Course Updates

http://www.phys.hawaii.edu/~varner/PHYS272-Spr10/physics272.html

Notes for today:

1) Complete Chap 26, Problem session Friday
2) Assignment 6 (Mastering Physics) online and separate, written problems due Monday
3) Quiz 3 on Friday
4) Review Midterm 1
Kirchhoff’s Rules

Kirchhoff’s rules are statements used to solve for currents and voltages in complicated circuits. The rules are

Rule I. Sum of currents into any junction is zero.
\[ \sum I_i = 0 \]

Why? Since charge is conserved.

Rule II. Sum of potential differences in any loop is zero. (This includes emfs)
\[ \sum V_i = 0 \]

Why? Since potential (energy) is conserved.
The diagram illustrates an RC circuit with a capacitor and a resistor, showing the charge $q$ over time $t$. The charge $q$ is given by the equation:

$$q = C\varepsilon \left( 1 - e^{-t/RC} \right)$$
RC Circuits (Time-varying currents, charging)

- Charge capacitor:
  - $C$ initially uncharged; connect switch to $a$ at $t=0$
  - Calculate current and charge as function of time.

- Loop theorem $\Rightarrow$
  - $\varepsilon - IR - \frac{Q}{C} = 0$
  - Would it matter where $R$ is placed in the loop??

- Convert to differential equation for $Q$:
  - $I = \frac{dQ}{dt} \Rightarrow \varepsilon = R\frac{dQ}{dt} + \frac{Q}{C}$
Charging Capacitor

- Charge capacitor:
  \[ \varepsilon = R \frac{dQ}{dt} + \frac{Q}{C} \]

- Guess solution:
  \[ Q = C \varepsilon (1 - e^{-t/RC}) \]

- Check that it is a solution:
  \[ \frac{dQ}{dt} = C \varepsilon e^{-t/RC} \left( -\frac{1}{RC} \right) \]
  \[ R \frac{dQ}{dt} + \frac{Q}{C} = -\varepsilon e^{-t/RC} + \varepsilon (1 - e^{-t/RC}) = \varepsilon \]

Note that this “guess” fits the boundary conditions:

\( t = 0 \Rightarrow Q = 0 \)

\( t = \infty \Rightarrow Q = C \varepsilon \)
Charging Capacitor

- Charge capacitor:
  \[ Q = C\varepsilon \left(1 - e^{-t/RC}\right) \]

- Current is found from differentiation:
  \[ I = \frac{dQ}{dt} = \frac{\varepsilon}{R} e^{-t/RC} \]

Conclusion:
- Capacitor reaches its final charge \( Q = C\varepsilon \) exponentially with time constant \( \tau = RC \).
- Current decays from max \( = \varepsilon /R \) with same time constant.
Charging Capacitor

**Charge on** $C$

$$Q = C \varepsilon \left(1 - e^{-t/RC}\right)$$

Max $= C \varepsilon$

63% Max at $t = RC$

**Current**

$$I = \frac{dQ}{dt} = \frac{\varepsilon}{R} e^{-t/RC}$$

Max $= \varepsilon / R$

37% Max at $t = RC$
Discharging Capacitor

\[ R \frac{dQ}{dt} + \frac{Q}{C} = 0 \]

- **Guess solution:**
  \[ Q = Q_0 e^{-t/\tau} = C \varepsilon e^{-t/RC} \]

- **Check that it is a solution:**
  \[ \frac{dQ}{dt} = C \varepsilon e^{-t/RC} \left(-\frac{1}{RC}\right) \]
  \[ \Rightarrow R \frac{dQ}{dt} + \frac{Q}{C} = -\varepsilon e^{-t/RC} + \varepsilon e^{-t/RC} = 0 \]

Note that this “guess” fits the boundary conditions:
- \( t = 0 \Rightarrow Q = C \varepsilon \)
- \( t = \infty \Rightarrow Q = 0 \)
Conclusion:

- Capacitor discharges exponentially with time constant $\tau = RC$
- Current decays from initial max value ($= -\frac{\mathcal{E}}{R}$) with same time constant.

Discharging Capacitor

Discharge capacitor:

$$Q = Q_0 e^{-t/\tau} = C \mathcal{E} e^{-t/RC}$$

Current is found from differentiation:

$$I = \frac{dQ}{dt} = -\frac{\mathcal{E}}{R} e^{-t/RC}$$

Minus sign: Current is opposite to original definition, i.e., charges flow away from capacitor.
Discharging Capacitor

**Charge on** \( C \)

\[ Q = C \epsilon e^{-t/RC} \]

Max = \( C \epsilon \)

37% Max at \( t = RC \)

**Current**

\[ I = \frac{dQ}{dt} = -\frac{\epsilon}{R} e^{-t/RC} \]

“Max” = \(-\epsilon/R\)

37% Max at \( t = RC \)
Midterm 1 statistics

Entries: 69
Mean: 60.78
RMS: 16.92

PHYS272 Spr. 10
Cumulative statistics

PHYS272 Spr. 10

Midterm 1 stats

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Midterm Review

• Not many systematic, consistent problems

• However, some recurrent themes:
  – Electric Potential (V) vs. Potential Energy (U)
  – Electric field sums as vector quantity
  – Potential field (V) sums as a scaler
  – Area of a circle is $\pi r^2$

• Detailed points breakdown (by roster ID) linked from course webpage

• Will go over in outline form problems next…
For next time

• HW #6 Assigned ➔ due next Monday

• Quiz #3 on Friday

• Problem session Friday prior to quiz