

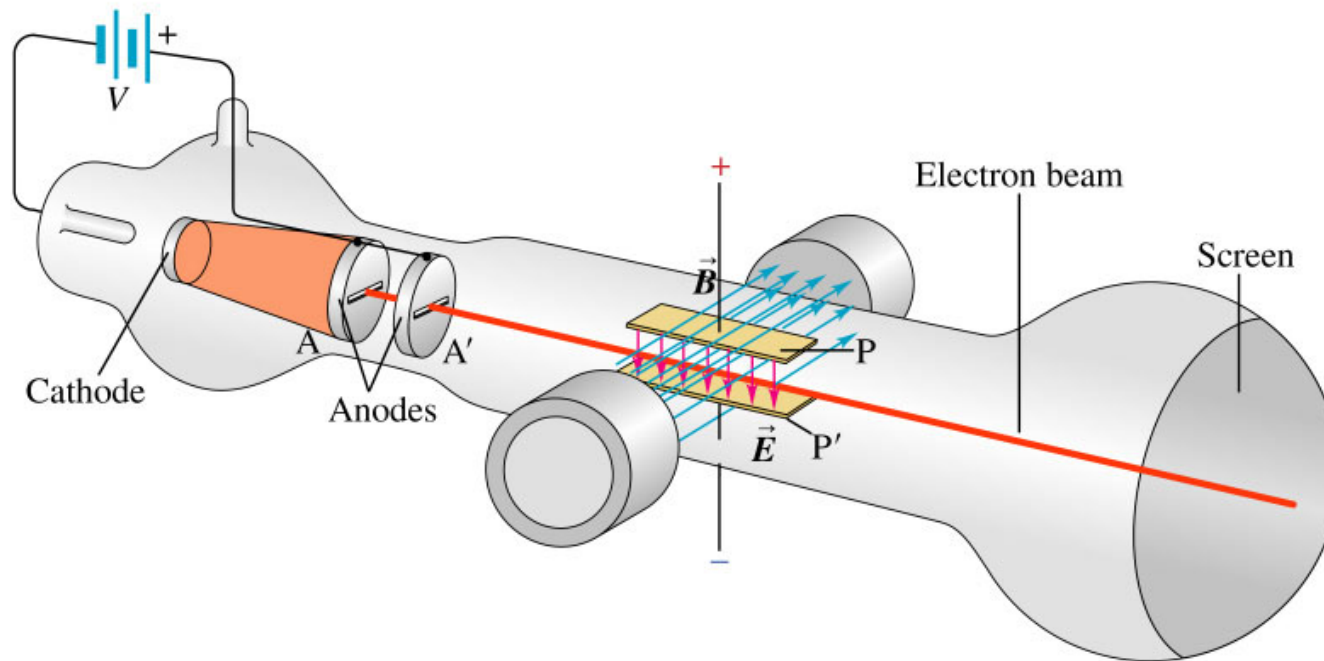
# Course Updates

<http://www.phys.hawaii.edu/~varner/PHYS272-Spr10/physics272.html>

Notes for today:

- 1) Assignment #7 due Monday
- 2) This week: Finish Chap 27 (magnetic fields and forces)
- 3) Next week Chap 28 (Sources of B fields)

# Thompson's charge/mass ratio of electron measurement

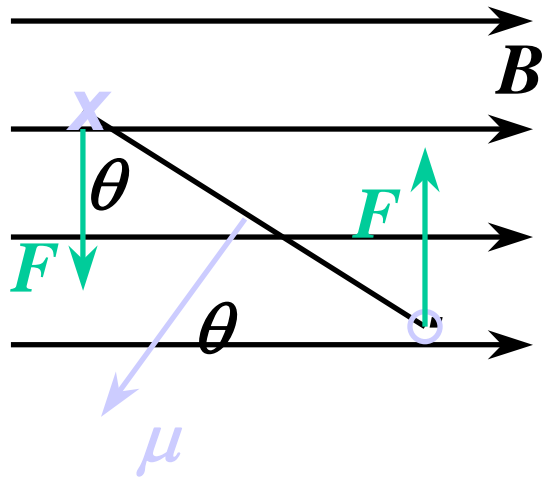


Found single value of  $e/m$   
independent of cathode material.  
This led to discovery of electrons.

$$e/m = 1.7588 \times 10^{11} \text{ C/kg}$$

# Magnetism

## Forces & Magnetic Dipoles



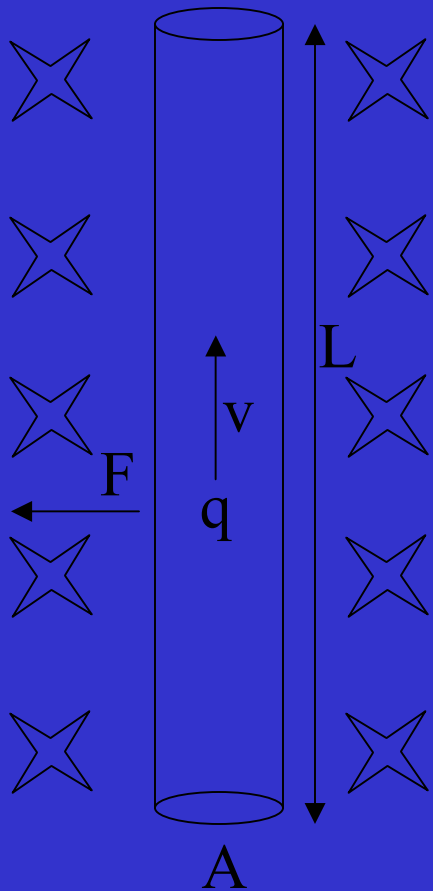
$$\mu = AI$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$U = -\vec{\mu} \cdot \vec{B}$$



## Magnetic force on current carrying wire



Suppose we have a straight wire with current where charges  $q$  are moving upwards and there is a  $B$  field pointing into this slide. There is a  $\vec{F} = q\vec{v} \times \vec{B}$  force tending to push the wire to the left.

Recall that current is  $I = n q v A$  (Eqn 25.2) with  $n$ =#charge/vol,  $v$ =velocity and  $A$ =area. In a length,  $L$ , the #charges =  $n L A$ , so the total force magnitude is,

$$F = (nLA)qvB = (nqvA)LB = ILB$$

If we consider a small line segment,  $dl$ , we can write the vector force eqn. as,

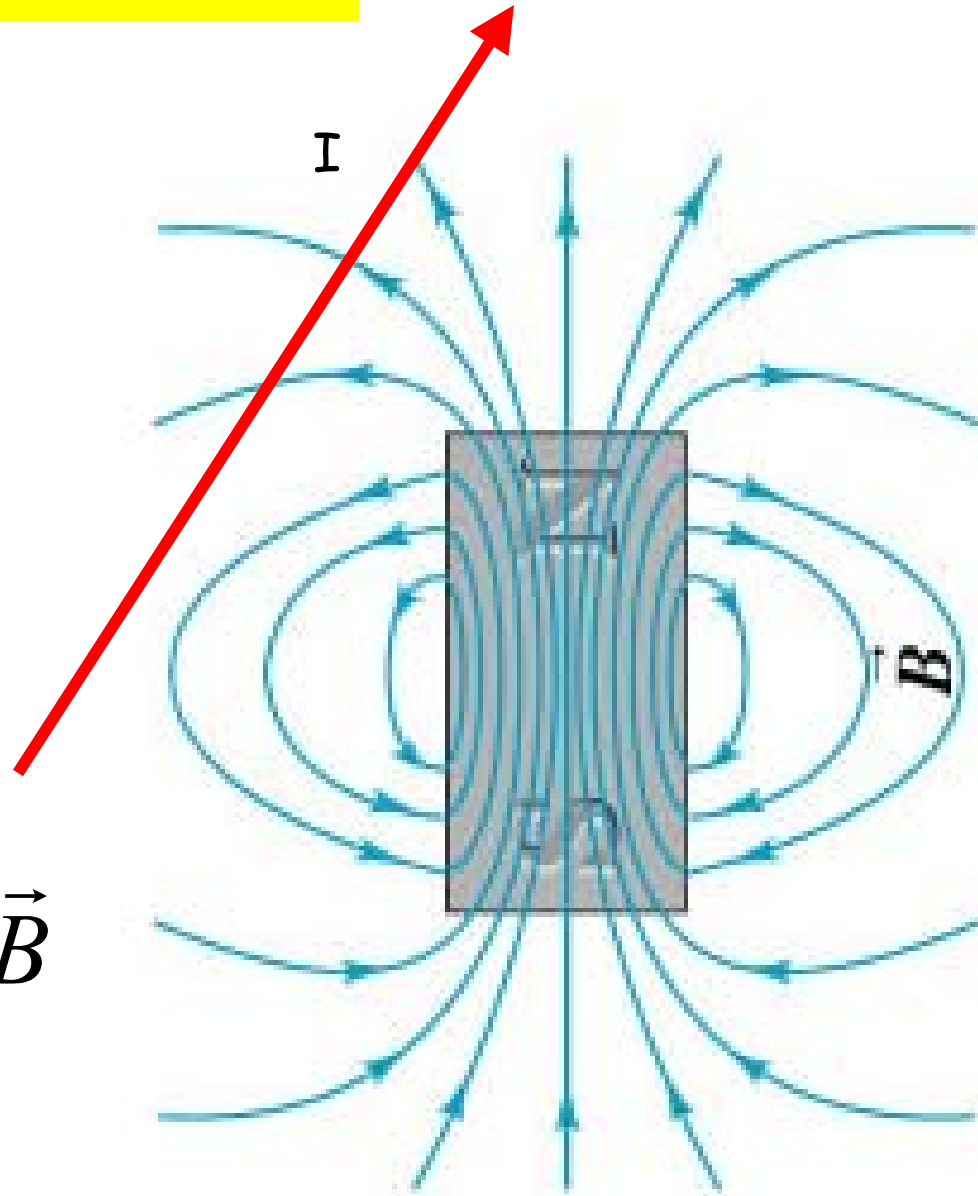
$$d\vec{F} = Id\vec{l} \times \vec{B}$$

More general.

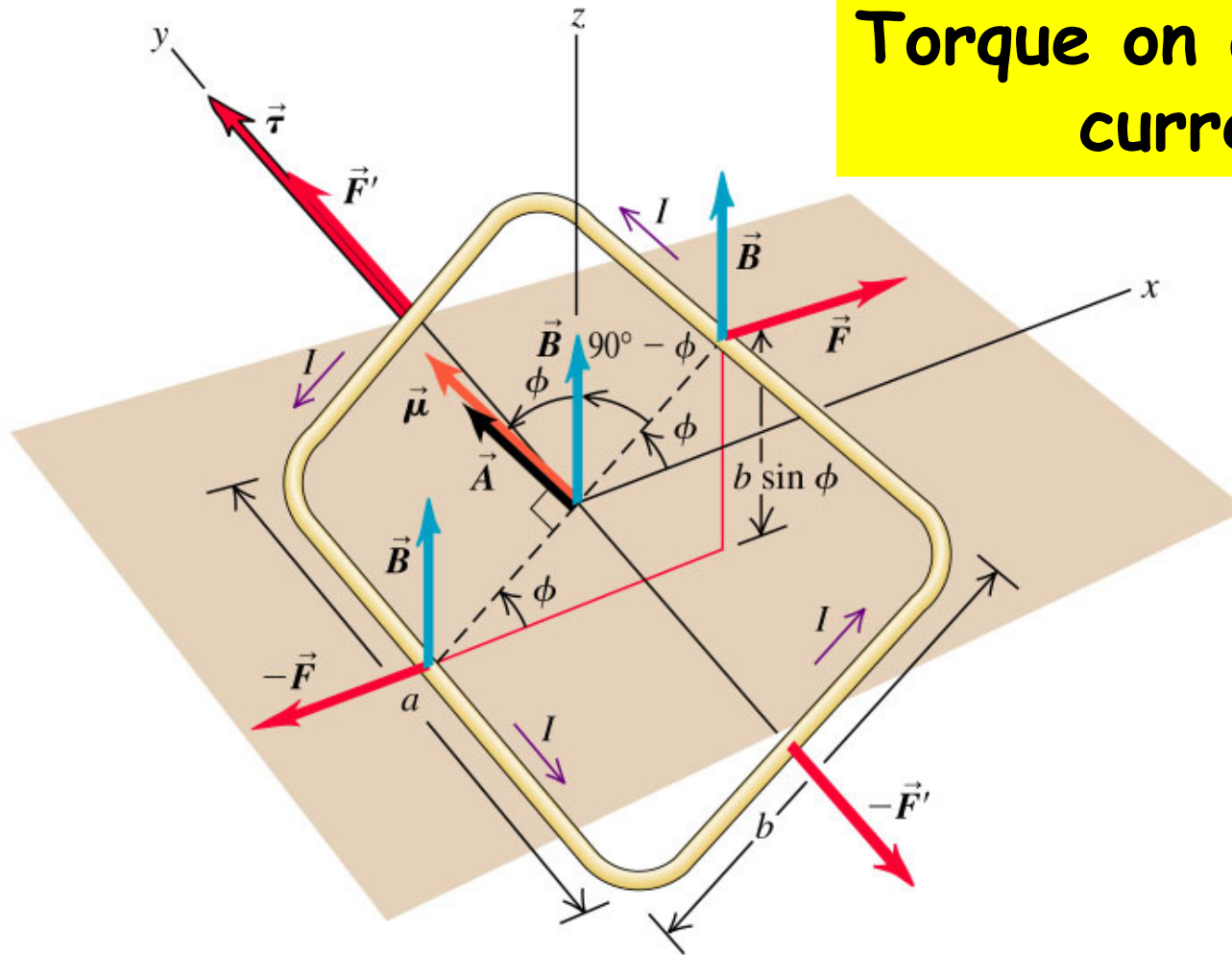
# Faraday's motor

Wire with current  
rotates around a  
Permanent magnet

$$d\vec{F} = I d\vec{l} \times \vec{B}$$



## Torque on a rectangular current loop



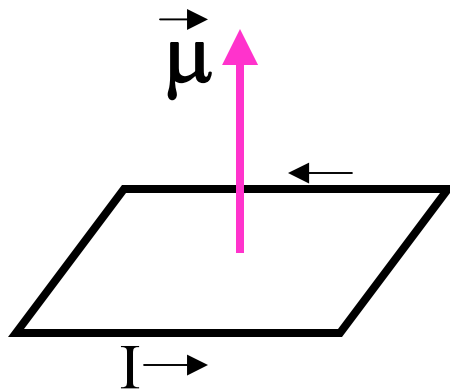
The loop will feel a torque which tends to rotate the loop, back into plane

Consider a wire loop dimensions  $a \times b$  whose plane is at an angle  $\phi$  relative to a constant  $B$  field. There will be a net torque whose magnitude on this loop is given by,

$$|\vec{\tau}| = IBab \sin \phi = IB(\text{area}) \sin \phi$$

# Magnetic Moment, $\mu$ , of a rectangular current loop

Definition ;  $\mu = \text{current} \times \text{area} = I A$



$\mu$  is vector quantity, whose direction is normal to loop plane, use right hand rule to define direction.

We can define vector torque more conveniently in terms of **vector magnetic moment** crossed by B field

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Another name for a current loop is “magnetic dipole”

# Derivation of $\vec{\tau} = \vec{\mu} \times \vec{B}$

Forces on wires of length  $b$  are collinear and cancel.

$$\vec{F} = I \vec{l} \times \vec{B}$$

Forces on wires of length  $a$  cancel **but are not collinear**.  
**There will be a torque.**

$$F = I |\vec{l} \times \vec{B}| = IaB$$

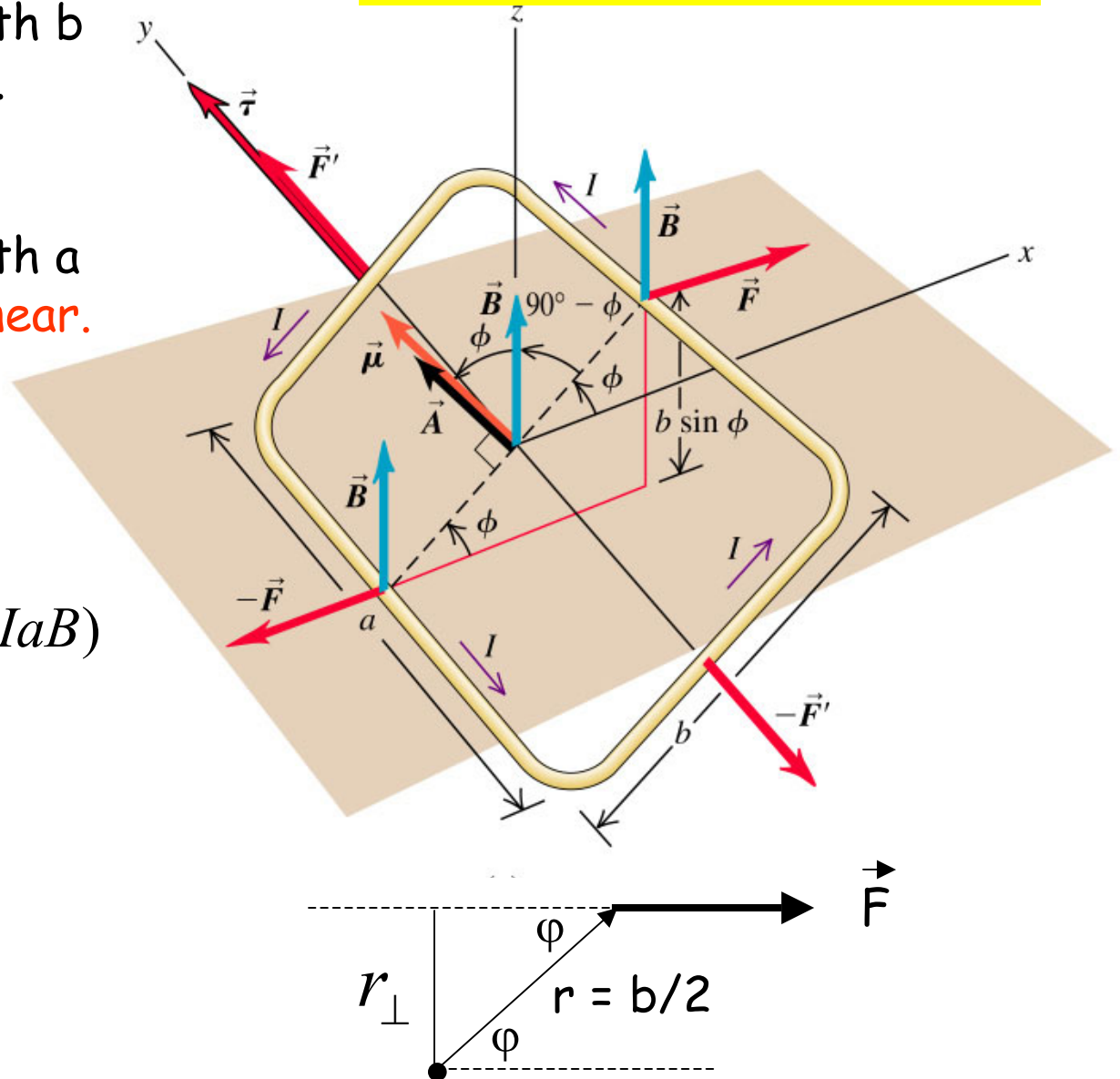
$$\tau = 2r_{\perp} F = 2\left(\frac{b}{2} \sin \phi\right)(IaB)$$

$$|\vec{\tau}| = IB(ab) \sin \phi$$

$$|\vec{\tau}| = \mu B \sin \phi = |\vec{\mu} \times \vec{B}|$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

## Torque on a rectangular current loop

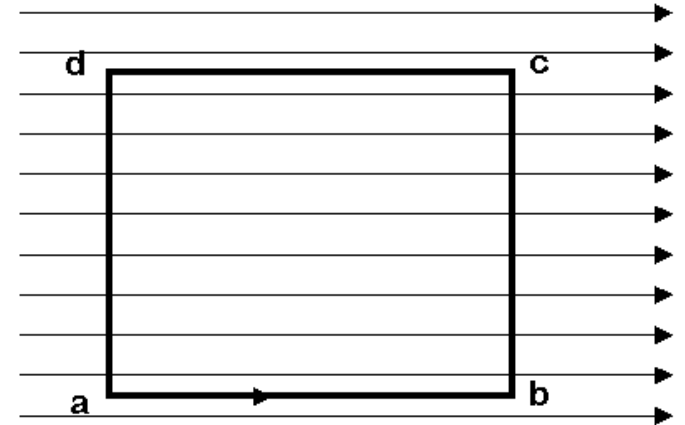




### Question 1:

$$\vec{F} = I\vec{L} \times \vec{B}$$

A square loop of wire is carrying current in the counterclockwise direction. There is a horizontal uniform magnetic field pointing to the right.



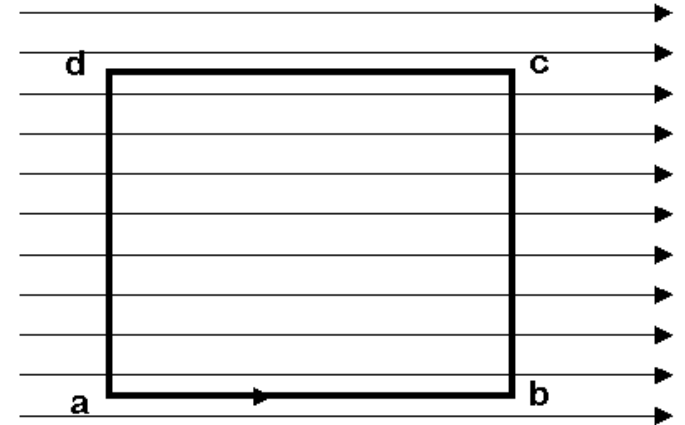
What is the force on section a-b of the loop?

- a) zero      b) out of the page      c) into the page

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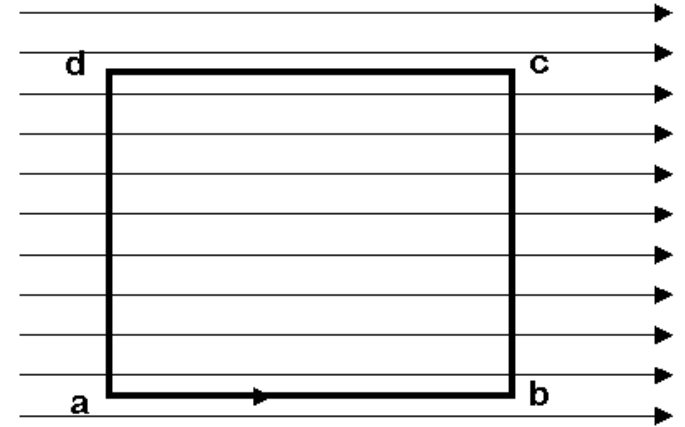
c) into the page

ab:  $F_{ab} = 0 = F_{cd}$  since the wire is parallel to  $B$ .

## Question 2:

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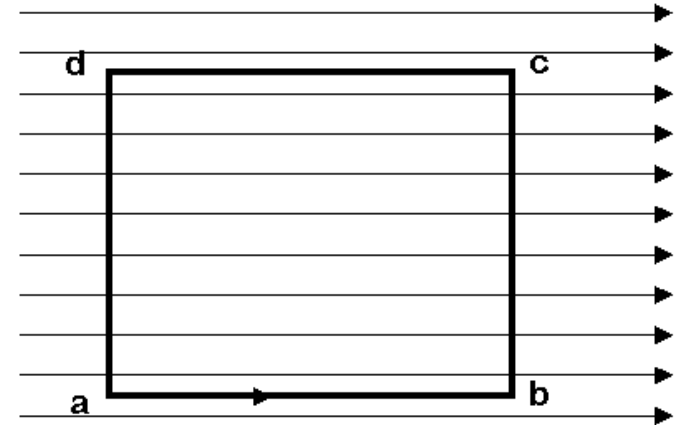
What is the force on section b-c of the loop?

- a) zero      b) out of the page      c) into the page

## Question 2:

$$\vec{F} = I\vec{L} \times \vec{B}$$

A square loop of wire is carrying current in the counterclockwise direction. There is a horizontal uniform magnetic field pointing to the right.



What is the force on section b-c of the loop?

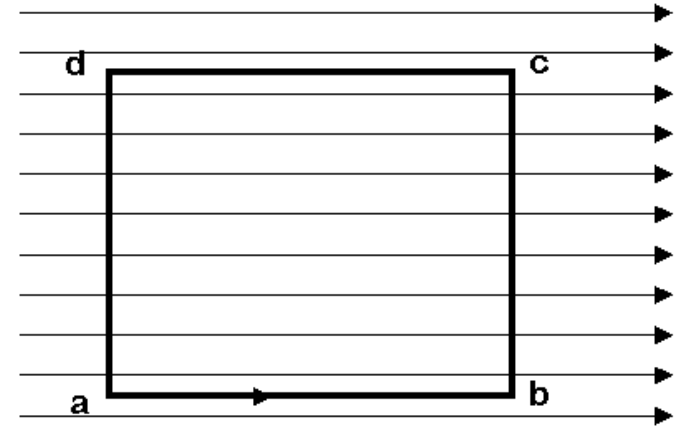
- a) zero      b) out of the page      c) into the page

bc:  $F_{bc} = ILB$  RHR:  $I$  is up,  $B$  is to the right, so  $F$  points into the screen.

### Question 3:

$$\vec{F} = I\vec{L} \times \vec{B}$$

A square loop of wire is carrying current in the counterclockwise direction. There is a horizontal uniform magnetic field pointing to the right.



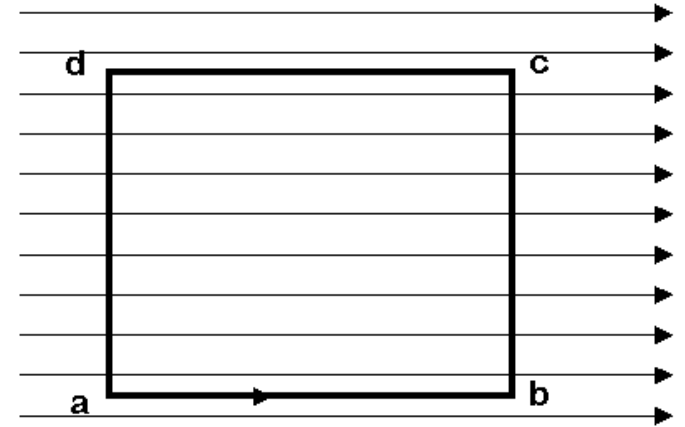
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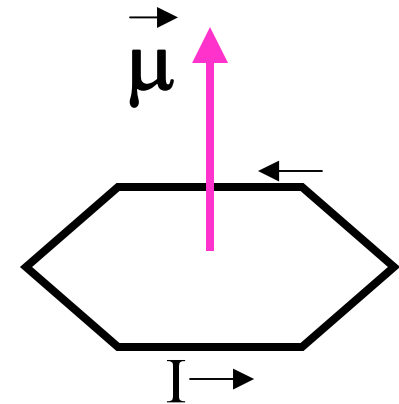
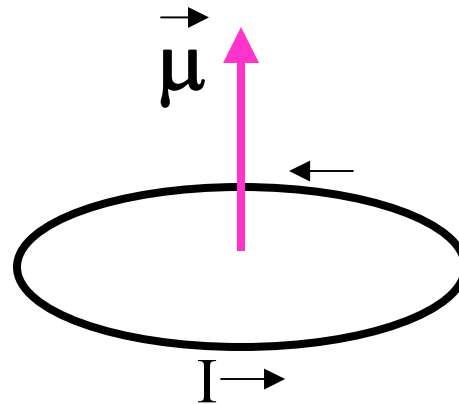
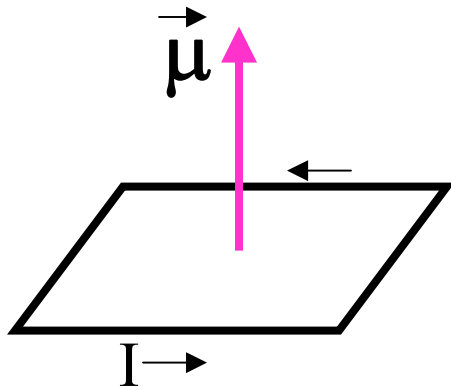
By symmetry:  $\vec{F}_{da} = -\vec{F}_{bc} \Rightarrow \vec{F}_{net} = \vec{F}_{ab} + \vec{F}_{bc} + \vec{F}_{cd} + \vec{F}_{da} = 0$

# Magnetic Moment, $\mu$ , of arbitrary loop

Definition ;  $\mu = \text{current} \times \text{area} = I A$

We can more generally define magnetic moments only by area and do not need to know the actual dimensions.

This is  
general

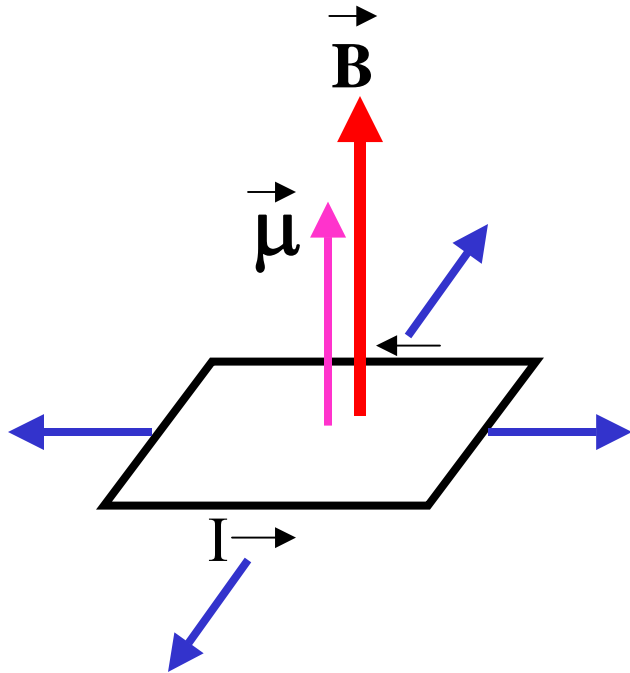


For any shape,  $\mu = \text{current} \times \text{area} = I A$   
and

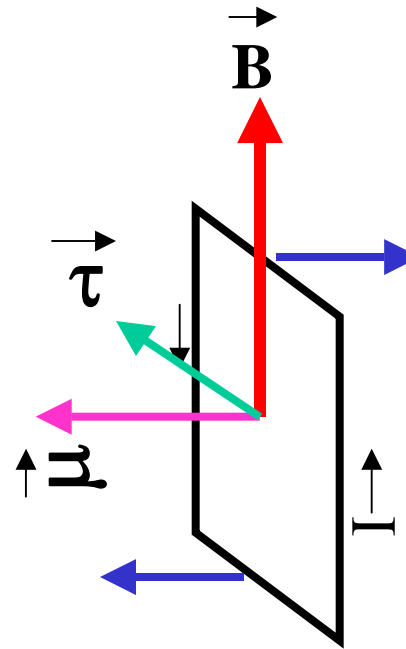
$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Note: if loop consists of  $N$  turns,  $\mu = NAI$

# Torque on loop in different angular positions



Net torque is **zero** when  $B$  is parallel to  $\mu$

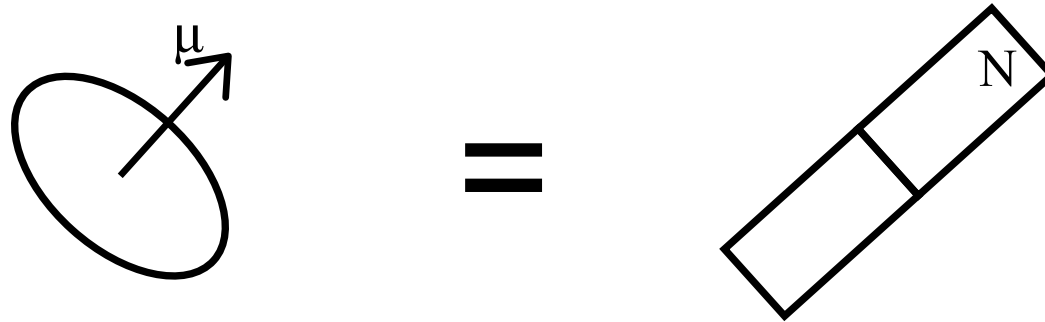


Net torque is **maximum** when  $B$  is perpendicular to  $\mu$



# Bar Magnet Analogy

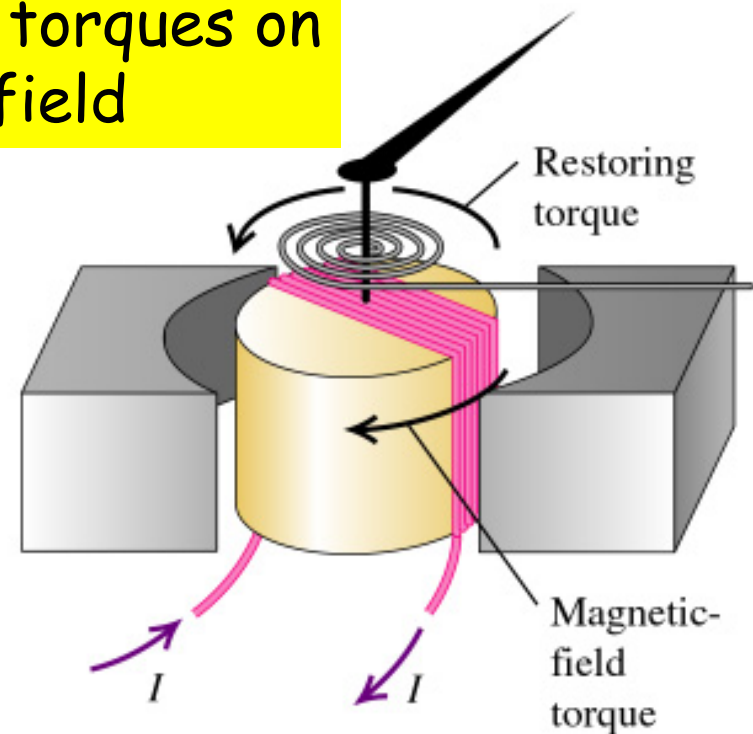
- You can think of a magnetic dipole moment as a bar magnet:



- In a magnetic field they both experience a torque trying to line them up with the field
  - As you increase  $I$  of the loop  $\rightarrow$  stronger bar magnet
- We will see that such a current loop does produce magnetic fields, similar to a bar magnet.

## Application; galvanometer uses torques on coiled loops in magnetic field

Torque is produced about the needle axis and this counteracts the restoring spring and enables the needle to rotate



Current increased

→  $\mu = I \cdot \text{Area}$  increases

→ Torque from  $\mathbf{B}$  increases

→ Angle of needle increases

Current decreased

→  $\mu$  decreases

→ Torque from  $\mathbf{B}$  decreases

→ Angle of needle decreases

This is how almost all dial meters work—voltmeters, ammeters, speedometers, RPMs, etc.

# Example. Loop in a B-Field

A circular loop has radius  $R = 5$  cm and carries current  $I = 2$  A in the counterclockwise direction. A magnetic field  $B = 0.5$  T exists in the negative  $z$ -direction. The loop is at an angle  $\theta = 30^\circ$  to the  $xy$ -plane.

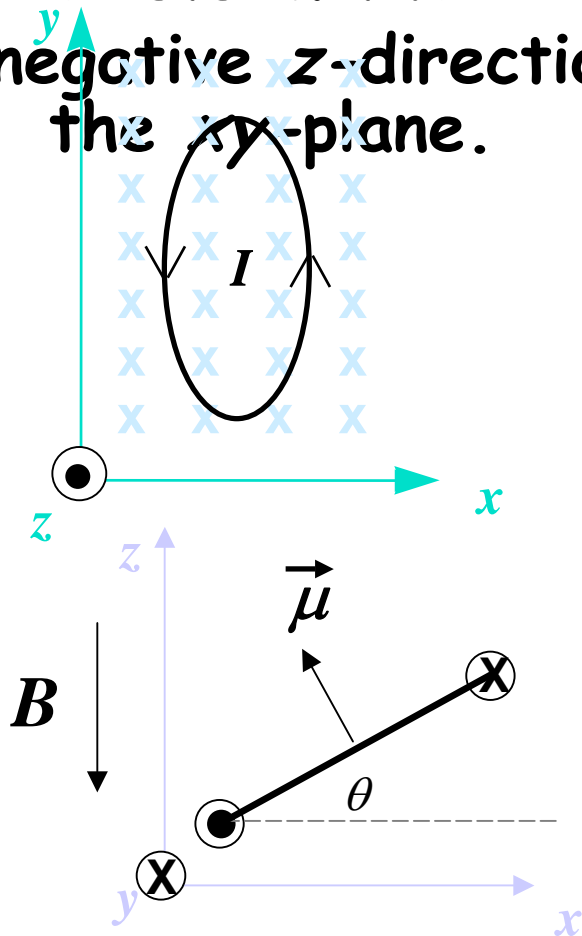
$$\mu = \pi r^2 I = .0157 \text{ Am}^2$$

The direction of  $\mu$  is perpendicular to the plane of the loop as in the figure.

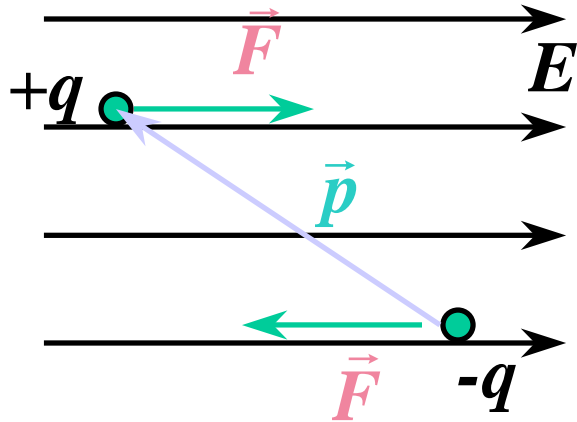
Find the  $x$  and  $z$  components of  $\mu$   $\Rightarrow$

$$\mu_x = -\mu \sin 30^\circ = -.0079 \text{ Am}^2$$

$$\mu_z = \mu \cos 30^\circ = .0136 \text{ Am}^2$$



# Electric Dipole Analogy

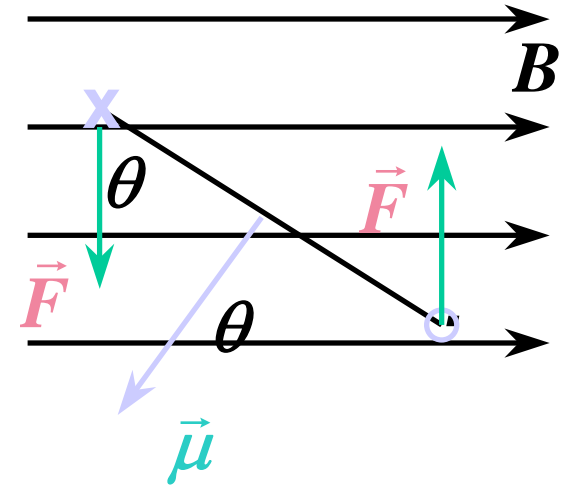


$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{F} = q\vec{E}$$

$$\vec{p} = 2q\vec{a}$$

$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$



$$\vec{\tau} = \vec{r} \times \vec{F}$$

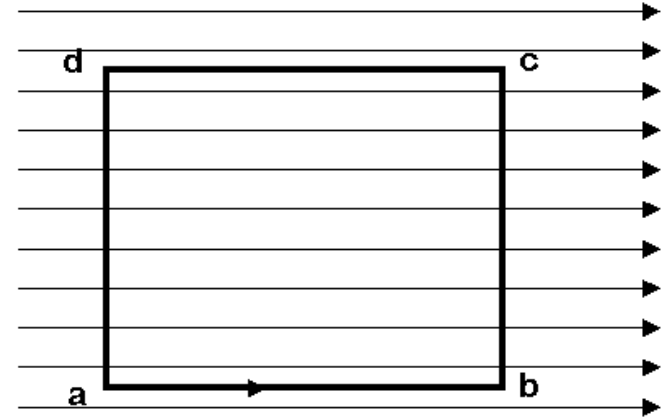
$$\vec{F} = I\vec{L} \times \vec{B} \text{ (per turn)}$$

$$\mu = NAI$$

$$\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$$

### Question 4:

A square loop of wire is carrying current in the counterclockwise direction. There is a horizontal uniform magnetic field pointing to the right.

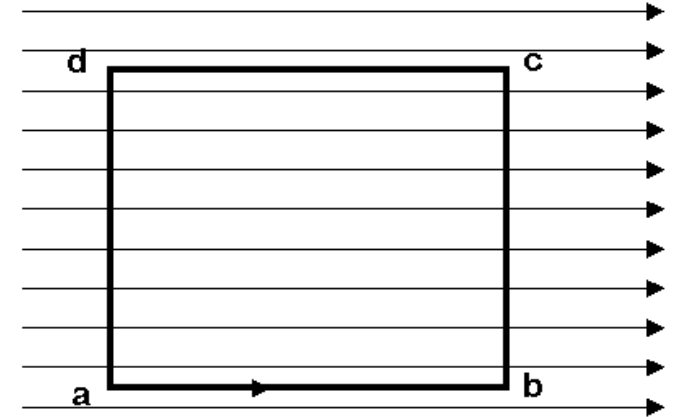


6) What is the net torque on the loop?

- a) zero    b) up    c) down    d) out of the page    e) into the page

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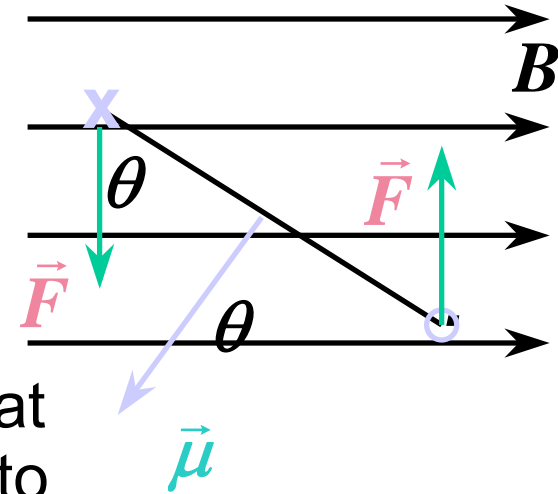
6) What is the net torque on the loop?

- a) zero   **b) up**   c) down   d) out of the page   e) into the page

$\vec{\tau} = \vec{\mu} \times \vec{B}$   $\mu$  points out of the page (curl your fingers in the direction of the current around the loop, and your thumb gives the direction of  $\mu$ ). Use the RHR to find the direction of  $\tau$  to be up.

# Potential Energy of Dipole

- **Work must be done to change the orientation of a dipole (current loop) in the presence of a magnetic field.**
- Define a potential energy  $U$  (with zero at position of max torque) corresponding to this work.

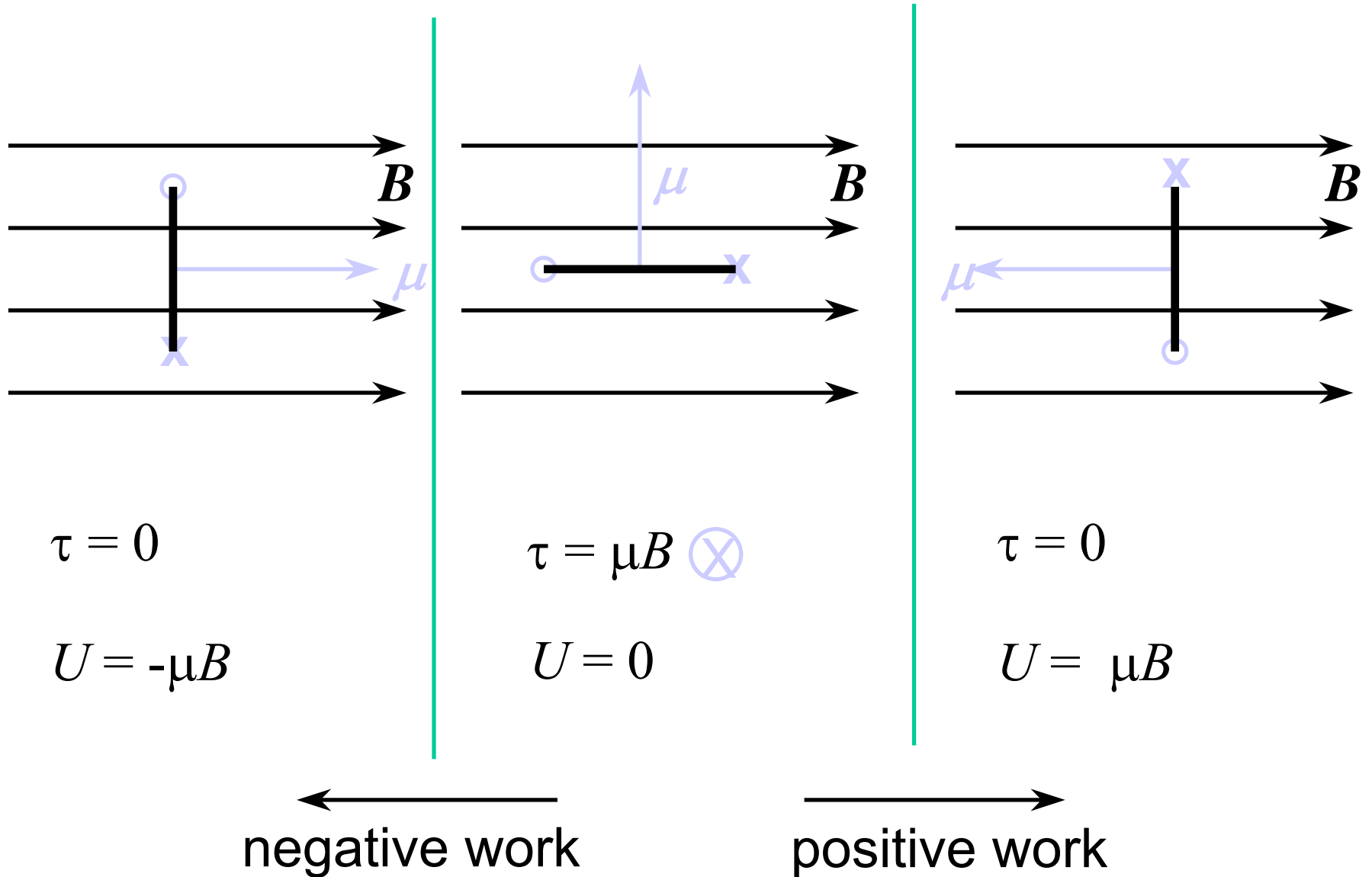


$$U \equiv \int_{90^\circ}^{\theta} \tau d\theta \quad \Rightarrow \quad U = \int_{90^\circ}^{\theta} \mu B \sin \theta d\theta$$

Therefore,

$$U = \mu B [-\cos \theta]_{90^\circ}^{\theta} \quad \Rightarrow \quad U = -\mu B \cos \theta \quad \Rightarrow \quad \boxed{U = -\vec{\mu} \cdot \vec{B}}$$

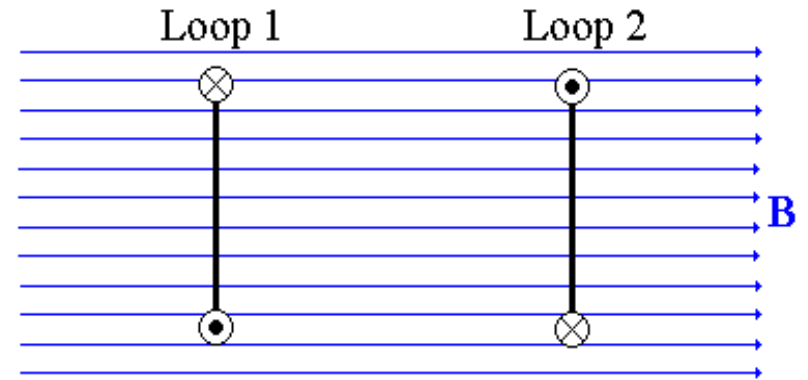
# Potential Energy of Dipole





### Question 5:

Two current carrying loops are oriented in a uniform magnetic field. The loops are nearly identical, except the direction of current is reversed.

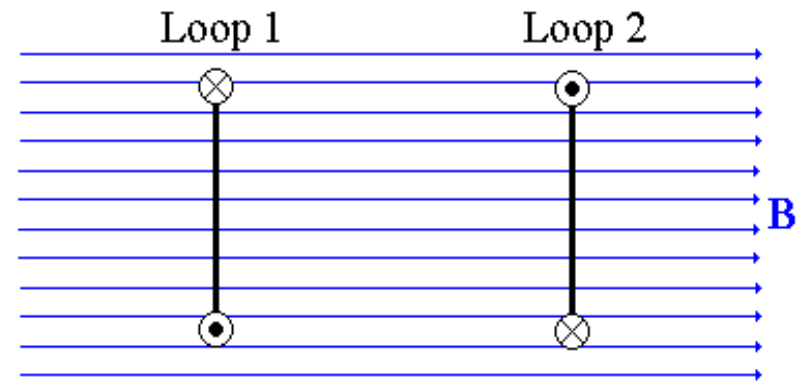


What is the torque on loop 1?

- a) clockwise      b) counter-clockwise      c) zero

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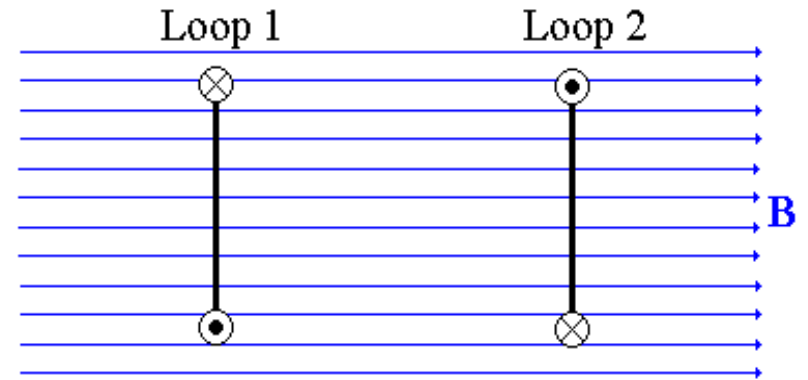
What is the torque on loop 1?

- a) clockwise      b) counter-clockwise      c) zero

Loop 1:  $\mu$  points to the left, so the angle between  $\mu$  and  $B$  is equal to  $180^\circ$ , hence  $\tau=0$ .

### Question 6:

Two current carrying loops are oriented in a uniform magnetic field. The loops are nearly identical, except the direction of current is reversed.



How does the torque on the two loops compare?

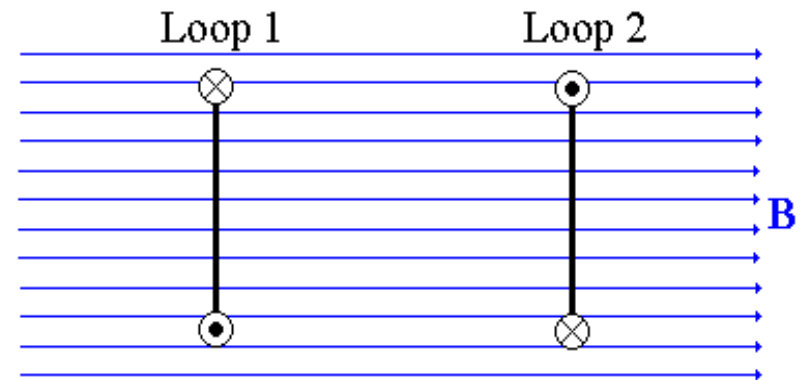
a)  $\tau_1 > \tau_2$

b)  $\tau_1 = \tau_2$

c)  $\tau_1 < \tau_2$

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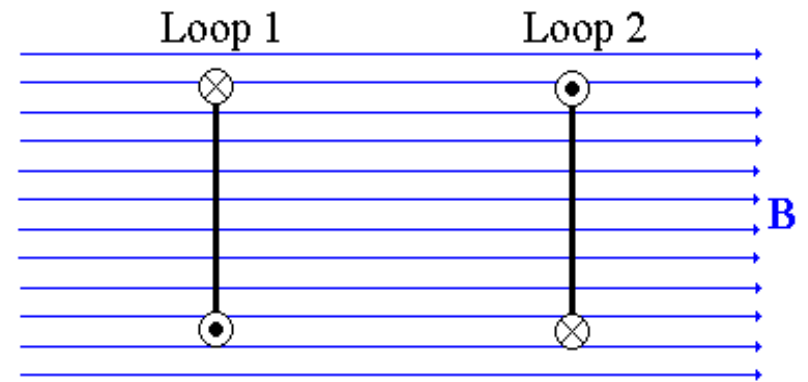
b)  $\tau_1 = \tau_2$

c)  $\tau_1 < \tau_2$

Loop 2:  $\mu$  points to the right, so the angle between  $\mu$  and  $B$  is equal to  $0^\circ$ , hence  $\tau = 0$ .

### Question 7:

Two current carrying loops are oriented in a uniform magnetic field. The loops are nearly identical, except the direction of current is reversed.



Which loop occupies a potential energy minimum, and is therefore stable?

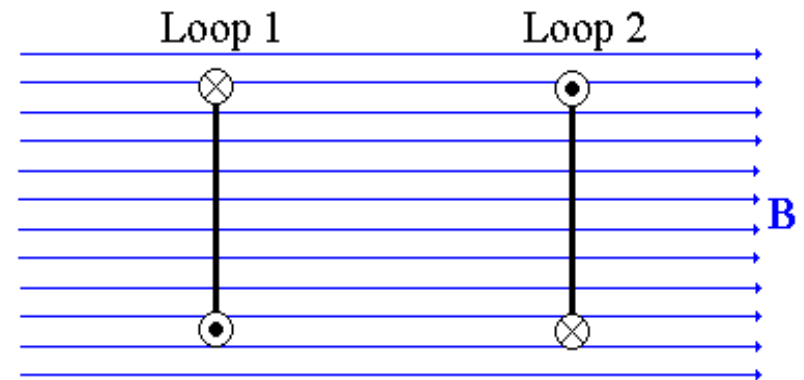
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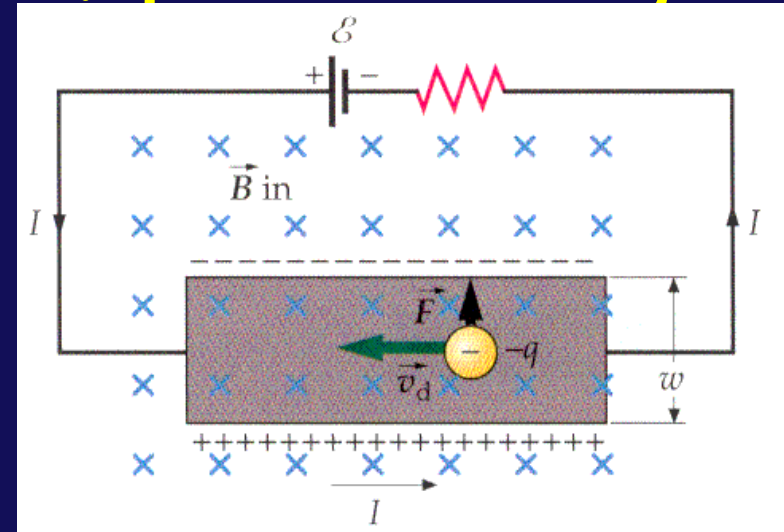
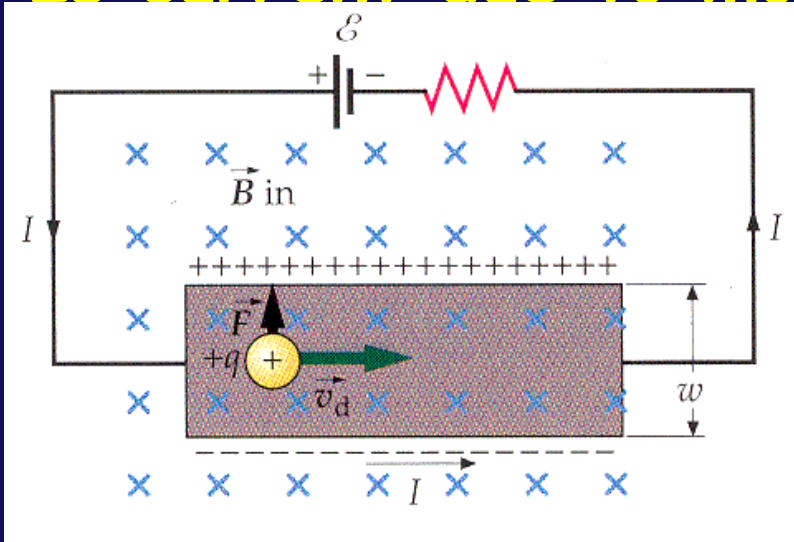
$$U = -\vec{\mu} \bullet \vec{B}$$

$$\text{Loop 1: } U_1 = +\mu B$$

$$\text{Loop 2: } U_2 = -\mu B \Rightarrow U_2 \text{ is a minimum.}$$

# The Hall Effect

- Is current due to motion of positive or negative



- Positive charges moving CCW experience upward force

Equilibrium between electrostatic & magnetic forces:

- Upper plate at higher potential

$$F_{\text{up}} = qv_{\text{drift}}B$$

$$F_{\text{down}} = qE_{\text{induced}} = q \frac{V_H}{w}$$

- Negative charges moving clockwise experience upward force

- Upper plate at lower potential

$$V_H = v_{\text{drift}}Bw = \text{"Hall Voltage"}$$

- This type of experiment led to the discovery (E. Hall, 1879) that current in conductors is carried by negative charges (not always so in semiconductors).
- Can be used as a B-sensor.

# For next time

- HW #7 → due Monday
- Well into Magnetism – keep up on reading
- Decision time approaching

