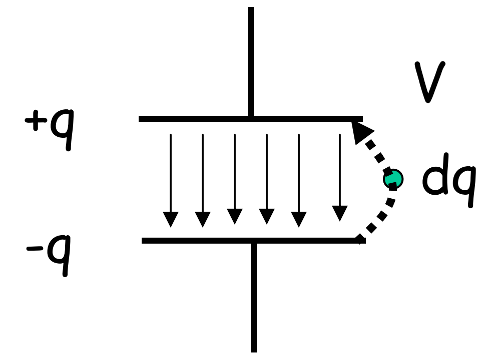
Energy storage in CAPACITORs

Charge capacitor by transferring bits of charge dq at a timefrom bottom to top plate. Can use a battery to do this. Battery does work which increase potential energy of capacitor.



q is magnitude of charge on plates

V= q/C V across plates

dU = V dq increase in potential energy

$$U = \int_0^U dU = \int_0^Q \frac{q}{C}\,dq = \frac{Q^2}{2C} = \frac{(CV)^2}{2C} = \frac{1}{2}CV^2$$

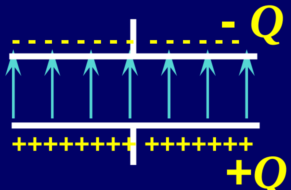
Y&F, eqn. 24.9

two ways to write

Where is the Energy Stored?

- **Claim: energy is stored in the electric field itself. Think of the energy needed to charge the capacitor as being the energy needed to create the field.**

- **To calculate the energy density in the field, first consider the constant field generated by a parallel plate capacitor, where**



$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q^2}{(A\varepsilon_0 / d)}$$

- **The electric field is given by:**

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \quad \Rightarrow \quad U = \frac{1}{2}\varepsilon_0 E^2 Ad$$

- **The energy density *u* in the field is given by:**

$$\boxed{u = \frac{U}{volume} = \frac{U}{Ad} = \frac{1}{2}\varepsilon_0 E^2}$$

This is the energy density, *u*, of the electric field….

Units: $\frac{\mathrm{J}}{\mathrm{m}^3}$

Energy Density

Claim: the expression for the energy density of the electrostatic field

$$\boxed{u = \frac{1}{2}\varepsilon_0 E^2}$$

is general and is not restricted to the special case of the constant field in a parallel plate capacitor.

- **Example**
  - **Consider *E*- field between surfaces of cylindrical capacitor:**
  - **Calculate the energy in the field of the capacitor by integrating the above energy density over the volume of the space between cylinders.**

$$U = \frac{1}{2}\varepsilon_0 \int E^2 dV = \frac{1}{2}\varepsilon_0 \int\int E^2 \pi r\, dr\, dl = etc.$$

  - **Compare this value with what you expect from the general expression:**

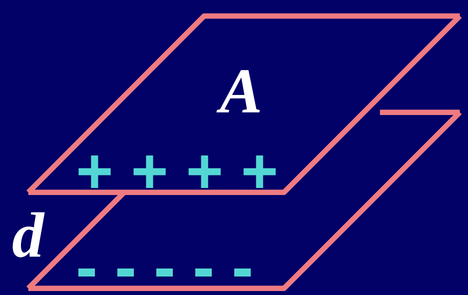
$$\boxed{W = \frac{1}{2} C V^2}$$

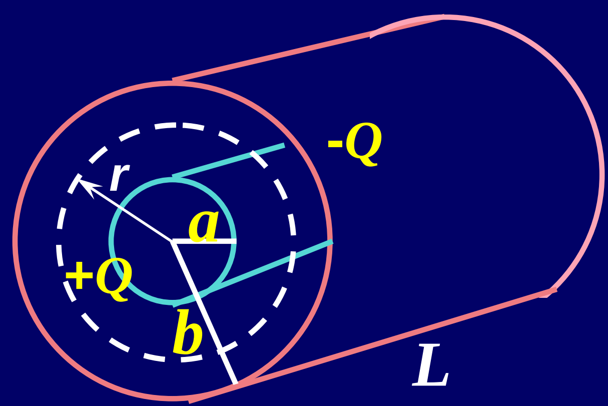
Capacitor Summary

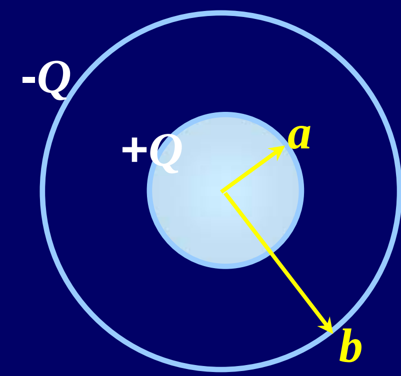
- A Capacitor is an object with two spatially separated conducting surfaces.
- The definition of the capacitance of such an object is:

$$\boxed{C \equiv \frac{Q}{V}}$$

- The capacitance depends on the geometry :







Parallel Plates

$$C = \frac{A\varepsilon_0}{d}$$

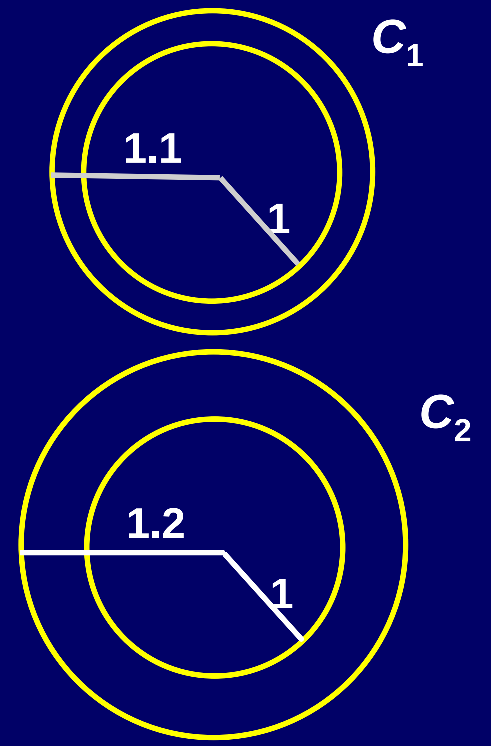
Cylindrical

$$C = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{b}{a}\right)}$$

Spherical

$$C = \frac{4\pi\varepsilon_0 ab}{b-a}$$

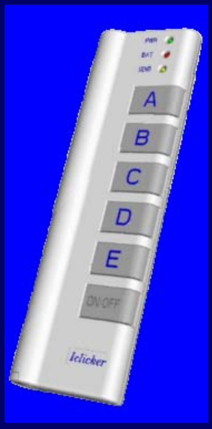
Example 1



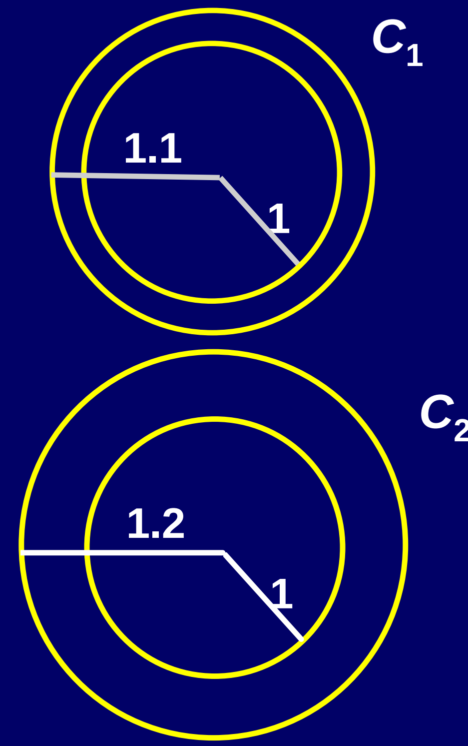
- **Consider two cylindrical capacitors, each of length L.**
  - C_1 has inner radius 1 cm and outer radius 1.1cm.
  - C_2 has inner radius 1 cm and outer radius 1.2cm.

If both capacitors are given the same amount of charge, what is the relation between U_1, the energy stored in C_1, and U_2, the energy stored in C_2?

(a) $U_2 < U_1$ **(b)** $U_2 = U_1$ **(c)** $U_2 > U_1$



Example 1



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(a) $U_2 < U_1$ **(b) $U_2 = U_1$** $\boxed{\textbf{(c) } U_2 > U_1}$

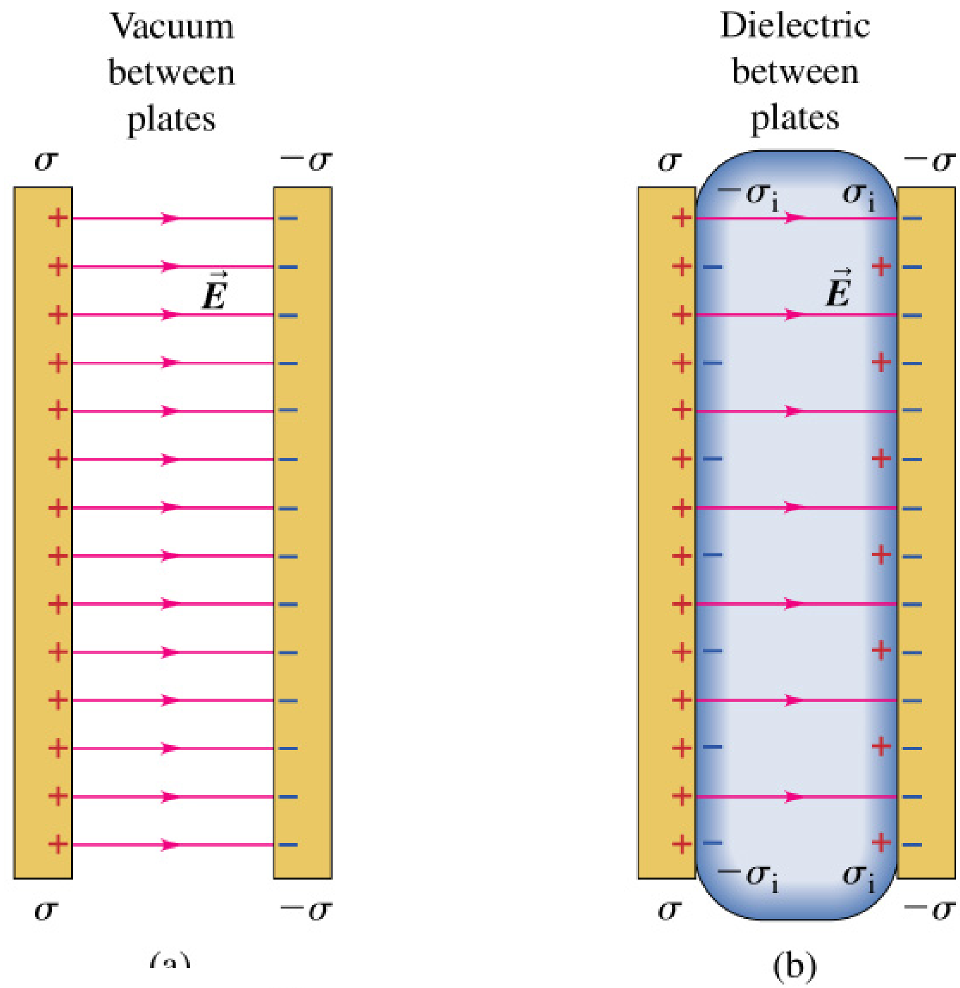
The magnitude of the electric field from r = 1 to 1.1 cm is the same for C_1 and C_2. But C_2 also has electric energy density in the volume 1.1 to 1.2 cm. In formulas:

$$C = \frac{2\pi\varepsilon_o L}{\ln\left(\frac{r_{outer}}{r_{inner}}\right)} \qquad C_1 \sim \frac{1}{\ln\left(\frac{1.1}{1}\right)} \qquad C_2 \sim \frac{1}{\ln\left(\frac{1.2}{1}\right)} \qquad \frac{U_2}{U_1} = \frac{Q^2/2C_2}{Q^2/2C_1} = \frac{C_1}{C_2} = \ln\left(\frac{1.2}{1.1}\right)$$

DIELECTRICS

Consider parallel plate capacitor with vacuum separating plates (left)

Suppose we place a material called a dielectric in between the plates (right)



Y&F Figure 24.13

The charge on the plates remain the same, but a dielectric has a property of having induced charges on its surface that <u>REDUCE</u> the electric field in between and the voltage difference.

Since C = Q/V, the resulting capacitance will <u>INCREASE</u>.

DIELECTRICS

Suppose the charges on the plate and the dielectric are, s and s_i. The electric Fields before and after are

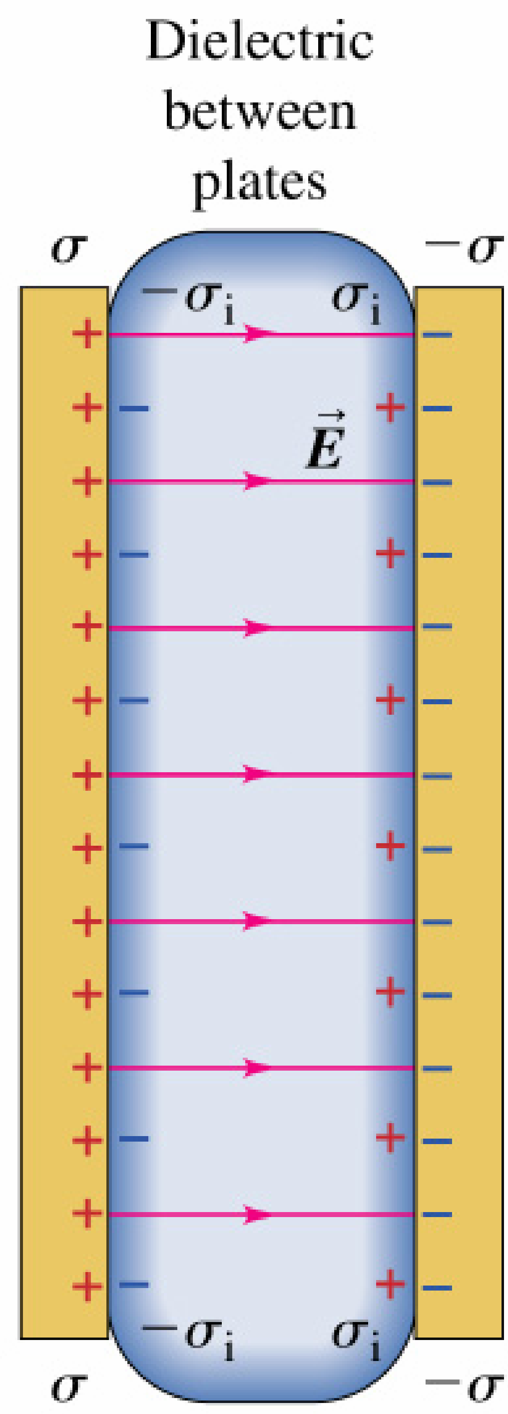
$$E_0 = \frac{\sigma}{\varepsilon_0} \quad ; \quad E = \frac{\sigma - \sigma_i}{\varepsilon_0} \quad ; \quad K \equiv \frac{E_0}{E} = \frac{\sigma}{\sigma - \sigma_i}$$

We define the ratio of the original field over the new field as the dielectric constant, **K**.

Hence, the voltage difference changes by 1/K and the capacitance, C_o=Q/V, changes by C=KQ/V=K C_o

For same Q: $C = KC_o$ $E = E_o/K$ $V = Vo/K$

But $\boxed{C = KC_o}$ General



DIELECTRICS Materials

Table 24.1 Values of Dielectric Constant K at 20°C

Material	K	Material	K
Vacuum	1	Polyvinyl chloride	3.18
Air (1 atm)	1.00059	Plexiglas	3.40
Air (100 atm)	1.0548	Glass	5–10
Teflon	2.1	Neoprene	6.70
Polyethylene	2.25	Germanium	16
Benzene	2.28	Glycerin	42.5
Mica	3–6	Water	80.4
Mylar	3.1	Strontium titanate	310

Glass, mica, plastics are very good dielectrics

DIELECTRICS and permittivity

We introduce a convenient redefinition of ε_0, called **permittivity**, as

$$\varepsilon = K\,\varepsilon_0$$

Consider a parallel plate capacitor with no dielectric

$$C_0 = \varepsilon_0 \frac{A}{d}$$

A capacitor with a dielectric becomes simply,

$$C = KC_0 = K\varepsilon_0 \frac{A}{d} = \varepsilon \frac{A}{d}$$

The change in capacitance can be accounted for by changing permittivity.

EXAMPLE of parallel plate capacitor problem

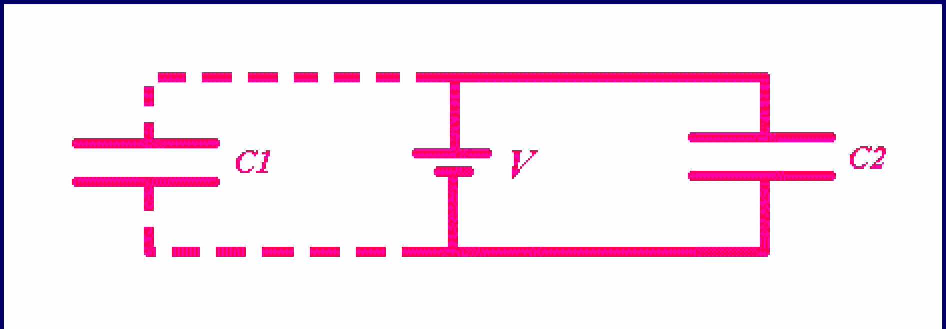
A parallel plate capacitor is made by placing polyethylene ($K = 2.3$) between two sheets of aluminum foil. The area of each sheet is 400 cm^2, and the thickness of the polyethylene is 0.3 mm. Find the capacitance.

$$C = K \varepsilon_o A/d = \frac{(2.3)\,(8.85 \times 10^{-12}\ C^2/Nm^2)\,(400\ cm^2)(1m^2/10^4\ cm^2)}{0.3 \times 10^{-3}\ m}$$

$$= 2.71\ nF$$

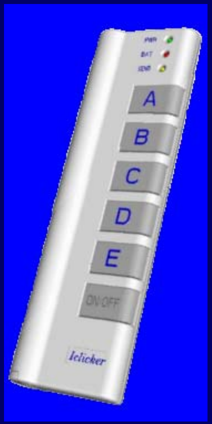
Example 2:

Two identical parallel plate capacitors are connected to a battery. Remaining connected, C_2 is filled with a dielectric.



➔ Compare the voltages of the two capacitors.

a) $V_1 > V_2$ b) $V_1 = V_2$ c) $V_1 < V_2$



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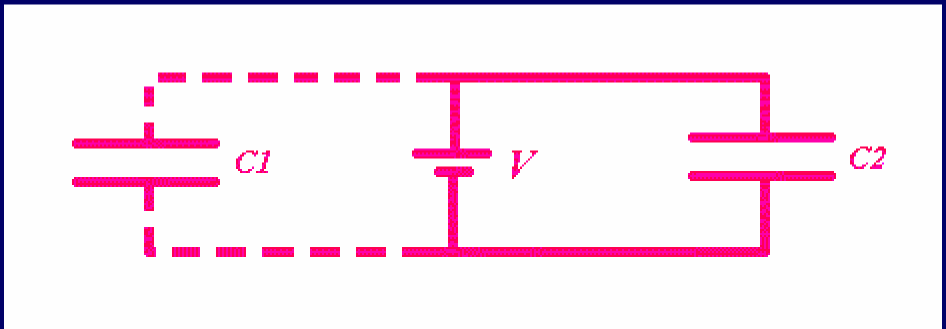

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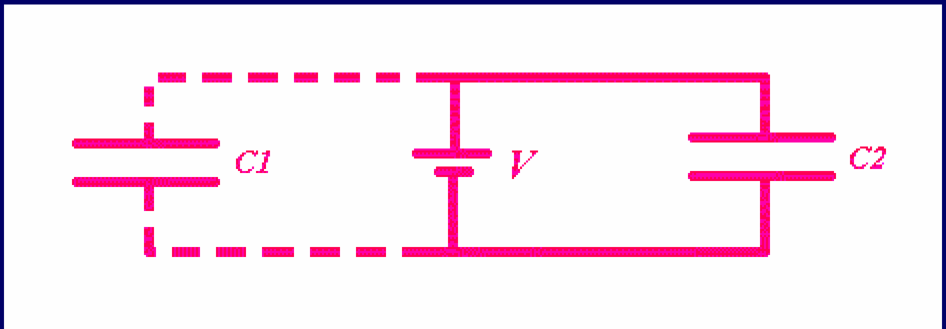
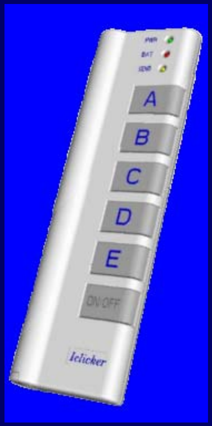
Example 3:

Two identical parallel plate capacitors are connected to a battery. Remaining connected, C_2 is filled with a dielectric.



➔ Compare the charges on the plates of the capacitors.

a) $Q_1 > Q_2$ b) $Q_1 = Q_2$ c) $Q_1 < Q_2$



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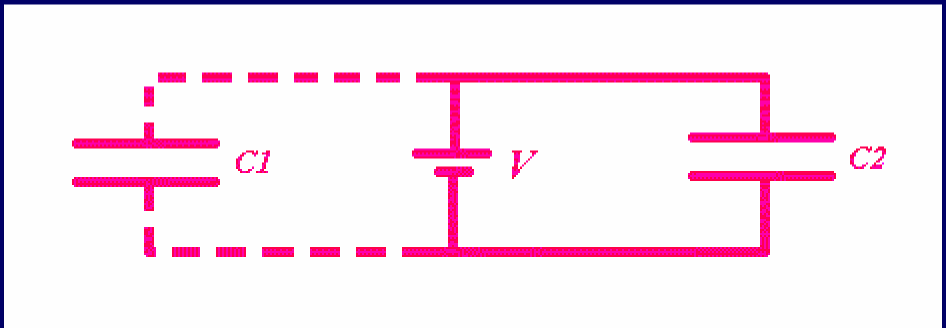
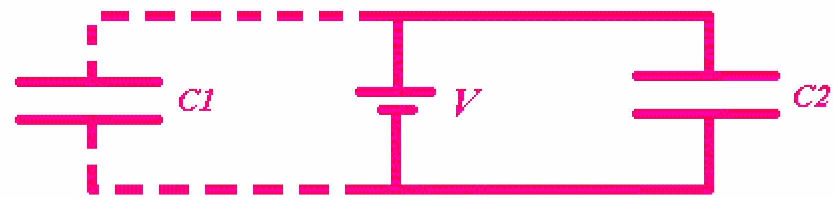


➔ Compare the charges on the plates of the capacitors.

a) $Q_1 > Q_2$ b) $Q_1 = Q_2$ c) $Q_1 < Q_2$

Note: Unlike constant Q case, here V and E remain the same but $C = K\ C_o$ still.

EXAMPLE



Two parallel plate capacitors, $C_1 = C_2 = 2\ \mu F$, are connected across a 12 V battery in parallel.

a.) What energy is stored?

$$U_1 = U_2 = \frac{1}{2}CV^2 = 144\mu J \quad U_T = 288\mu J$$

b.) A dielectric (K = 2.5) is inserted between the plates of C_2. Energy?

$$C_2' = KC_2 = 2.5 \times 2\mu F = 5\mu F$$

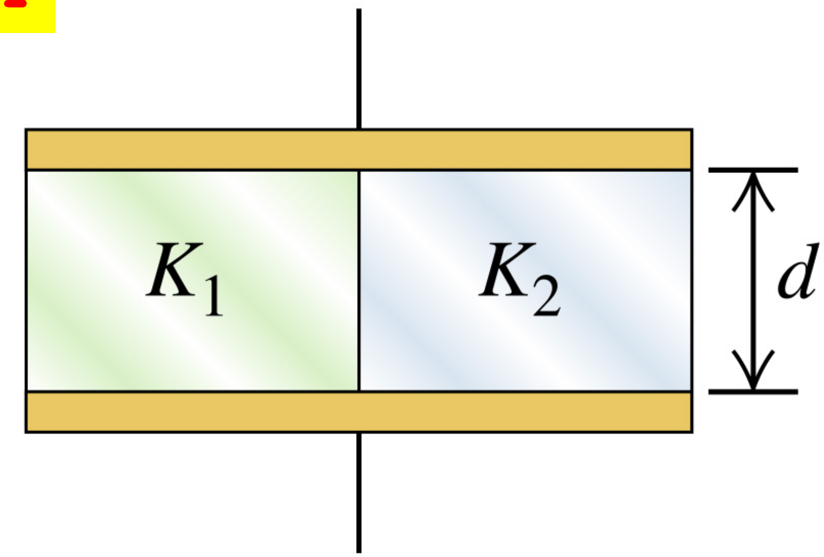
$$U_2' = \frac{1}{2}C_2'V^2 = 360\mu J \quad U_T = 504\mu J$$

Note: a dielectric increases amount of energy stored in C_2.

Y&F Problems 24.72 and 24.71

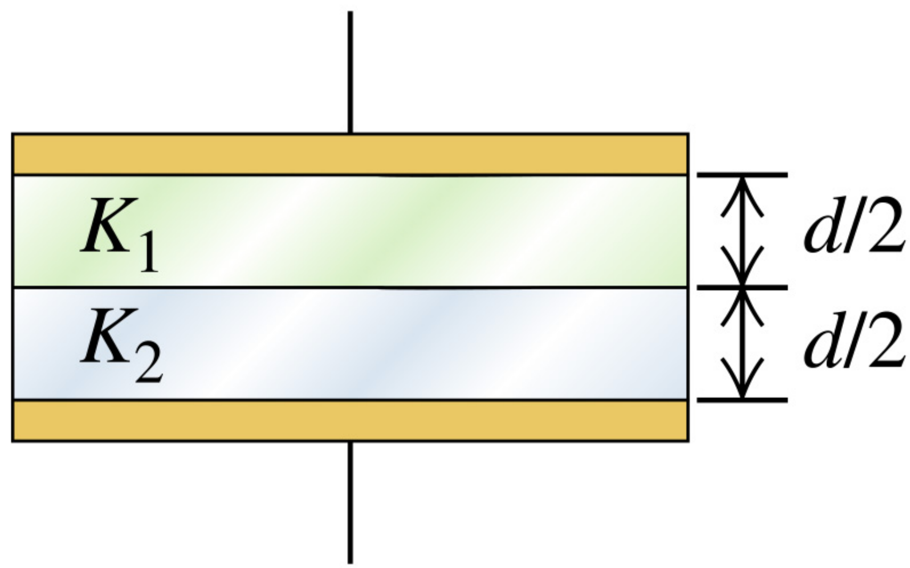
A parallel plate capacitor has two dielectrics, side by side, show the capacitance is,

$$C = \varepsilon_0 \frac{A}{d} \frac{K_1 + K_2}{2}$$



A parallel plate capacitor has two dielectrics, stacked, show the capacitance is,

$$C = \varepsilon_0 \frac{A}{d} \frac{2K_1K_2}{K_1 + K_2}$$



More weekend Fun

- HW #4 → get cracking (Hints on Monday)
- Office Hours immediately after this class (9:30 – 10:00) in WAT214 [1-1:30pm today]
- 2nd Quiz Now