

# Course Updates

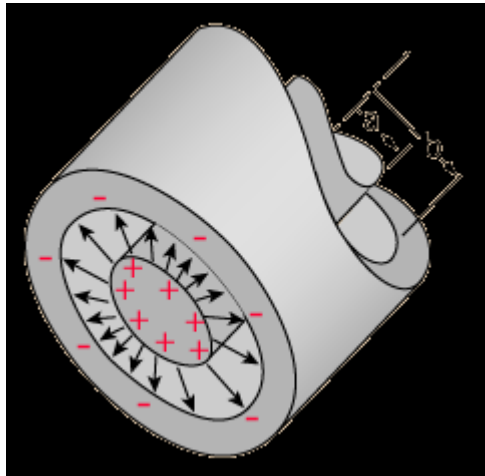
<http://www.phys.hawaii.edu/~varner/PHYS272-Spr10/physics272.html>

Notes for today:

- 1) Brief review of Quiz 2 (→ take home) due **Wednesday**
- 2) Assignment 4 (Mastering Physics) online and separate, written problems due **Wednesday**
- 3) Complete Chapter 25 this week (**review all of Chap 21-24 for Midterm next week**)
- 4) Schedule for next week:
  - 1) Monday: **holiday**
  - 2) Wednesday: **review**
  - 3) Friday: **Midterm #1**

# Hints\_HWK4 (24.70)

The inner cylinder of a long cylindrical capacitor has radius  $r_a$  and linear charge density  $\lambda$ . It is surrounded by a coaxial conducting cylinder with inner radius  $r_b$  and linear charge density  $-\lambda$ . Find the energy density at a distance  $r$  from the axis

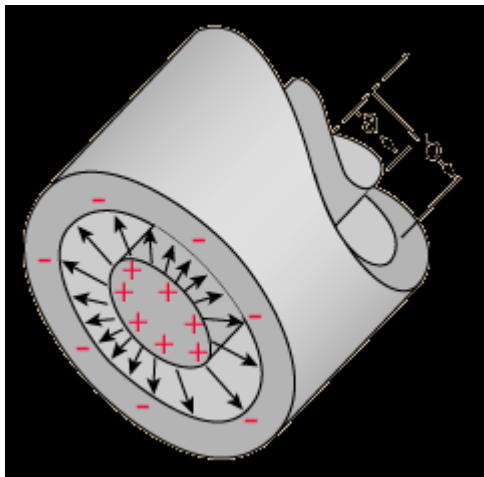


$$u = \frac{1}{2} \epsilon_0 E^2$$

Let's insert the electric field  $E$

$$u = \frac{1}{2} \epsilon_0 \left( \frac{\lambda}{2\pi\epsilon_0 r} \right)^2 = \frac{\lambda^2}{8\pi^2 \epsilon_0^2 r^2}$$

$$u = \frac{1}{2} \epsilon_0 \left( \frac{\lambda}{2\pi\epsilon_0 r} \right)^2 = \frac{\lambda^2}{8\pi^2 \epsilon_0^2 r^2}$$



Now let's find the total potential energy

$$U = \int u dV$$

What is the differential volume element for a cylinder of length  $L$ , radius  $r$  ??

$$V = \pi r^2 L \quad \longrightarrow \quad dV = L 2\pi r dr$$

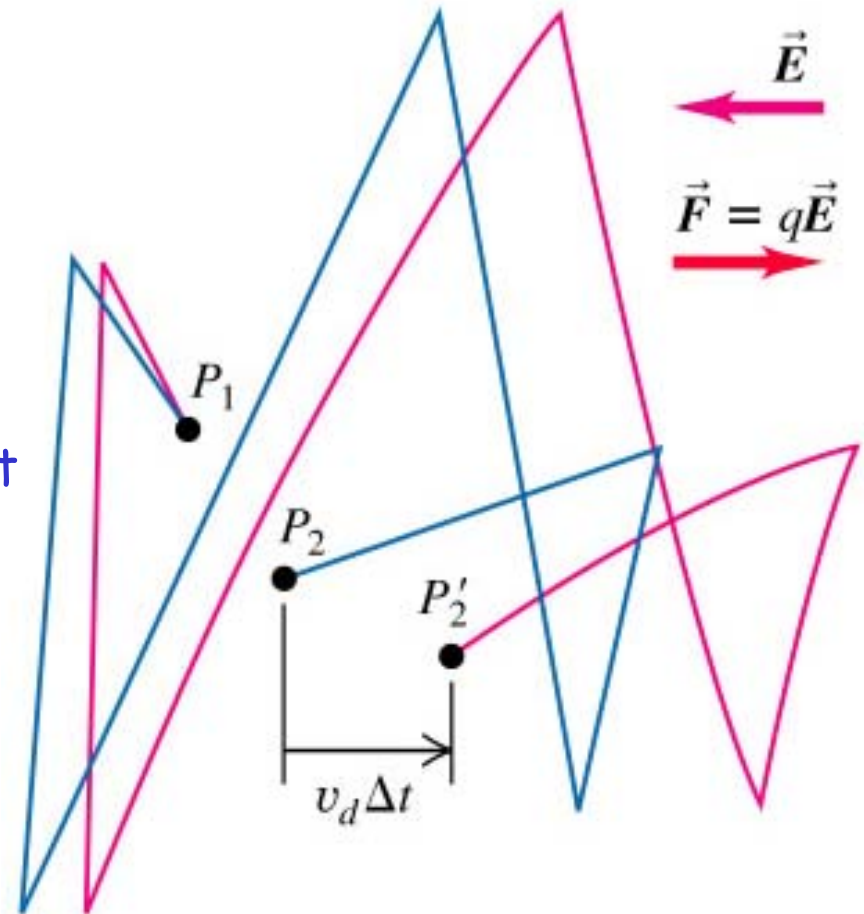
# Current

NO longer electrostatics!  
Can have  $E$  in conductor and a flow of charge.

Consider free electron motion without any external electric field. The net motion (blue) is random and the average displacement is zero. Electron moves from  $P_1 \rightarrow P_2$

Consider electron motion with an external electric field. The net motion (red) has a drift and the average displacement is opposite to the electric direction. Electron moves  $P_1 \rightarrow P_2'$ . The net drift velocity  $v_d$  is  $\sim 10^{-4}$  m/s.

The drift velocity is very small!



## Current in Wire; charge carriers

Current,  $I$ , is the rate of charge flow through cross section of wire or charge per unit time:

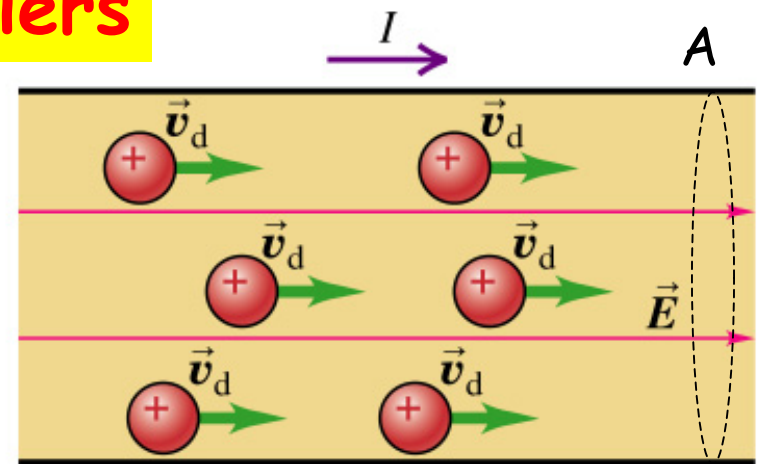
units of  $C/s = 1$  Ampere

$$I = \frac{dQ}{dt}$$

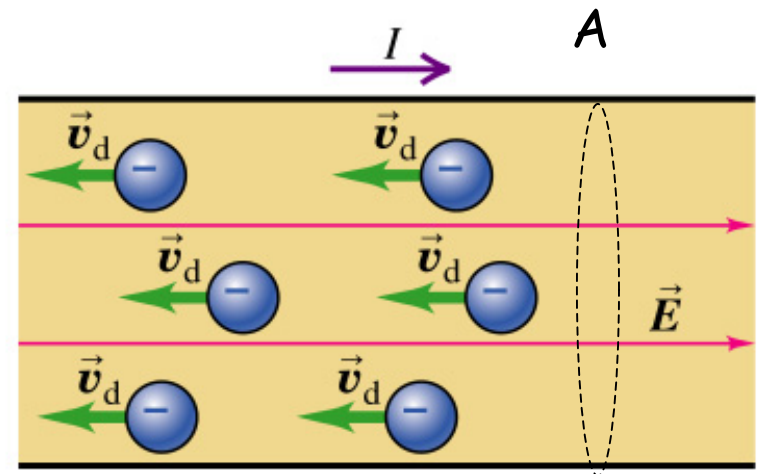
Current can be formed with positive Charges moving in positive direction or with negative charges moving in the opposite direction.

Electron flow is opposite to the current flow. This is a historical choice made by Ben Franklin.

House outlets are fused at 15Amps  
Whole house circuit breaker is around 200Amps



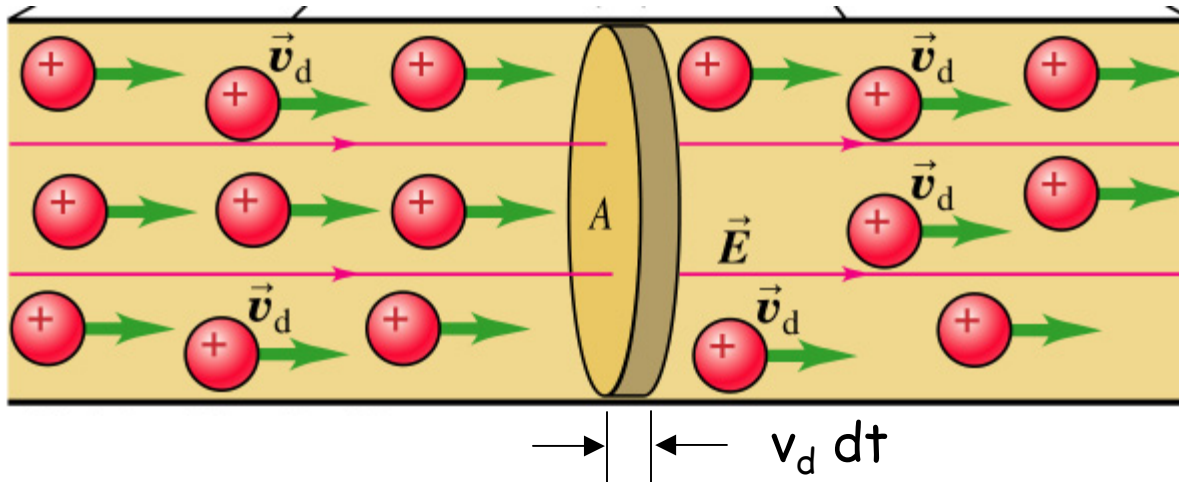
effective picture



(b) Y&F fig 25.2

# Current Density, $J$

Flow of + charges in a wire, through surface area  $A$



Y&F fig 25.3

Current Density,  $J$ , is the current flow per unit area (amp/m<sup>2</sup>)

$$J = \frac{I}{A} = \frac{1}{A} \frac{dQ}{dt}$$

If + charges,  $q$ , have velocity  $v_d$  and a volume density,  $n$  (#/volume). Then in a time  $dt$ , a volume,  $A v_d dt$ , is swept out and the differential amount of charge is

$$dQ = q n A v_d dt$$

We can write current as,  $I = \frac{dQ}{dt} = nq v_d A$

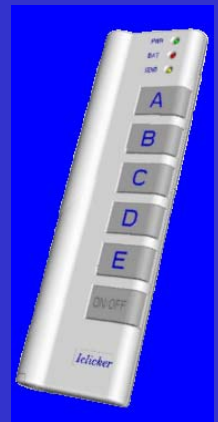
### Example 1:

A light switch connects to a bulb 1 meter away. How long will it take for an electron just leaving the switch to reach the bulb?

What do we need to know?

The wire is made of copper and has a radius 0.815 mm. Calculate the drift velocity assuming 1 free electron per atom for  $I = 1\text{A}$ .

- A) 7.8 hours
- B) 7.8 s
- C) 7.8 ms
- D) 7.8  $\mu\text{s}$
- E) 7.8 ns



### Example 1:

A light switch connects to a bulb 1 meter away. How long will it take for an electron just leaving the switch to reach the bulb?

What do we need to know?

The wire is made of copper and has a radius 0.815 mm. Calculate the drift velocity assuming 1 free electron per atom for  $I = 1\text{A}$ .

A) 7.8 hours



B) 7.8 s

C) 7.8 ms

D) 7.8  $\mu\text{s}$

E) 7.8 ns

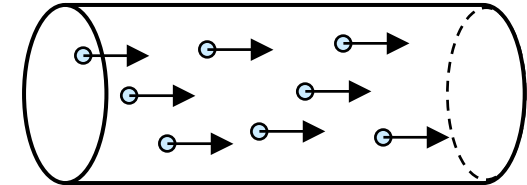
What? I thought the speed of light was very fast?



# Example 1

A wire is made of copper and has a radius 0.815 mm. Calculate the drift velocity assuming 1 free electron per atom for  $I = 1\text{A}$ .

$$I = \frac{dQ}{dt} = nqv_d A \quad v_d = \frac{I}{nqA}$$



$$n = n_a = \frac{\rho N_A}{M} = \frac{(8.93 \text{ g/cm}^3)(6.02 \times 10^{23} \text{ atoms/mole})}{63.5 \text{ g/mole}}$$

$$= 8.47 \times 10^{28} \text{ atoms/m}^3$$

$n_a$  = number density of copper atoms

$\rho$  = density of copper

$N_A$  = Avogadro's number

$M$  = atomic mass of Cu

$$v_d = \frac{I}{nqA} = \frac{1\text{A}}{(8.47 \times 10^{28} \text{ m}^{-3})(1.6 \times 10^{-19} \text{ C}) \pi (8.15 \times 10^{-4} \text{ m})^2}$$

$$= 3.54 \times 10^{-5} \text{ m/s}$$

Very slow.

7.8 hours to travel 1 m!

Why does the light come on so quickly when switch is thrown?

## Current Density, $J$

Since the current is,  $I = \frac{dQ}{dt} = nqv_d A$

We can write the current density as,  $J = \frac{I}{A} = nqv_d$

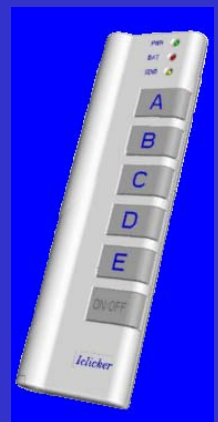
Technically current density is a vector quantity since velocity is a vector quantity,

$$\vec{J} = nq\vec{v}_d$$

### Example 2:

A current of 3.60 A flows through an automobile headlight. How many coulombs of charge flow through the headlight in a time of 2.60 hrs?

- A)  $3.37 \times 10^{-6} \text{ C}$
- B)  $3.37 \text{ C}$
- C)  $3.37 \times 10^4 \text{ C}$
- D)  $3.37 \times 10^{17} \text{ C}$



### Example 2:

A current of 3.60 A flows through an automobile headlight. How many coulombs of charge flow through the headlight in a time of 2.60 hrs?

A)  $3.37 \times 10^{-6} \text{ C}$

B)  $3.37 \text{ C}$

C)  $3.37 \times 10^4 \text{ C}$  ←

D)  $3.37 \times 10^{17} \text{ C}$

$$I = dQ / dt$$

$$dQ = I dt$$

$$Q = \int I dt \quad I \text{ constant}$$

$$\begin{aligned} Q &= It = (3.6 \text{ A})(2.6 \text{ hr})(3600 \text{ s} / \text{hr}) \\ &= 3.37 \times 10^4 \text{ C} \end{aligned}$$

$$(A = C/s)$$

### Example 2:

A current of 3.60 A flows through an automobile headlight. How many coulombs of charge flow through the headlight in a time of 2.60 hrs?

Dimensional analysis:

3.6 C/s and so we want in the end an answer in C

How?

Figure out number of seconds in 2.6 hours

$$60\text{s/min} * 60\text{min/hr} * 2.6 \text{ hr} = 3600 * 2.6 \\ = 9,360 \text{ s}$$

$$3.6 \text{ C/s} * 9,360\text{s} = 3.37 \times 10^4 \text{ C}$$

# Resistivity $\rho$

The current density at a point in a material depends on the material and on  $E$ . For some materials,  $J$  is proportional to  $E$  at a given temperature.

These materials (metals for example) are ohmic and are said to obey "Ohm's Law".

Other materials (like semiconductors) are non Ohmic.

Define resistivity is the ratio of electric field to current density ( $V \cdot m / \text{Amp}$ ). The symbol for resistivity is the Greek letter rho,  $\rho$ .

$$\rho = \frac{|\vec{E}|}{|\vec{J}|} \quad \text{OR} \quad \vec{E} = \rho \vec{J}$$

For ohmic materials,  $\rho$  at a given temperature is nearly constant.

New notation for  $V/\text{Amp}$  is unit, Ohm, represented by Greek letter, capital omega,  $\Omega$ . So  $\rho$  has units,  $\Omega \cdot m$ . Insulators have *large* values of  $\rho$ . For Glass,  $\rho > 10^{10} \Omega \cdot m$ .

# Resistivity $\rho$

The inverse of resistivity is called conductivity. Conductors have *large* values of *conductivity* or very small values of  $\rho$ .

$$\sigma = \frac{1}{\rho}$$

For copper  $\rho = 2.44 \times 10^{-8} \Omega \cdot \text{m}$ .

## Temperature dependence of $\rho$ :

For ohmic materials:

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)]$$

$\rho_0$  = resistivity at room temp (20° C)

$\rho(T)$  = resistivity at T

$\alpha$  = temperature coefficient of resistivity

Other materials (non-ohmic) more complicated.

# Resistance R

Consider a uniform, straight section of wire of length  $L$  and cross section  $A$  and with current  $I$ .

$$\vec{E} = \rho \vec{J}$$

Multiply by length  $L$

$$|\vec{E}|L = V = \rho |\vec{J}| L = \rho \frac{I}{A} L = \left( \rho \frac{L}{A} \right) I = R I$$

We define resistance,  $R$  as  $R = \rho \frac{L}{A}$

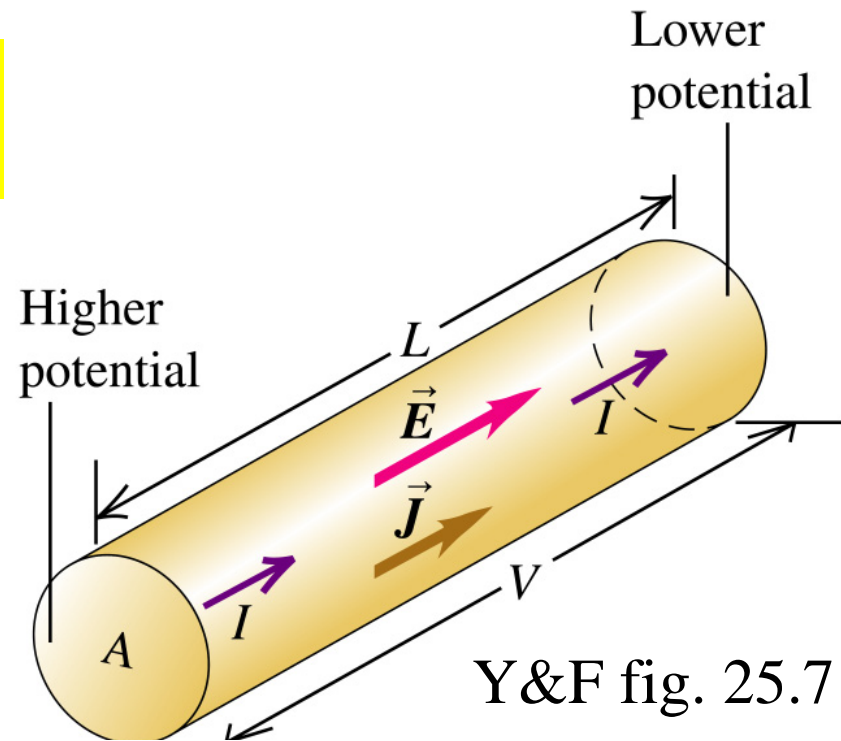
And we have for ohmic materials Ohm's Law:

$$V = I R$$

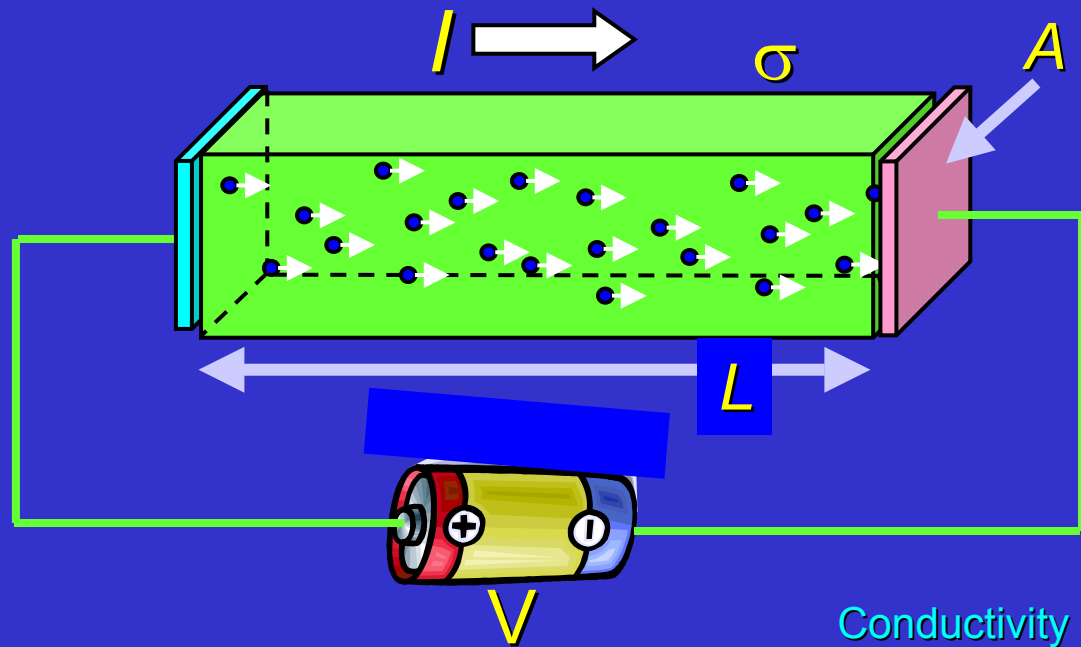
Resistance increases with bigger  $L$  and decreases with bigger  $A$

Units of  $R$ :  $\Omega$

$V$ ,  $R$ ,  $I$  easier to measure than  $E$ ,  $\rho$ , and  $J$ .







Conductivity – high for good conductors.

**Ohm's Law:  $J = \sigma E$**

Observables:

$$V = EL$$

$$I = JA$$



$$I/A = \sigma V/L$$



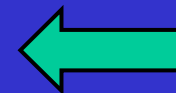
$$I = V/(L/\sigma A)$$



$R = \text{Resistance}$

$$\rho = 1/\sigma$$

**$I = V/R$**



$$R = \frac{L}{\sigma A}$$

# Analogy to plumbing

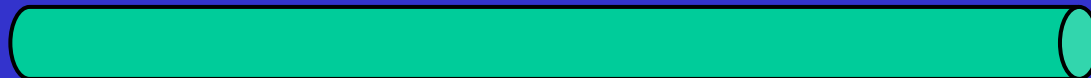
$I$  is like flow rate of water

$V$  is like pressure

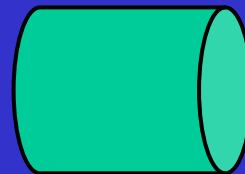
$R$  is resistance to water flow through pipe

$$R = \frac{L}{\sigma A}$$

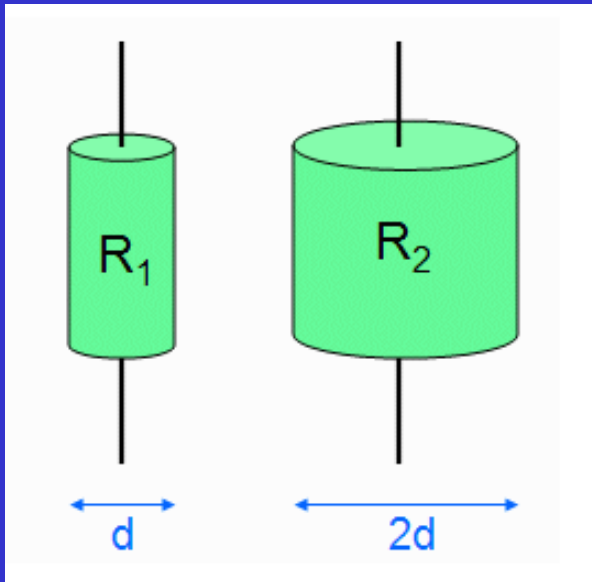
To increase  $R$ , make  $L$  longer or  $A$  smaller



To reduce  $R$ , shorten  $L$  or increase  $A$



# Example 3

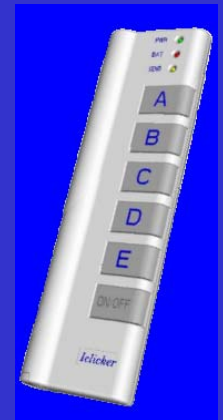


Same current through  
both resistors

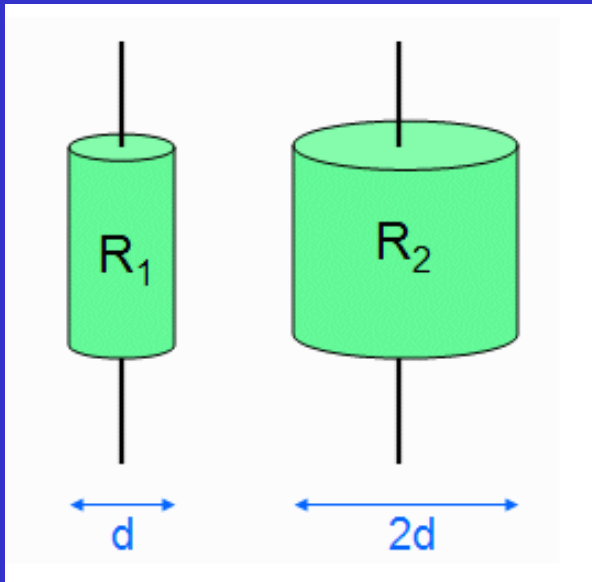
Compare voltages  
across resistors

☐  $V_1 > V_2$    ☐  $V_1 = V_2$    ☐  $V_1 < V_2$

Hint: Ohm's law



# Example 3



Same current through  
both resistors

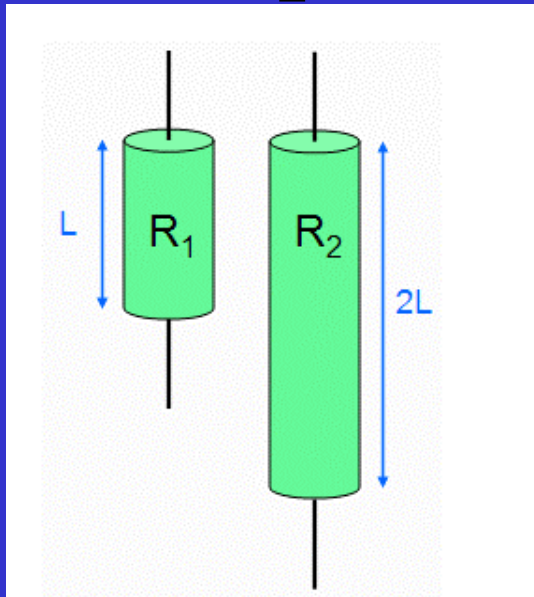
Compare voltages  
across resistors

☒  $V_1 > V_2$    ☐  $V_1 = V_2$    ☐  $V_1 < V_2$

$$R \propto \frac{L}{A}$$

$$V = IR \propto \frac{L}{A}$$

# Example 4



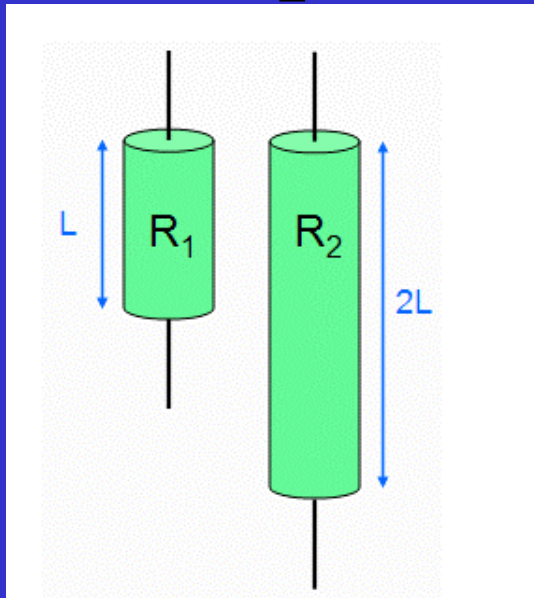
☐  $V_1 > V_2$    ☐  $V_1 = V_2$    ☐  $V_1 < V_2$

Same current through  
both resistors

Compare voltages  
across resistors

Hint: Ohm's law

# Example 4



☐  $V_1 > V_2$  ☐  $V_1 = V_2$  ☒  $V_1 < V_2$

Same current through  
both resistors

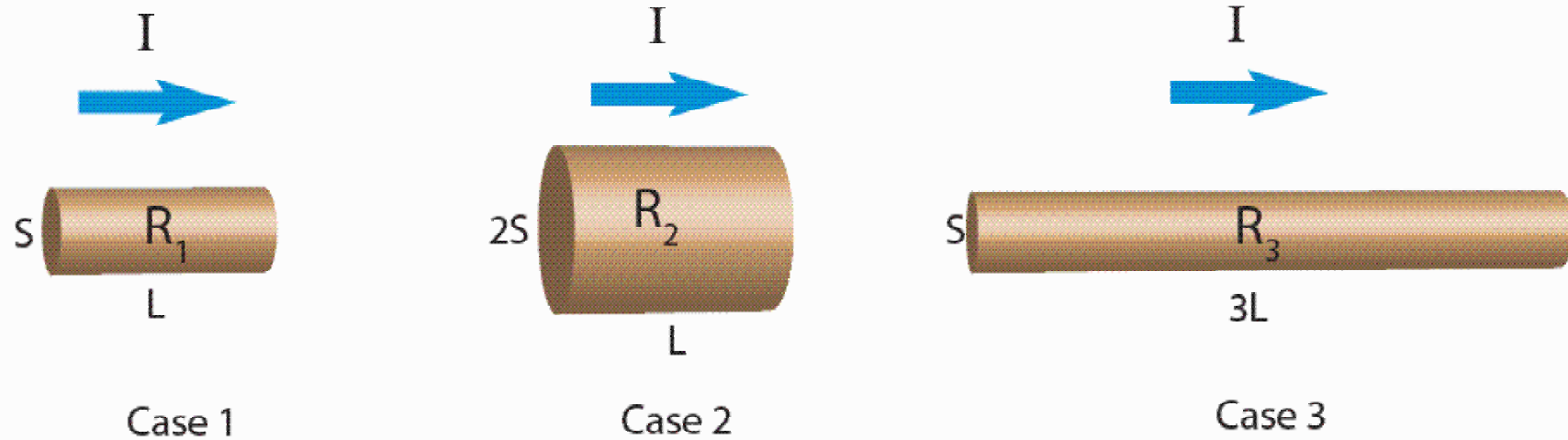
Compare voltages  
across resistors

$$R \propto \frac{L}{A}$$

$$V = IR \propto \frac{L}{A}$$

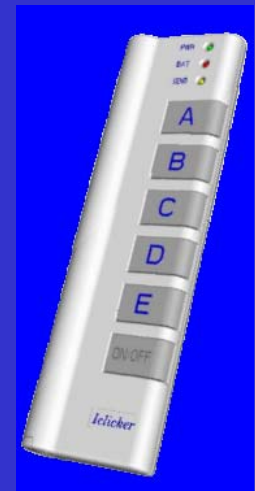
# Example 5

12) The SAME amount of current  $I$  passes through three different resistors.  $R_2$  has twice the cross-sectional area and the same length as  $R_1$ , and  $R_3$  is three times as long as  $R_1$  but has the same cross-sectional area as  $R_1$ .



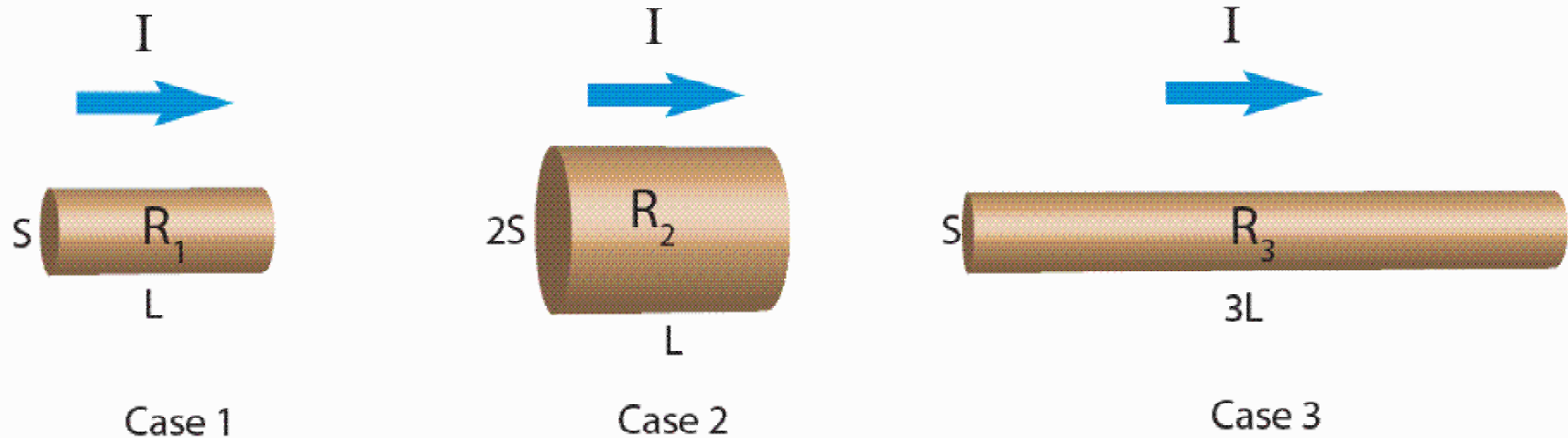
In which case is the CURRENT DENSITY through the resistor the smallest?

- ☐ Case 1   ☐ Case 2   ☐ Case 3



# Example 5

12) The SAME amount of current  $I$  passes through three different resistors.  $R_2$  has twice the cross-sectional area and the same length as  $R_1$ , and  $R_3$  is three times as long as  $R_1$  but has the same cross-sectional area as  $R_1$ .

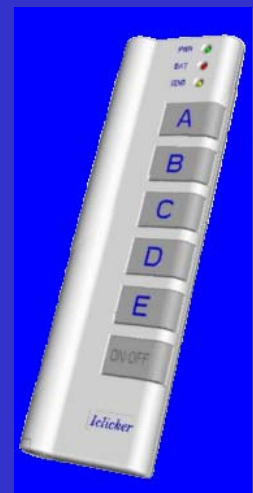


In which case is the CURRENT DENSITY through the resistor the smallest?

- ☐ Case 1 
 ☒ Case 2 
 ☐ Case 3

$$J \equiv \frac{I}{A} \quad \Rightarrow \quad J_1 = J_3 = 2J_2$$

Same Current  $\Rightarrow J \propto \frac{1}{A}$



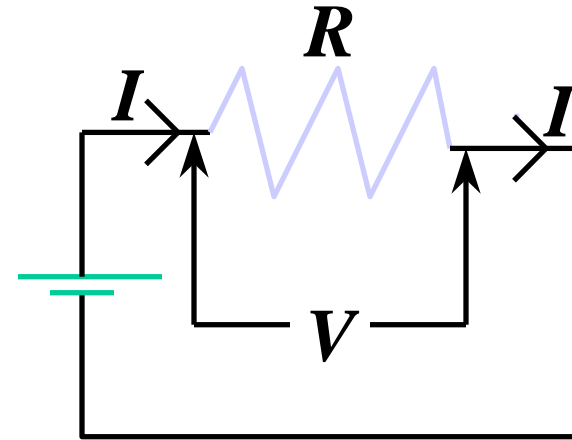




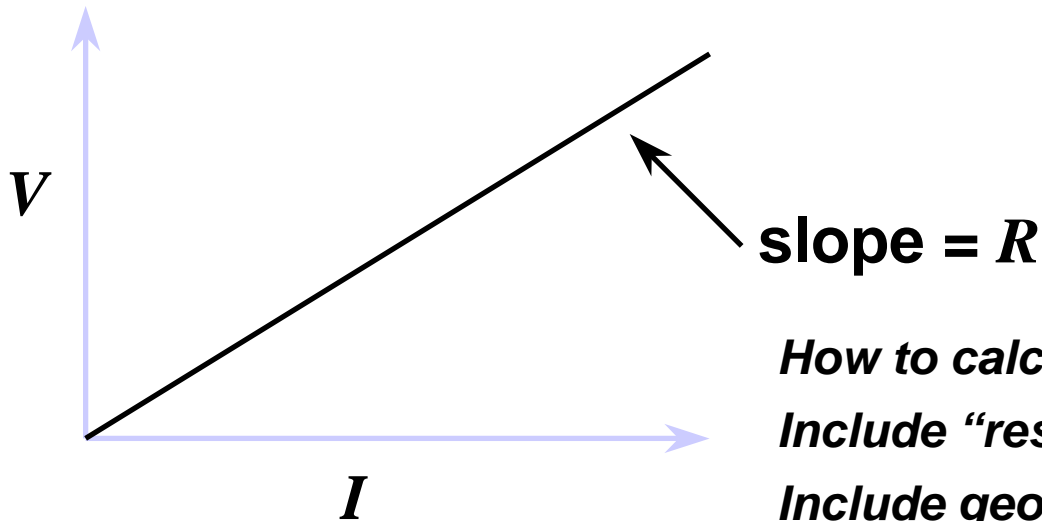
# Ohm's Law

- **Demo:**

- Vary applied voltage  $V$ .
- Measure current  $I$
- Does ratio  $\frac{V}{I}$  remain constant?



$$R \equiv \frac{V}{I}$$



*How to calculate the resistance?*  
*Include "resistivity" of material*  
*Include geometry of resistor*

# Resistance R

We define resistance, R as  $R = \rho \frac{L}{A}$

And we have for ohmic materials Ohm's Law:

$$V = I R$$

Units of R:  $\Omega$

Temperature dependence (ohmic materials):

$$R(T) = R_0[1 + \alpha(T - T_0)]$$

# For next time

- HW #4 due next time
- Now is time to resolve any questions you may have about previous HW, Quiz
- Office Hours usually after this class (9:30 – 10:00) in WAT214 – today (1-1:30pm)
- HW #5 → due next Wednesday

