/~varner/PHYS272-Spr10/physics272.html

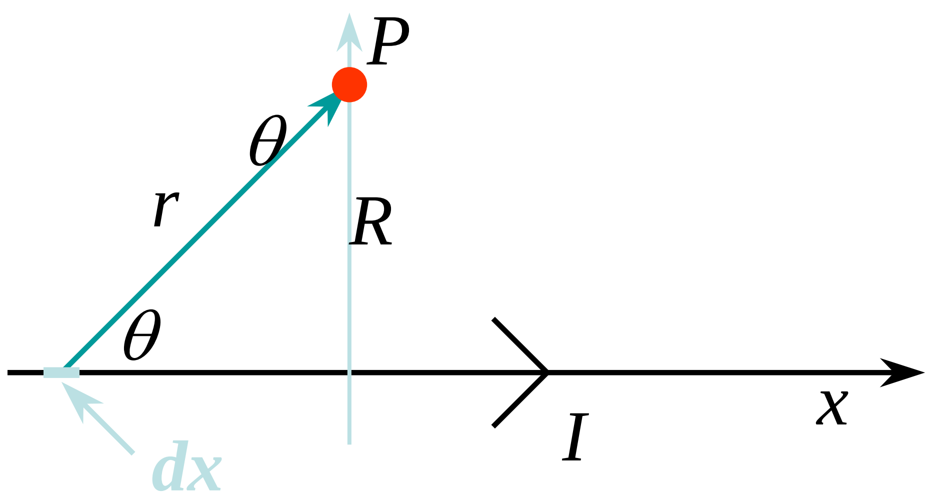
Reminders:

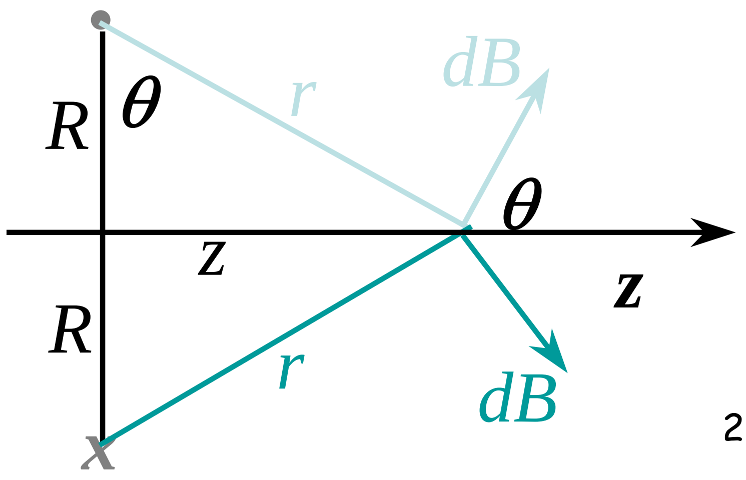
1) Assignment #8 available

2) Chapter 28 this week

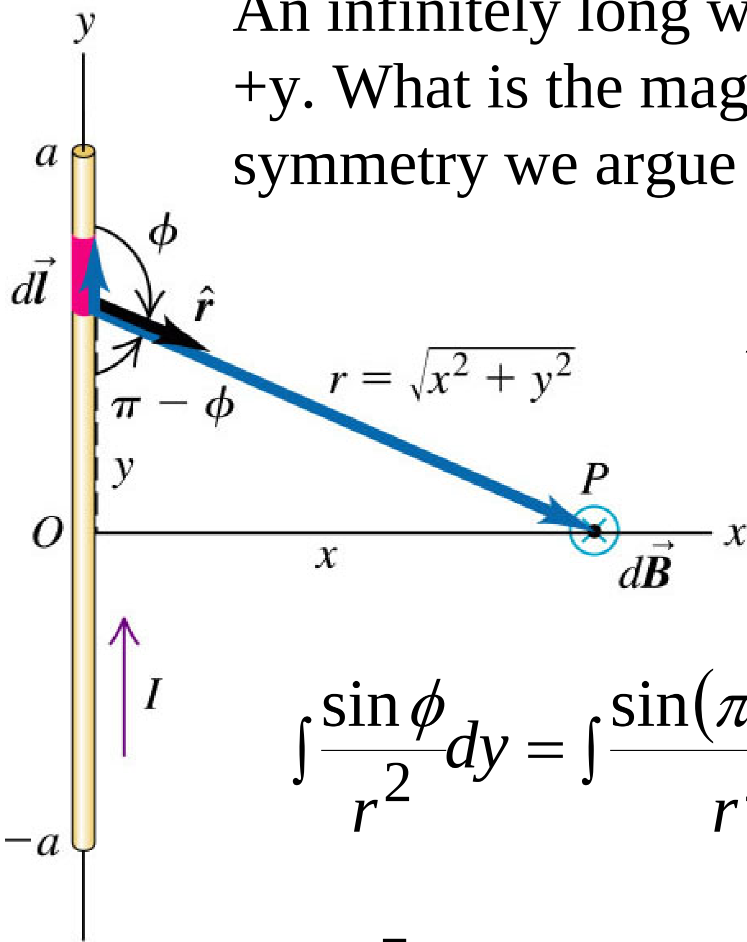
Magnetism

Biot-Savart's Law (Continued)





Magnetic Fields from a long wire



An infinitely long wire along the y-axis with the current moving +y. What is the magnetic field at position x on the x-axis? By symmetry we argue the field must be in the –z direction only.

$$\vec{B} = I\frac{\mu_0}{4\pi}\int\frac{d\vec{l}\times\hat{r}}{r^2} = I\frac{\mu_0}{4\pi}\int\frac{\sin\phi}{r^2}dl(-\hat{z})$$

$$= \frac{-\hat{z}I\mu_0}{4\pi}\int\frac{\sin\phi}{r^2}dy;$$

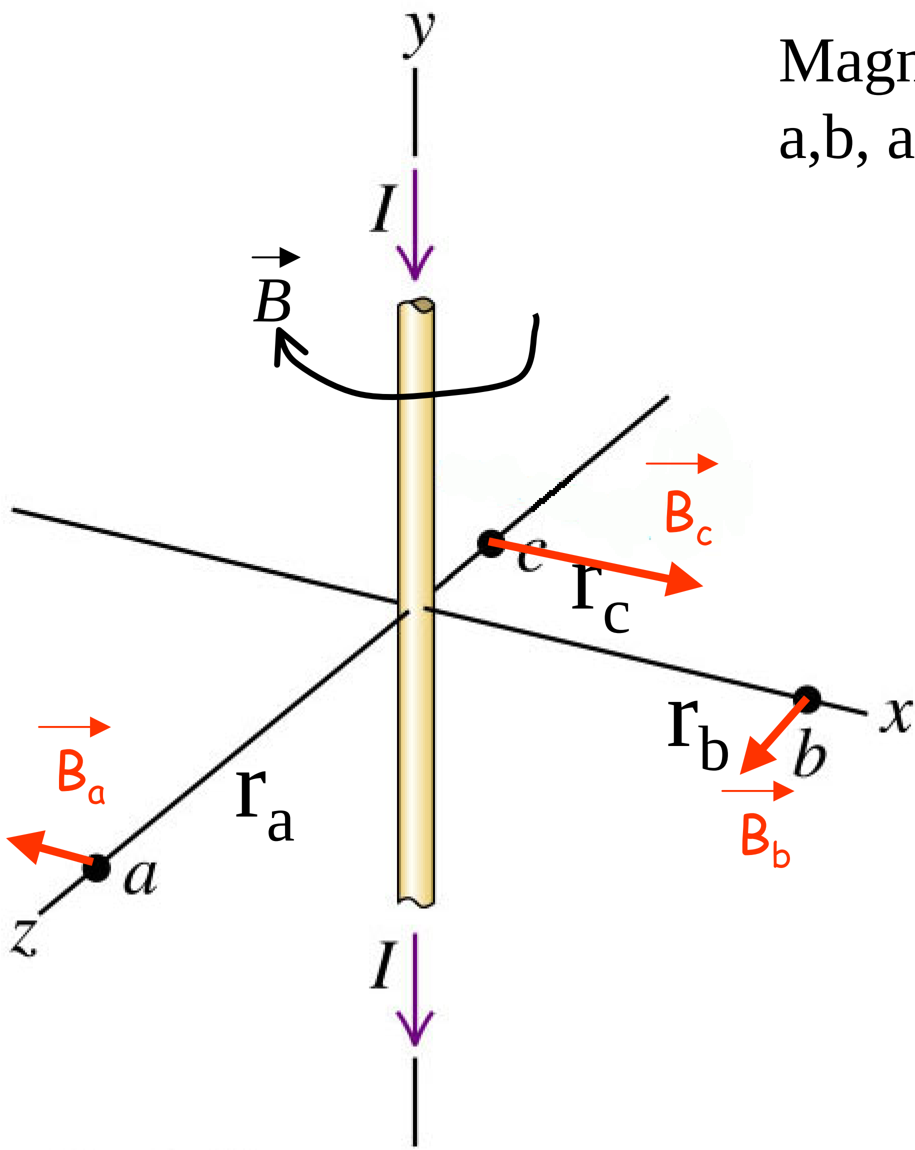
Since, sin(ϕ)= sin(π−ϕ), we have

$$\int\frac{\sin\phi}{r^2}dy = \int\frac{\sin(\pi-\phi)}{r^2}dy = \int_{-\infty}^{+\infty}\frac{x}{r^3}dy = x\int_{-\infty}^{+\infty}\frac{1}{\left(y^2+x^2\right)^{3/2}}dy$$

$$= x\left[\frac{1}{x^2}\frac{y}{\sqrt{y^2+x^2}}\right]_{-\infty}^{+\infty} = \frac{2}{x}$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2\pi\ x}(-\hat{z})}$$

Magnetic Fields from a long wire

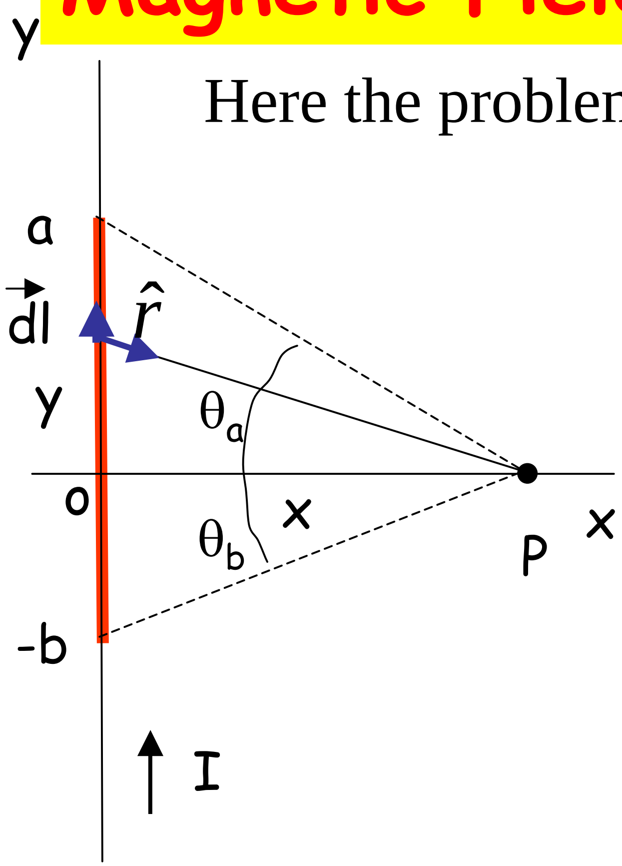


Magnetic fields at points a,b, and c loop around

$$\left|\vec{B}\right| = \frac{\mu_0 I}{2\pi\, r}$$

Magnetic Fields from a wire segment

Here the problem is the same except wire not infinitely long!



$$\vec{B} = I\frac{\mu_0}{4\pi}\int\frac{d\vec{l}\times\hat{r}}{r^2} = \frac{-\hat{z}I\mu_0}{4\pi}\int\frac{\sin\phi}{r^2}dy;$$

$$\int\frac{\sin\phi}{r^2}dy = \int\frac{\sin(\pi-\phi)}{r^2}dy = x\int_{-b}^{+a}\frac{1}{\left(y^2+x^2\right)^{3/2}}dy$$

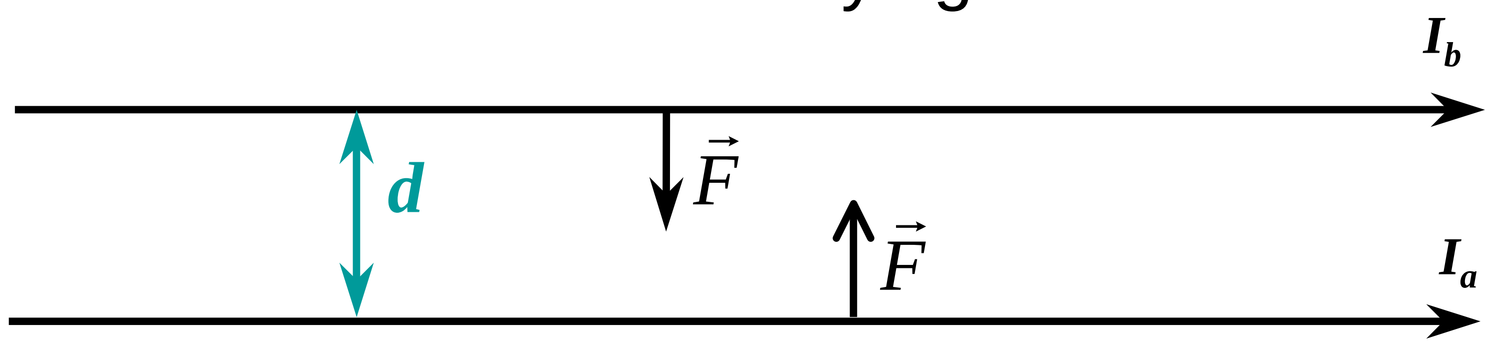
$$= x\left[\frac{1}{x^2}\frac{y}{\sqrt{y^2+x^2}}\right]_{-b}^{+a} = \frac{1}{x}\left\{\frac{a}{\sqrt{a^2+x^2}}+\frac{b}{\sqrt{b^2+x^2}}\right\}$$

$$\boxed{\begin{aligned}\vec{B} &= \frac{\mu_0 I}{4\pi\ x}\left\{\frac{a}{\sqrt{a^2+x^2}}+\frac{b}{\sqrt{b^2+x^2}}\right\}(-\hat{z}) \\ &= \frac{\mu_0 I}{4\pi\ x}\{\sin\theta_a+\sin\theta_b\}(-\hat{z})\end{aligned}}$$

But must be careful of signs.

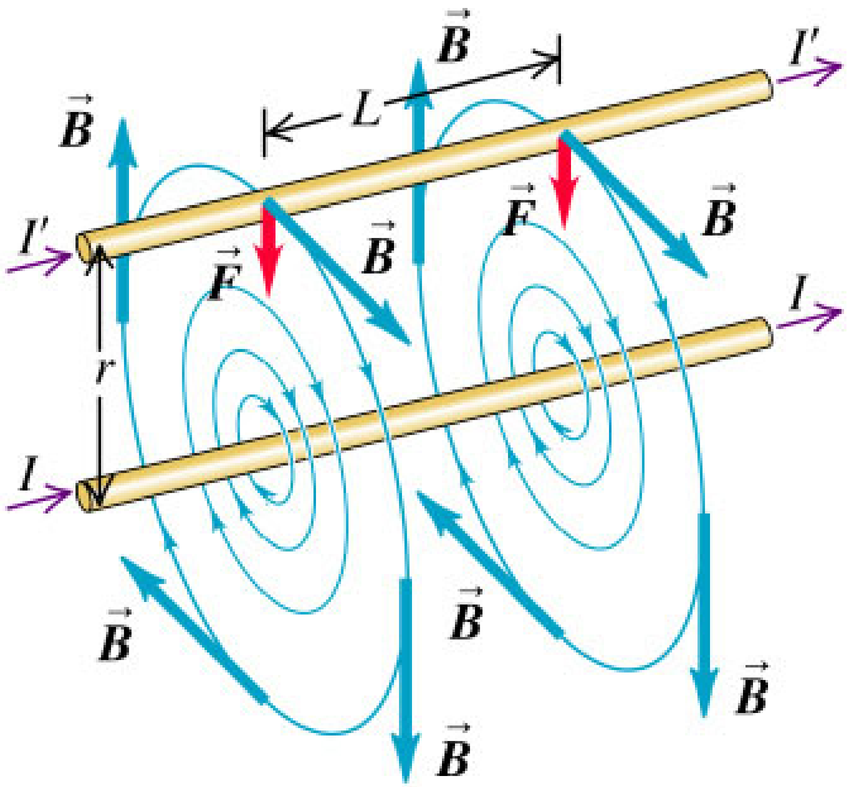
Putting it all together

- We know that a current-carrying wire can experience force from a ***B***-field.
- We know that a a current-carrying wire *produces* a ***B***-field.
- Therefore: We expect one current-carrying wire to exert a force on another current-carrying wire:



- Current goes together → wires come together
- Current goes opposite → wires go opposite

Force between two parallel long wires



Suppose we have two // wires with Currents I′ and I which are distance r apart. The bottom wire will produce a magnetic field of

$$\left|\vec{B}\right| = \frac{\mu_0 I}{2\pi r}$$

on the top wire. The force on a length L on the top wire will be

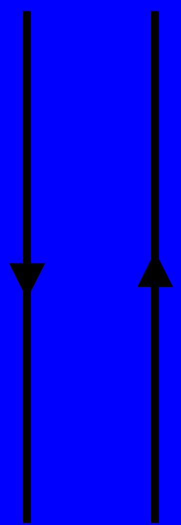
$$\left|F\right| = I' L B = I' L \frac{\mu_0 I}{2\pi r}$$

The force is downward in the direction of the other wire. Force/unit length is

$$\frac{\left|F\right|}{L} = \frac{\mu_0 \ I \ I'}{2 \ \pi \ r}$$

(1st derived by Mr. Ampere)

Question 1:



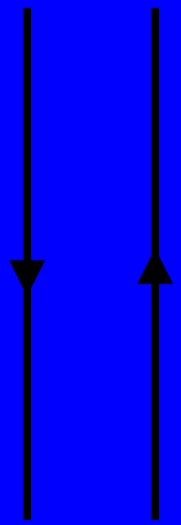
→ Two slack wires are carrying current in opposite directions. What will happen to the wires? They will:

a) attract

b) repel

c) twist due to torque

Question 1:



→ Two slack wires are carrying current in opposite directions. What will happen to the wires? They will:

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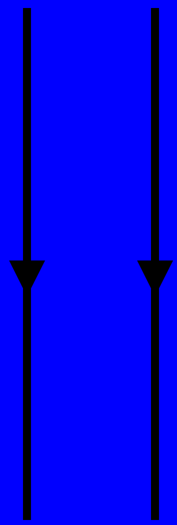
Question 2:

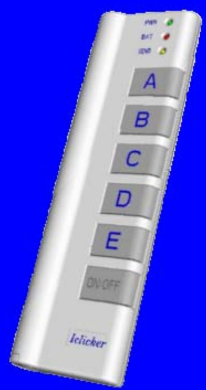


➔ Now, two slack wires are carrying current in the same direction. What will happen to the wires? They will:

a) attract

b) repel

c) twist due to torque



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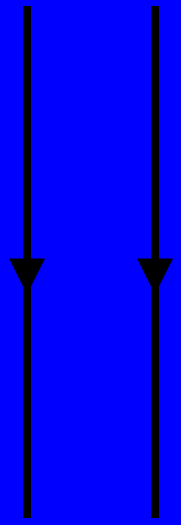

➔ Now, two slack wires are carrying current in the same direction. What will happen to the wires? They will:

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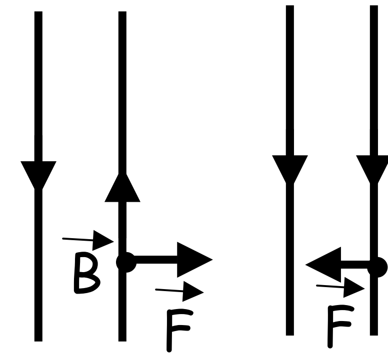


a) Find B due to one wire at the position of the other wire

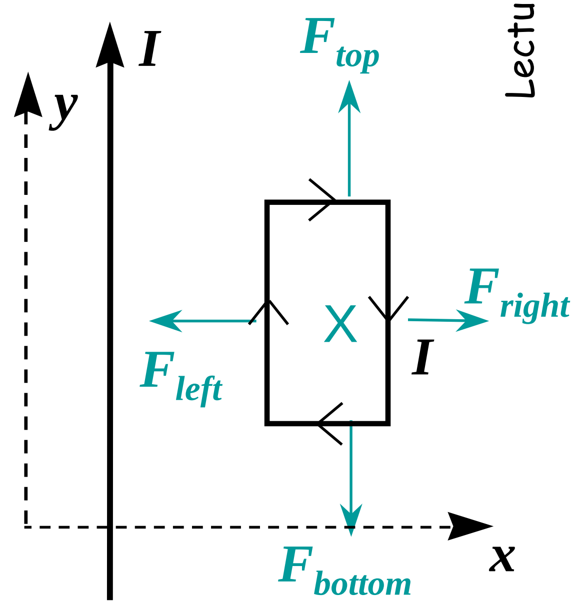
b) Use $\vec{F} = I\vec{L} \times \vec{B}$ to find the direction of F in each case

a: Point your thumb down, your fingers wrap in the direction of B around the wire: The direction of B due to the left wire at the position of the right wire is out of the screen.

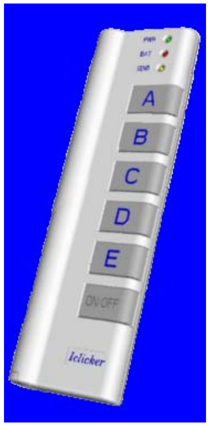
b: I is up, B is out of the screen, so F is to the right. $\Rightarrow$ the force is repulsive.

Question 3

- A current *I* flows in the +*y* direction in an infinite wire; a current *I* also flows in the loop as shown in the diagram.
  - What is F_x, the net force on the loop in the *x*-direction?

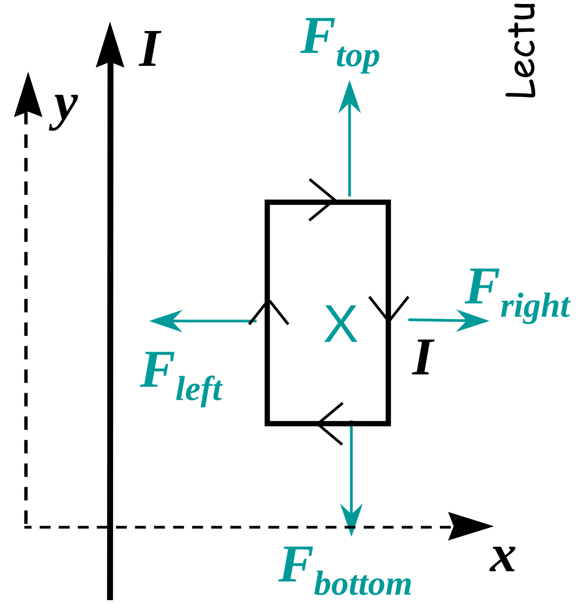


(a) $F_x < 0$ (b) $F_x = 0$ (c) $F_x > 0$



Question 3

- A current I flows in the $+y$ direction in an infinite wire; a current I also flows in the loop as shown in the diagram.
  - What is F_x, the net force on the loop in the x-direction?



$\boxed{\text{(a) } F_x < 0}$ (b) $F_x = 0$ (c) $F_x > 0$

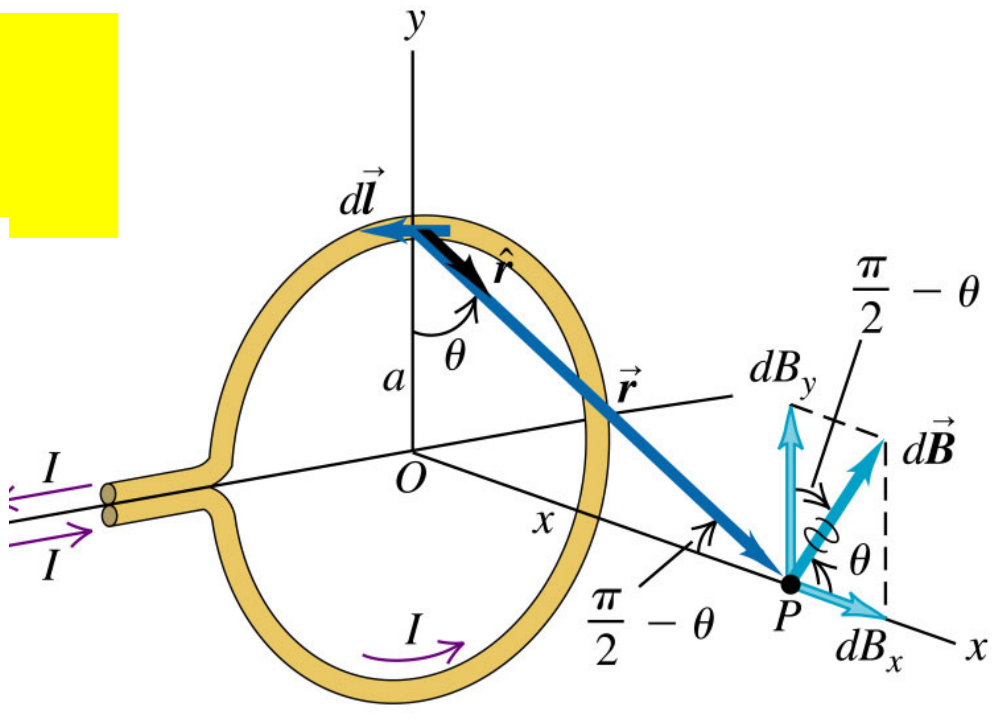
- Forces cancel on the top and bottom of the loop.
- Forces **do not** cancel on the left and right sides of the loop.

- The left segment is in a larger magnetic field than the right

- Therefore, $F_{\text{left}} > F_{\text{right}}$

Magnetic Field of a wire loop

Suppose a wire loop is centered at the origin in the y-z plane. What is the B field along the center line axis (x-axis)? By symmetry, the net B field must be only along the x-axis as the y and z components will cancel.



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Idl \times \hat{r}}{r^2}$$

$$dB_x = dB\cos\theta = dB\frac{a}{R}$$

$$dB = \frac{\mu_0 Idl}{4\pi\, r^2}$$

$$B_x = \frac{\mu_0 I}{4\pi}\int\left(\frac{1}{r^2}\right)\left(\frac{a}{r}\right)dl = \frac{\mu_0 I}{4\pi}\frac{a}{r^3}\int dl$$

$$= \frac{\mu_0 I}{4\pi}\frac{a}{r^3}2\pi a = \boxed{\frac{\mu_0 I a^2}{2\sqrt{x^2+a^2}^3}}$$

For x >> a: $\boxed{B_x = \frac{\mu_0 I a^2}{2x^3}}$

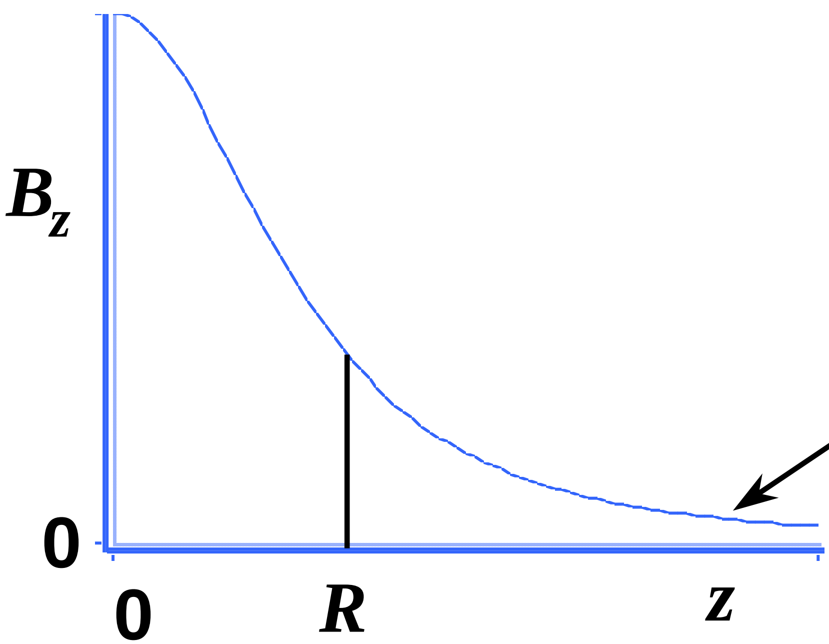
Circular Loop, anywhere on axis

$$\boxed{B_z = \frac{\mu_0 I R^2}{2\left(z^2 + R^2\right)^{3/2}}}$$

$$B_z\left(z >> R\right) \approx \frac{\mu_0 I R^2}{2z^3}$$

Expressed in terms of the magnetic moment $\mu = I\,\pi R^2$

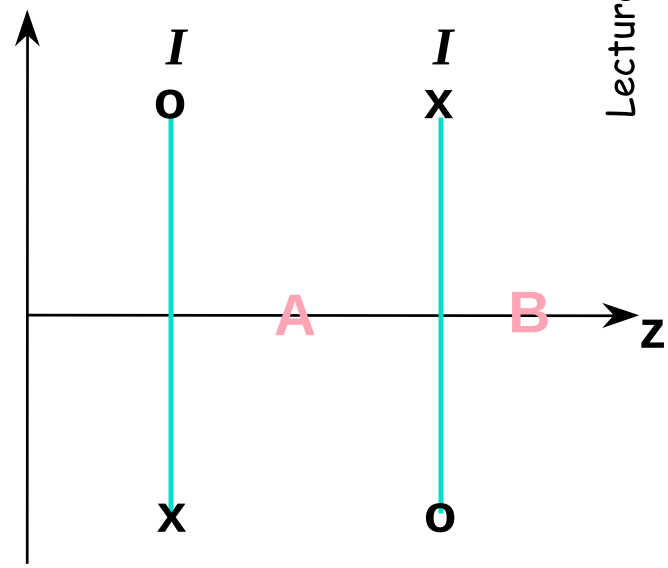
$$\boxed{B_z\left(z >> R\right) \approx \frac{\mu_0}{2\pi}\frac{\mu}{z^3}}$$



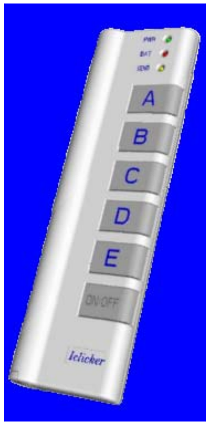
Note the typical $1/z^3$ dipole field behavior!

Question 4

- Equal currents I flow in identical circular loops as shown in the diagram. The loop on the right (left) carries current in the ccw (cw) direction as seen looking along the $+z$ direction.
  - What is the magnetic field $B_z(A)$ at point A, the midpoint between the two loops?

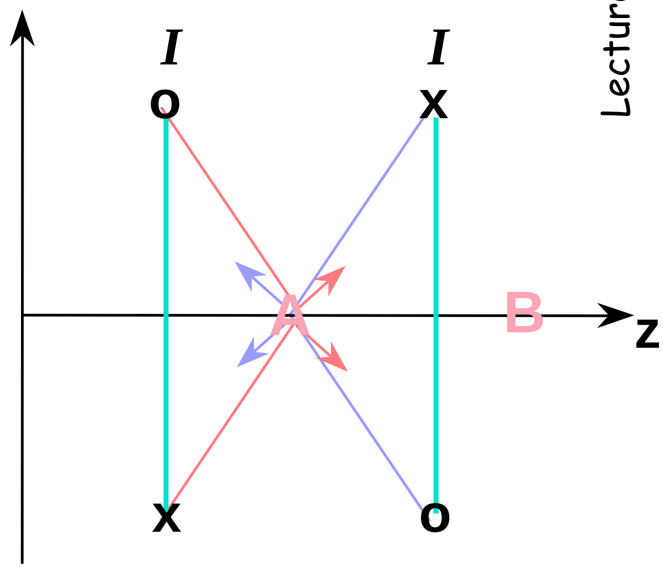


(a) $B_z(A) < 0$ (b) $B_z(A) = 0$ (c) $B_z(A) > 0$



Question 4

- **Equal currents *I* flow in identical circular loops as shown in the diagram. The loop on the right (left) carries current in the ccw (cw) direction as seen looking along the +*z* direction.**
  - **What is the magnetic field $B_z(A)$ at point *A*, the midpoint between the two loops?**

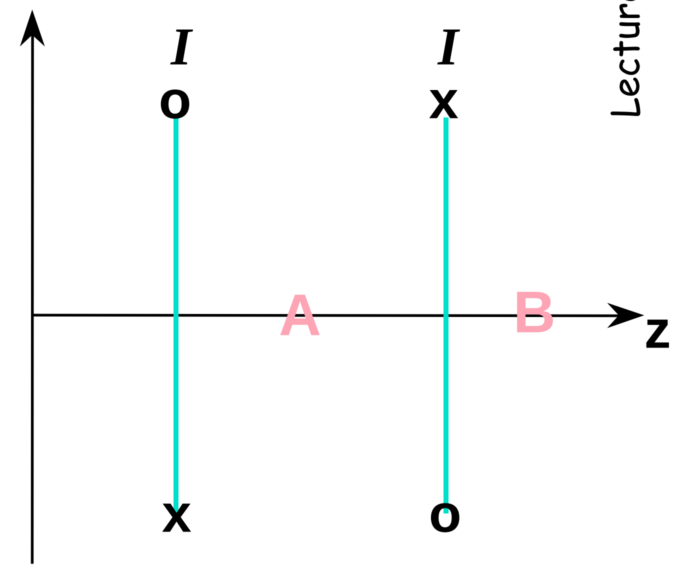


(a) $B_z(\mathrm{A}) < 0$ **(b) $B_z(\mathrm{A}) = 0$** **(c) $B_z(\mathrm{A}) > 0$**

- The right current loop gives rise to $B_z < 0$ at point A.
- The left current loop gives rise to $B_z > 0$ at point A.
- From symmetry, the magnitudes of the fields must be equal.
- Therefore, $B(\mathrm{A}) = 0$

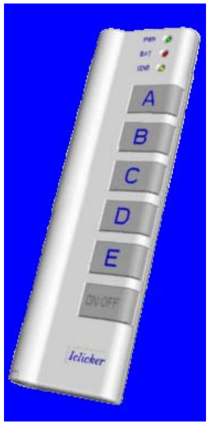
Question 5

- Equal currents I flow in identical circular loops as shown in the diagram. The loop on the right (left) carries current in the ccw (cw) direction as seen looking along the +z direction.



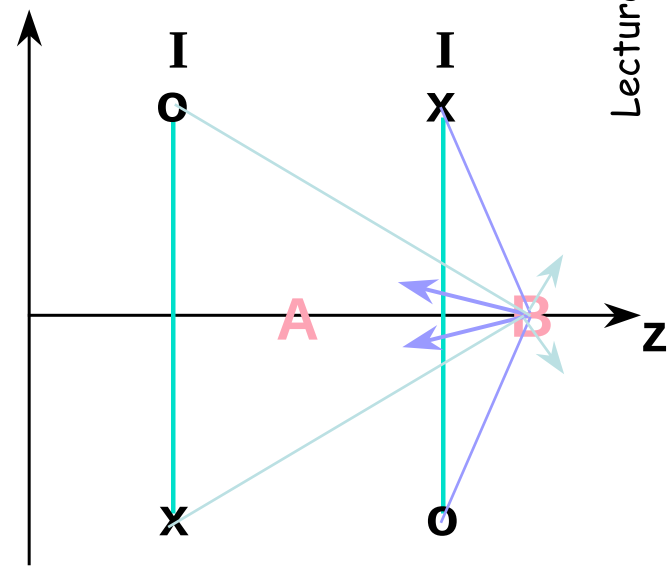
- What is the magnetic field B_z(B) at point B, just to the right of the right loop?

(a) $B_z(\mathrm{B}) < 0$ **(b)** $B_z(\mathrm{B}) = 0$ **(c)** $B_z(\mathrm{B}) > 0$



Question 5

- Equal currents *I* flow in identical circular loops as shown in the diagram. The loop on the right (left) carries current in the ccw (cw) direction as seen looking along the +*z* direction.



$\boxed{\textbf{(a)}\ B_z(\mathrm{B}) < 0}$ **(b)** $B_z(\mathrm{B}) = 0$ **(c)** $B_z(\mathrm{B}) > 0$

- The signs of the fields from each loop are the same at B as they are at A
- However, point B is closer to the right loop, so its field wins!

4 more lectures until Spring Break

- HW #8 → need magnetic equivalent of Gauss' Law, next time
- Office Hours immediately after this class (9:30 – 10:00) in WAT214 (1:30-2/1-1:30 M/WF)
- Last day to drop soon – grade feedback on webpage