Course Updates

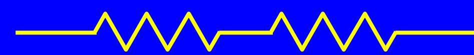
http://www.phys.hawaii.edu/~varner/PHYS272-Spr10/physics272.html

Notes for today:

1) Chapter 26 this week (DC, RC circuits)
2) Assignment 6 (Mastering Physics) online and separate, written problems due next Monday
3) Review Midterm 1 Wednesday
4) Quiz #3 on Friday

A Reminder

Resistors in series:

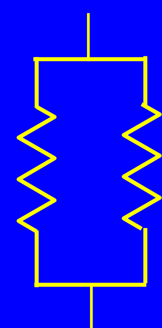


Current through is same.

Voltage drop across is IR_i

$$R_{effective} = R_1 + R_2 + R_3 + \ldots$$

Resistors in parallel:

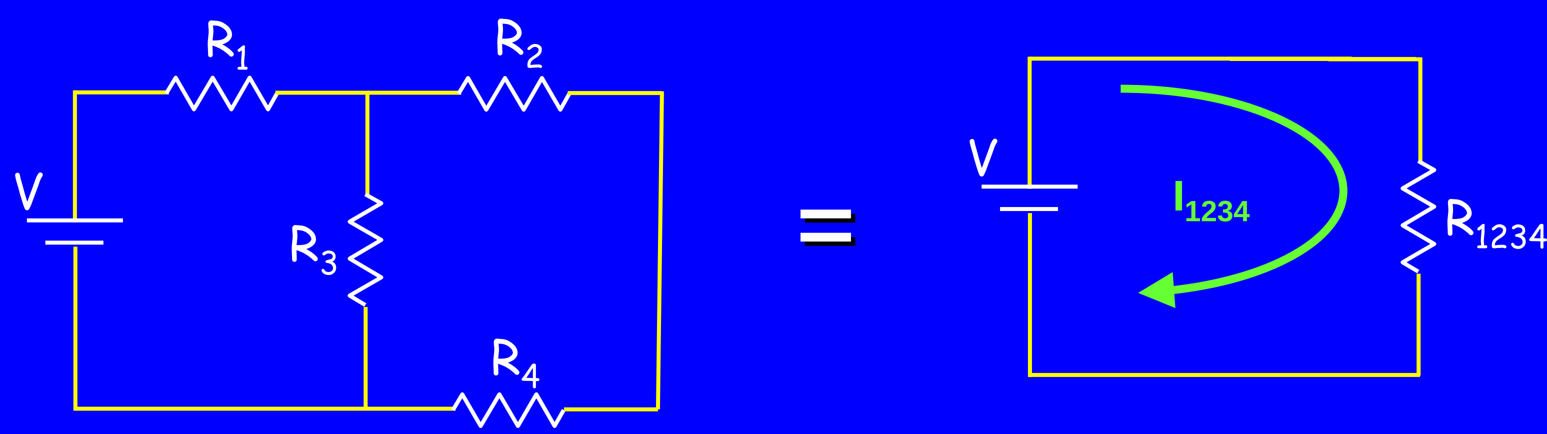


Voltage drop across is same.

Current through is V/R_i

$$\frac{1}{R_{effective}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

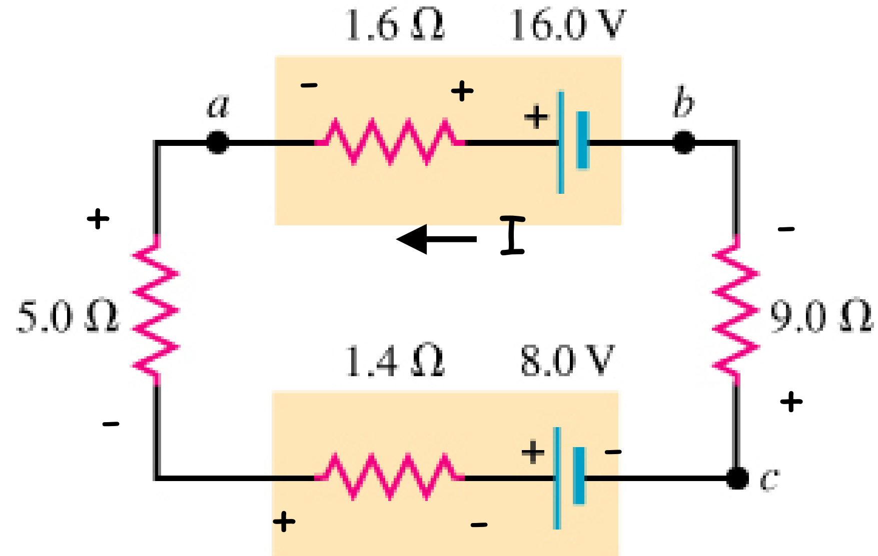
Solved Circuits



Y&F 25.36

The circuit shown in the figure contains two batteries, each with an emf and an internal resistance, and two resistors.

A) Find the direction and magnitude of the current in the circuit
B) Find the terminal voltage Vab of the 16.0-V battery.
C) Find the potential difference Vbc of point b with respect to point c .



A.) Use Kirchoff's loop law:

$$16V - 1.6\Omega I - 5.0\Omega I - 1.4\Omega I - 8V - 9.0\Omega I = 0$$

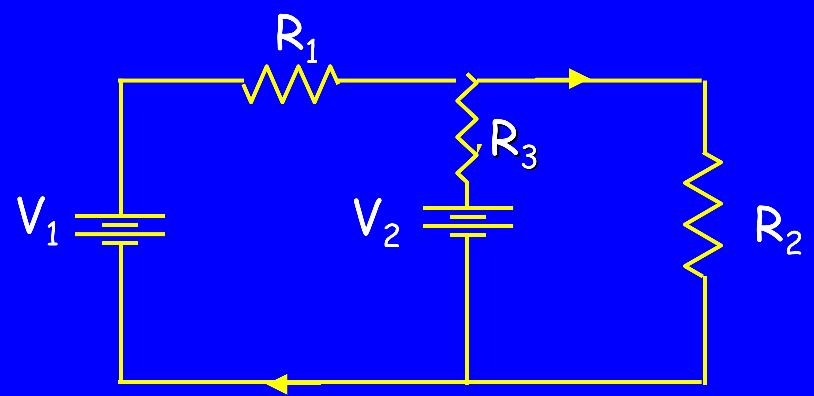
$$8V - 17\Omega I = 0$$

$$I = \frac{8}{17}A = 0.471A$$

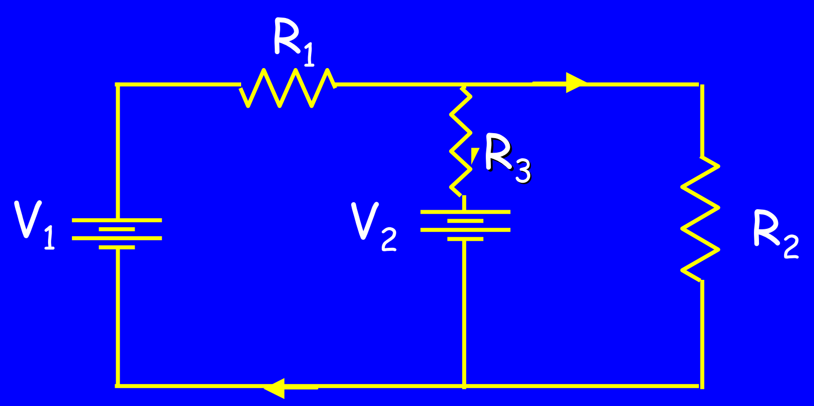
B.) V_{ab} = 16V – I(1.6 Ω) = 16V – (0.471 A)(1.6 Ω) = 15.2 V

C.) V_{bc} = – I(9.0 Ω) = – (0.471 A)(9.0 Ω) = -4.24 V

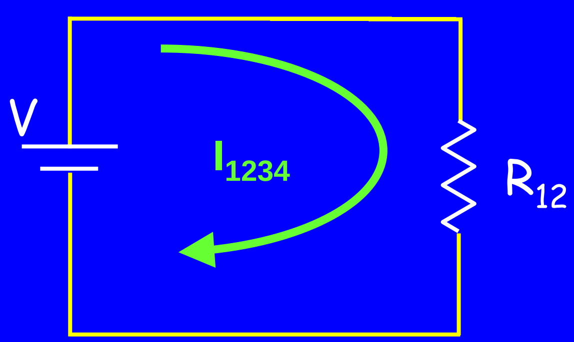
New Circuit



How Can We Solve This One?



$\neq$



THE ANSWER: Kirchhoff's Rules

$$\sum \Delta V_i = 0$$

Kirchoff's Voltage Rule

Kirchoff's Voltage Rule states that the sum of the voltage changes caused by any elements (like wires, batteries, and resistors) around a circuit must be zero.

If we model voltage as height above the ground floor, see if you can come up with the analogy to Kirchoff's Voltage Rule in terms of someone walking around in the hallways and stairways and elevators of a high-rise building.

If we start at the ground floor with a potential of 0, and walk around up and down stairs, take the elevator a few flights, go to the roof, parachute off, and end up back on the ground floor, our potential is still 0. Therefore, the potential difference is 0. We may have increased and decreased our potential as we traveled through the building, but we still start and end at a potential of 0.

OR:

The potential difference between a point and itself is zero !

Kirchoff's Current Rule

$$\sum I_{in} = \sum I_{out}$$

Kirchoff's Current Rule states that the sum of all currents entering any given point in a circuit must equal the sum of all currents leaving the same point.

If we model electrical current as water, see if you can come up with an analogy to Kirchoff's Current Rule in terms of household plumbing.

If you have a main water line coming into your house, it will split off to service all utilities, such as sink, toilet, shower, etc. The water in all of those lines must equal the amount of water coming out of the main line, and when all the household water drains out of the house into a main line again, all the smaller lines must include as much water as the drain carries out.

OR:

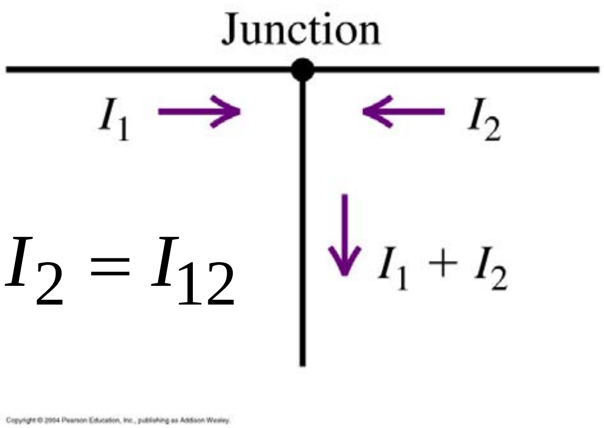
Electric Charge is Conserved

Kirchhoff's Rules

Kirchhoff's rules are statements used to solve for currents and voltages in complicated circuits. The rules are

Rule I. Sum of currents into any junction is zero.

$$\sum_i I_i = 0$$

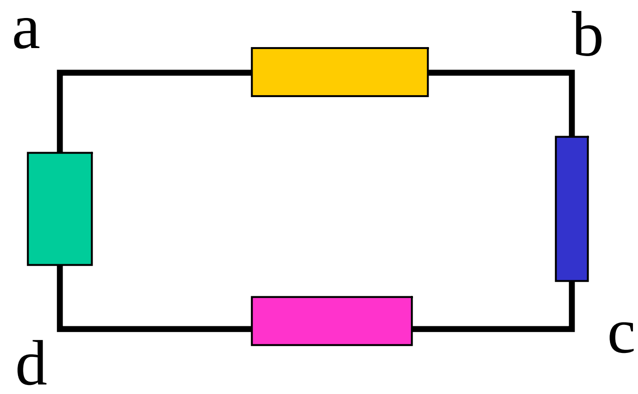


$$I_1 + I_2 = I_{12}$$

Why? Since charge is conserved.

Rule II. Sum of potential differences in any loop is zero. (This includes emfs)

$$\sum_i V_i = 0$$

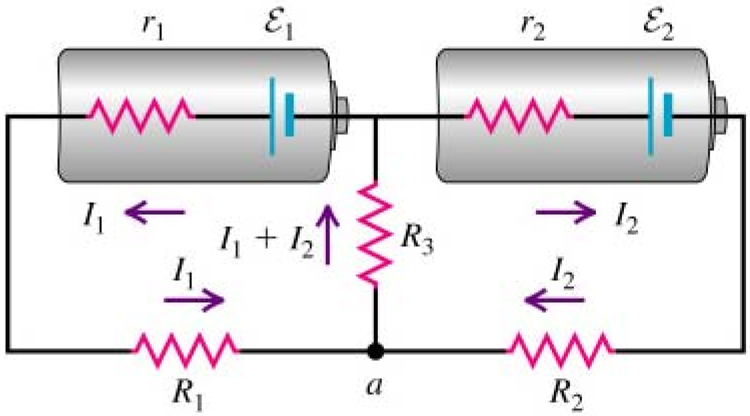


$$V_{ab} + V_{bc} + V_{cd} + V_{da} = 0$$

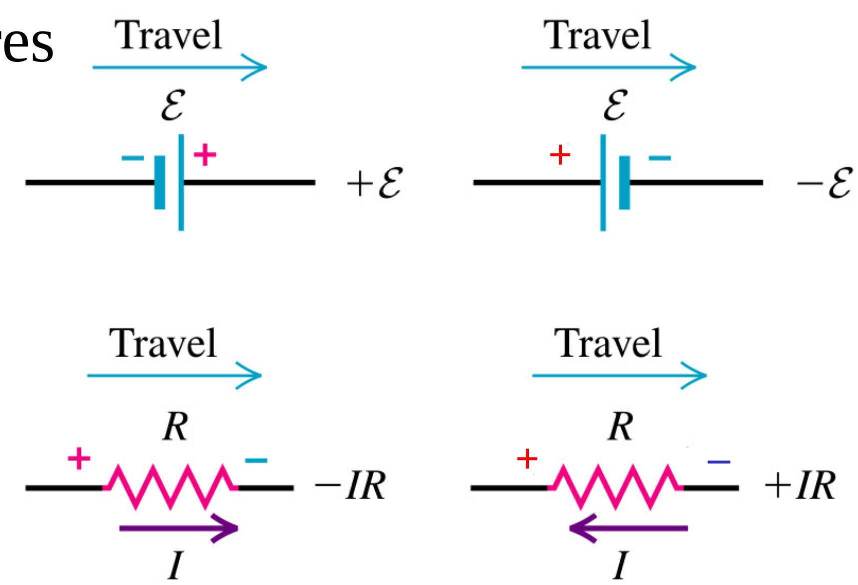
Why? Since potential (energy) is conserved

Kirchhoff's Rules

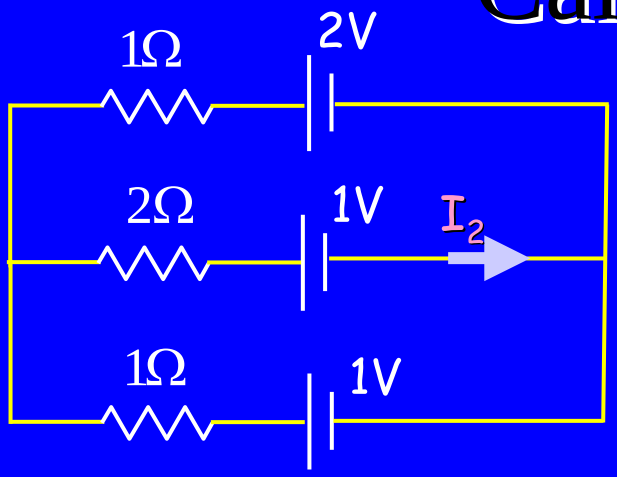
(1) Set up current directions. The current is the same along single path and at a junction the sum of 2 currents entering a junction equals the current exiting the junction.



(2) Getting potential differences requires setting up travel path through a loop, either clockwise or counter clockwise. Positive for current flow – to + across a battery and negative for flow + to -. For a resistor, negative voltage drop if travel & I in same direction and pos. voltage increase if travel & I opposite



Calculation

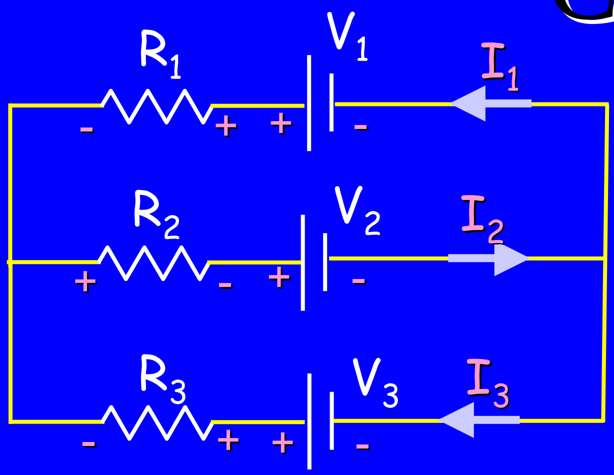


In this circuit, assume V_i and R_i are known.

What is I_2 ??

- Conceptual Analysis:
  - Circuit behavior described by Kirchhoff's Rules:
    - KVR: $\Sigma V_{drops} = 0$
    - KCR: $\Sigma I_{in} = \Sigma I_{out}$
- Strategic Analysis
  - Write down Loop Equations (KVR)
  - Write down Node Equations (KCR)
  - Solve

Calculation



In this circuit, assume V_i and R_i are known.

What is I_2 ??

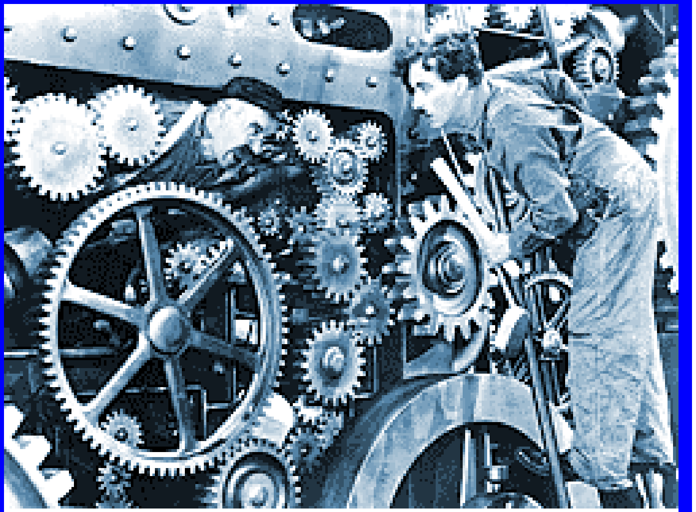
Label and pick directions for each current

Label the + and – side of each element

This is easy for batteries

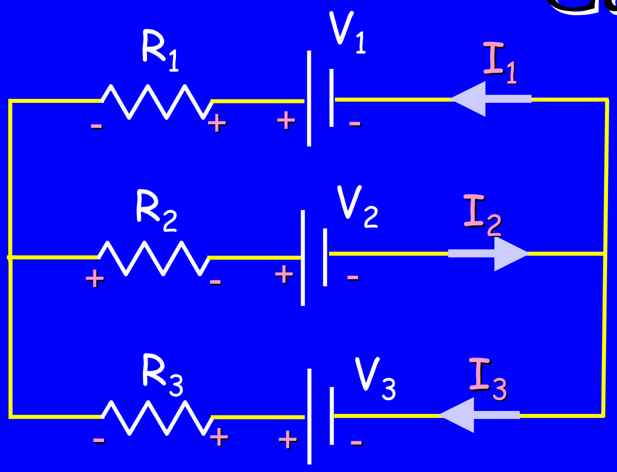
For resistors, the "upstream" side is +

Now write down loop and node equations



Just turn the crank.

Calculation 1

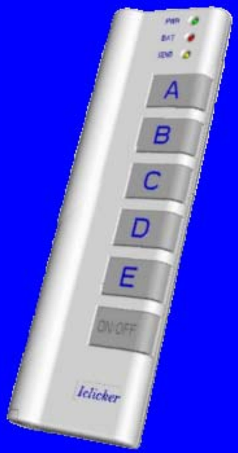


In this circuit, assume V_i and R_i are known.

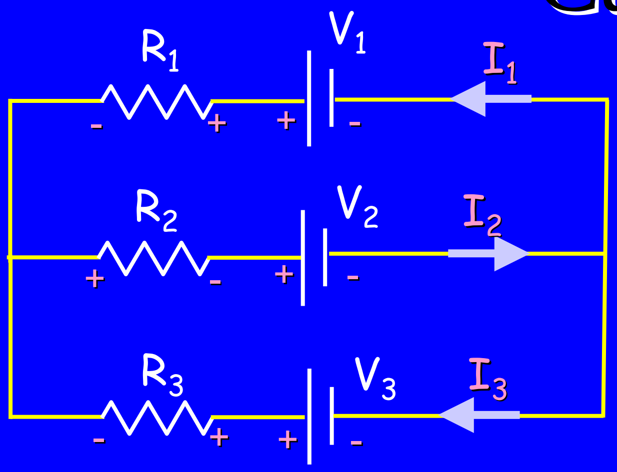
What is I_2 ??

- How many equations do we need to write down in order to solve for I_2?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5



Calculation 1



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What is I_2 ??

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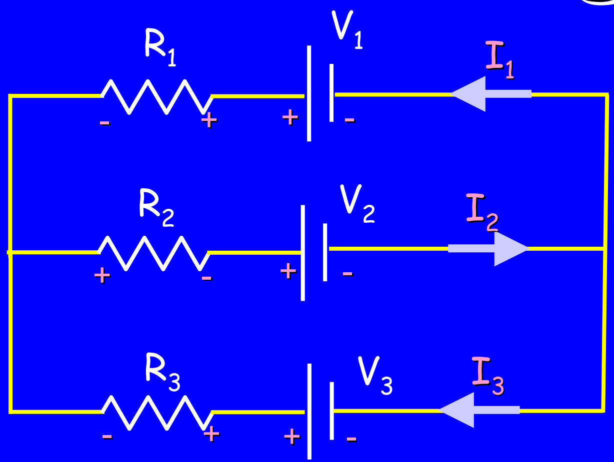
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

- Why??
  - We have 3 unknowns: I_1, I_2, and I_3
  - We need 3 independent equations to solve for these unknowns

Calculation 2

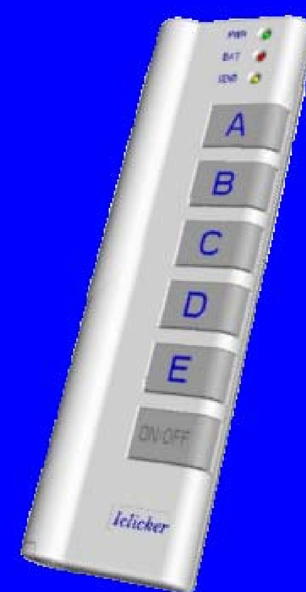


In this circuit, assume V_i and R_i are known.

What is I_2 ??

- Which of the following equations is NOT correct?

(A) $I_2 = I_1 + I_3$
(B) $-V_1 + I_1R_1 - I_3R_3 + V_3 = 0$
(C) $-V_3 + I_3R_3 + I_2R_2 + V_2 = 0$
(D) $-V_2 - I_2R_2 + I_1R_1 + V_1 = 0$



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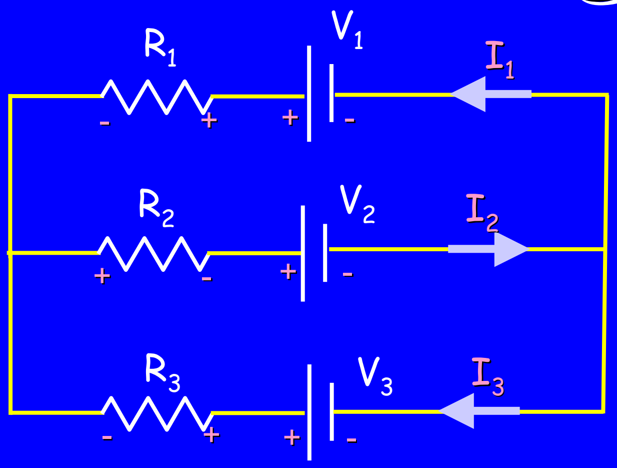

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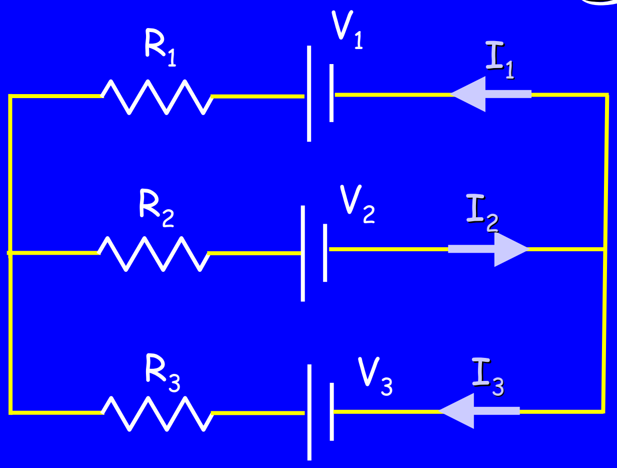
(C) $- V_3 + I_3R_3 + I_2R_2 + V_2 = 0$

(D) $- V_2 – I_2R_2 + I_1R_1 + V_1 = 0$

- Why??
  - (D) is an attempt to write down KVR for the top loop
  - Start at negative terminal of V_2 and go clockwise
    - V_{gain} $(-V_2)$ then V_{gain} $(-I_2R_2)$ then V_{gain} $(-I_1R_1)$ then V_{drop} $(+V_1)$

Calculation 3



In this circuit, assume V_i and R_i are known.

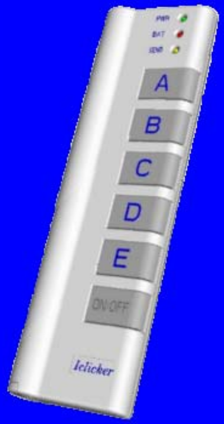
What is I_2 ??

- We have the following 4 equations:

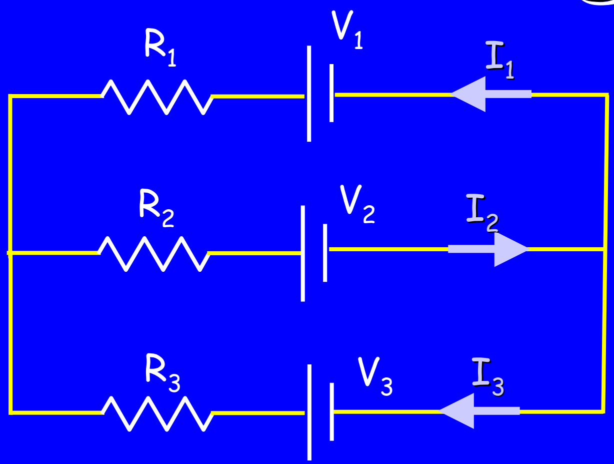
1. $I_2 = I_1 + I_3$
2. $- V_1 + I_1R_1 - I_3R_3 + V_3 = 0$
3. $- V_3 + I_3R_3 + I_2R_2 + V_2 = 0$
4. $- V_2 - I_2R_2 - I_1R_1 + V_1 = 0$

We need 3 equations:
Which 3 should we use?

A) Any 3 will do
B) 1, 2, and 4
C) 2, 3, and 4



Calculation 3



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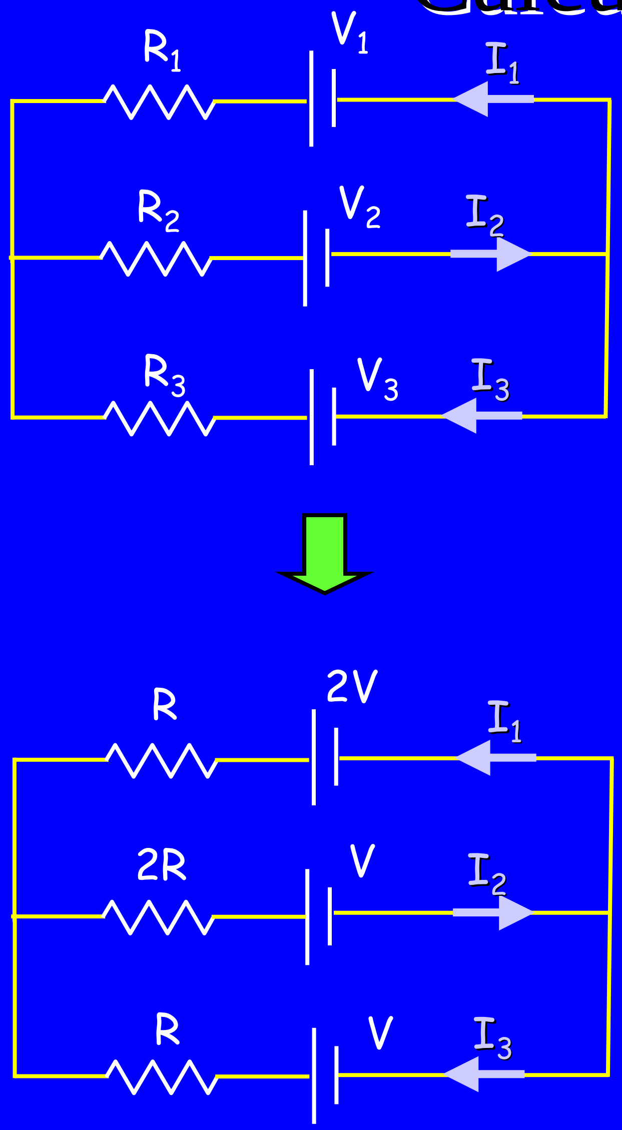
We need 3 equations:
Which 3 should we use?

A) Any 3 will do
B) 1, 2, and 4
C) 2, 3, and 4

EQ2 + EQ3
$-V_1 + I_1R_1 + I_2R_2 + V_2 = 0$

- Why??
  - We need 3 INDEPENDENT equations
  - Equations 2, 3, and 4 are NOT INDEPENDENT
    - Eqn 2 + Eqn 3 = - Eqn 4
  - We must choose Equation 1 and any two of the remaining (2, 3, and 4)

Calculation (cont'd)



In this circuit, assume V_i and R_i are known.

What is I_2 ??

- We have 3 equations and 3 unknowns.

$$I_2 = I_1 + I_3$$
$$V_1 + I_1R_1 - I_3R_3 + V_3 = 0$$
$$V_2 - I_2R_2 - I_1R_1 + V_1 = 0$$

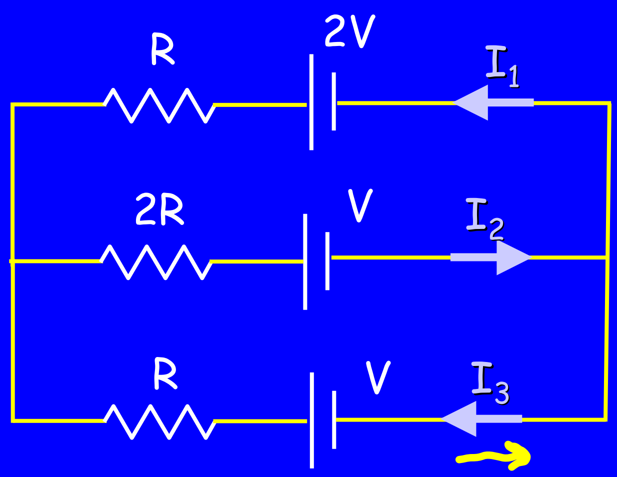
- The solution can get very messy!

Simplify: assume $V_2 = V_3 = V$

$$V_1 = 2V$$
$$R_1 = R_3 = R$$
$$R_2 = 2R$$

Calculation: Simplify

In this circuit, assume V and R are known. What is I_2 ??



- We have 3 equations and 3 unknowns.

$$I_2 = I_1 + I_3$$

$$-2V + I_1R - I_3R + V = 0 \quad \text{(outside)}$$

$$-V - I_2(2R) - I_1R + 2V = 0 \quad \text{(top)}$$

- With this simplification, you can verify:

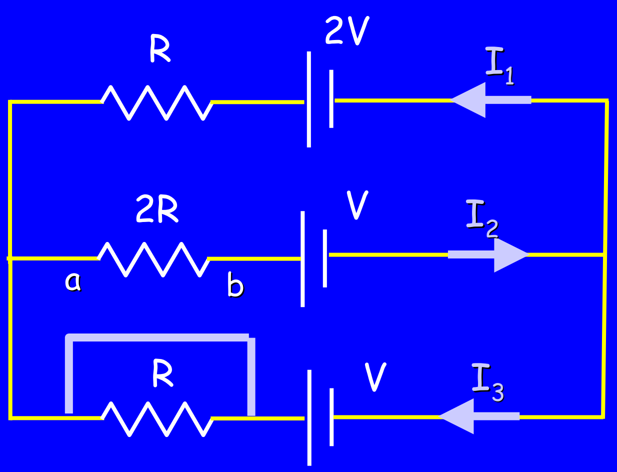
$$I_2 = (1/5)\ V/R$$

$$I_1 = (3/5)\ V/R$$

$$I_3 = (-2/5)\ V/R$$

$$\frac{1}{5} = 3/5 + (-2/5) = \frac{1}{5}$$

Follow-Up Check

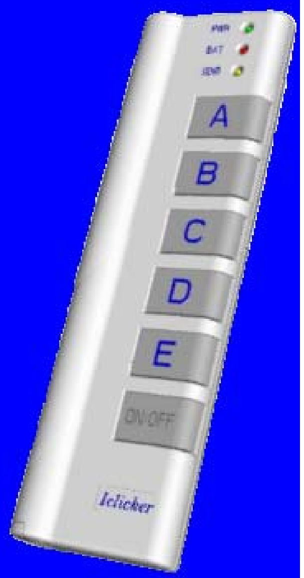


- We know:

  $I_2 = (\ 1/5)\ V/R$

  $I_1 = \ (\ 3/5)\ V/R$

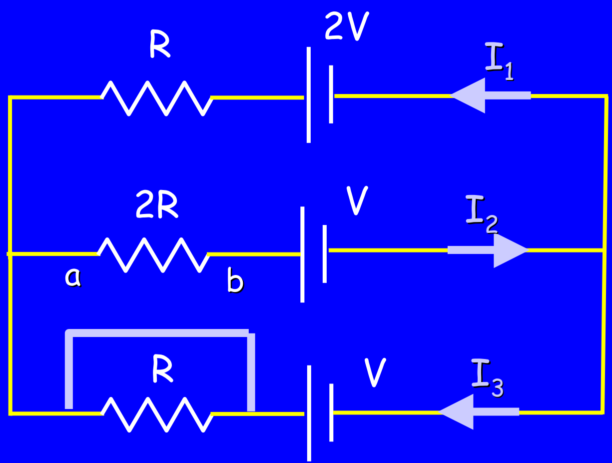
  $I_3 = (-2/5)\ V/R$



- Suppose we short R_3: What happens to V_{ab} (voltage across R_2?)

(A) V_{ab} remains the same
(B) V_{ab} changes sign
(C) V_{ab} increases
(D) V_{ab} goes to zero

Follow-Up Check



- We know:

  $I_2 = (1/5)\ V/R$

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- Suppose we short R_3: What happens to V_{ab} (voltage across R_2?)

(A) V_{ab} remains the same

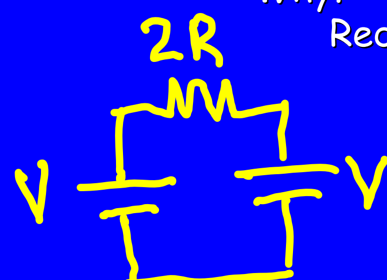
(B) V_{ab} changes sign

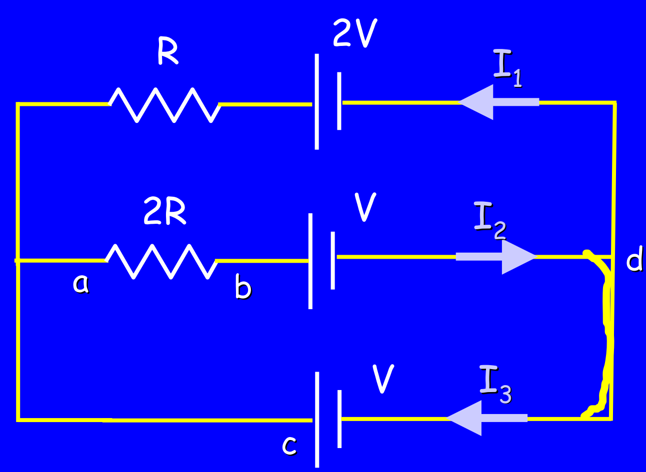
(C) V_{ab} increases

(D) V_{ab} goes to zero

Why?

Redraw:





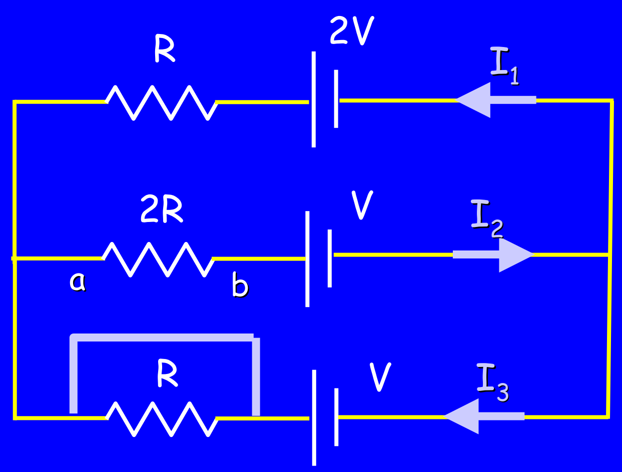
$V_{cd} = +V$

$V_{bd} = +V$

$V_{ad} = V_{cd} = +V$

$\Rightarrow$ $V_{ab} = V_{ad} - V_{bd} = V - V = 0$

Follow-Up Check



- We know:

  $I_2 = (1/5)\ V/R$

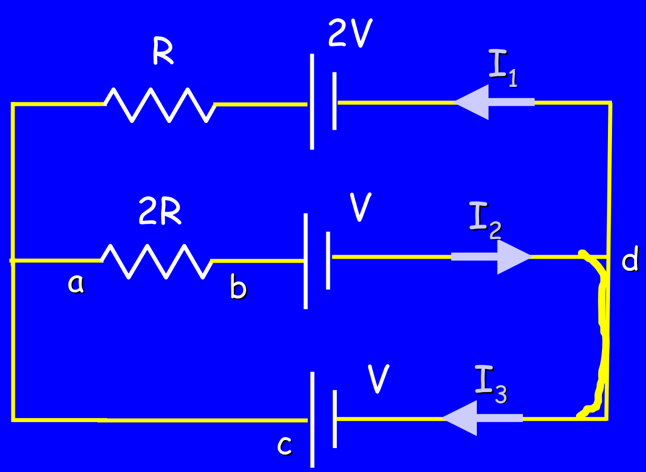
  $I_1 = (3/5)\ V/R$

  $I_3 = (-2/5)\ V/R$

- Suppose we short R_3: What happens to V_{ab} (voltage across R_2?)

(A) V_{ab} remains the same
(B) V_{ab} changes sign
(C) V_{ab} increases
(D) V_{ab} goes to zero

Why?
Redraw:



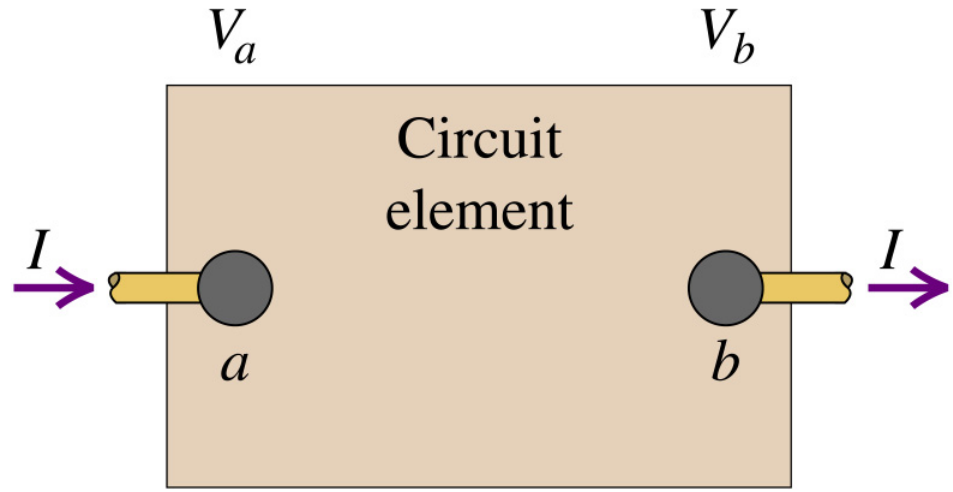
Then is I_2 zero?

Then is I_1 zero?

Power

Very important use of electricity.

Suppose we have a circuit element that has a voltage drop of $V_{ab} = V_a - V_b$ and a current flow of I.



What is the change in potential energy in this circuit element?

$$dU = (V_{ab})dq$$

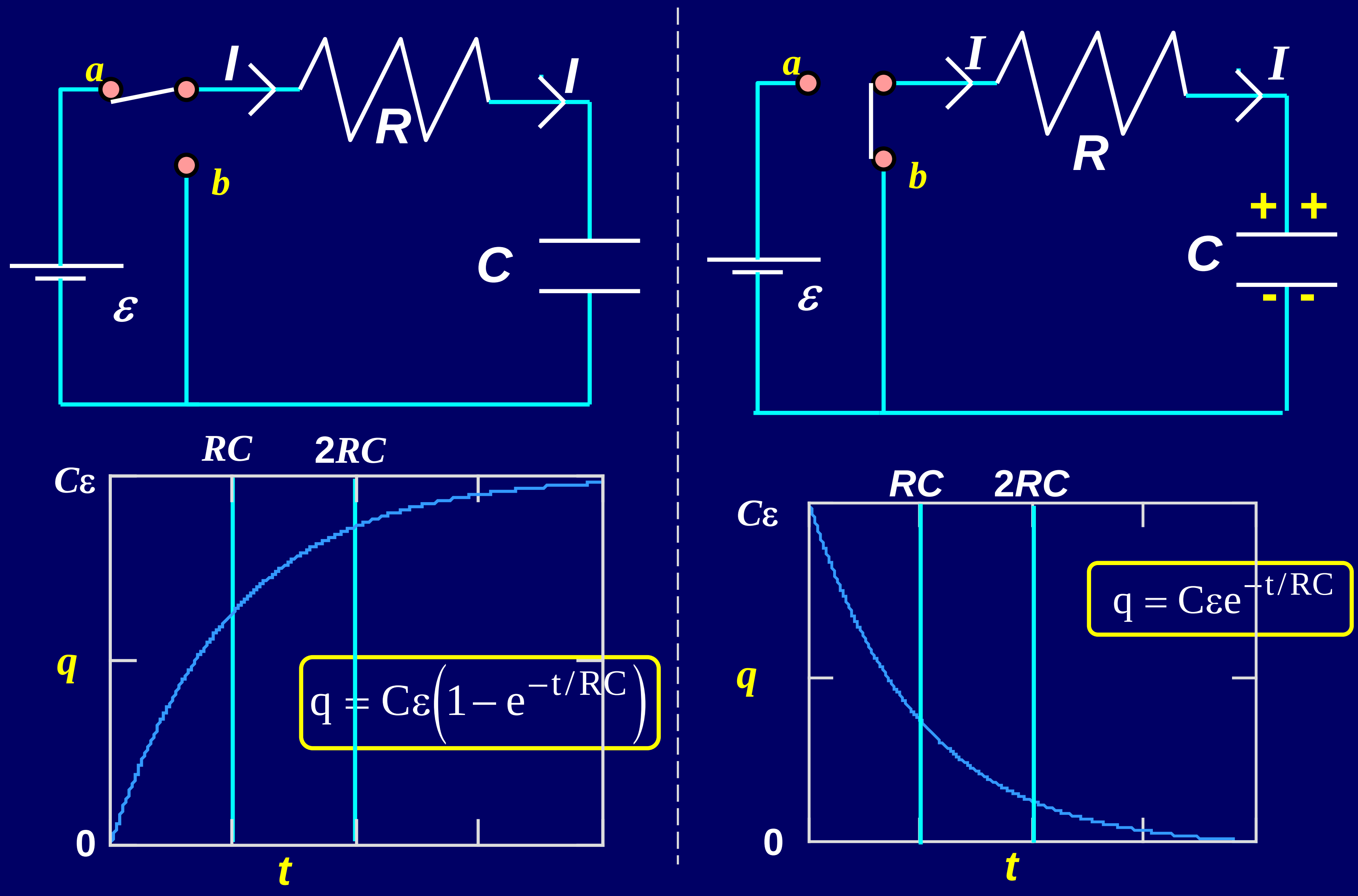
What is the time rate change in potential energy in this circuit element?

$$P = \frac{dU}{dt} = (V_{ab})\frac{dq}{dt} = (V_{ab})\ I$$

Power, P, is the time rate change in energy and equals voltage × current

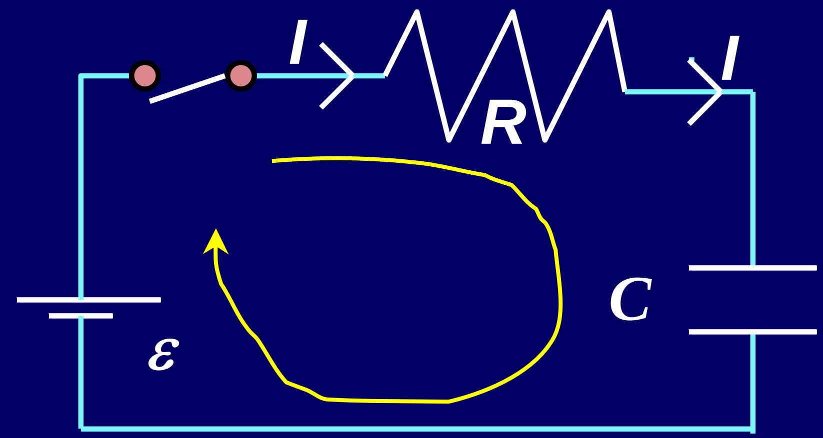
$$\boxed{P = (V_{ab})\ I \qquad \text{Units = voltage-amps = Watts}}$$

RC Circuits



a
I
R
b
I
C
ε
RC
2RC
Cε
q
0
t
$q = C\varepsilon\left(1 - e^{-t/RC}\right)$
a
I
R
I
b
+ +
C
- -
ε
RC
2RC
Cε
q
0
t
$q = C\varepsilon e^{-t/RC}$

Resistor-capacitor circuits



Let's add a Capacitor to our simple circuit

Recall voltage "drop" on C?

$$V = \frac{Q}{C}$$

Write KVL: $$\varepsilon - IR - \frac{Q}{C} = 0$$

Use $$I = \frac{dQ}{dt}$$ **Now eqn. has only "Q":**

KVL gives Differential Equation ! $$\varepsilon - R\frac{dQ}{dt} - \frac{Q}{C} = 0$$

We will solve this later. For now, look at qualitative behavior…

Capacitors Circuits, Qualitative

Basic principle: Capacitor resists change in Q →
resists changes in V

- **Charging (it takes time to put the final charge on)**
  - **Initially, the capacitor behaves like a wire ($\Delta V = 0$, since $Q = 0$).**
  - **As current continues to flow, charge builds up on the capacitor**
    → it then becomes more difficult to add more charge
    → the current slows down
  - **After a long time, the capacitor behaves like an open switch.**
- **Discharging**
  - **Initially, the capacitor behaves like a battery.**
  - **After a long time, the capacitor behaves like a wire.**

For next time

- HW #6 Assigned → due next Monday
- Quiz #3 on Friday
- Review Midterm 1 Wednesday – problem session Friday prior to quiz