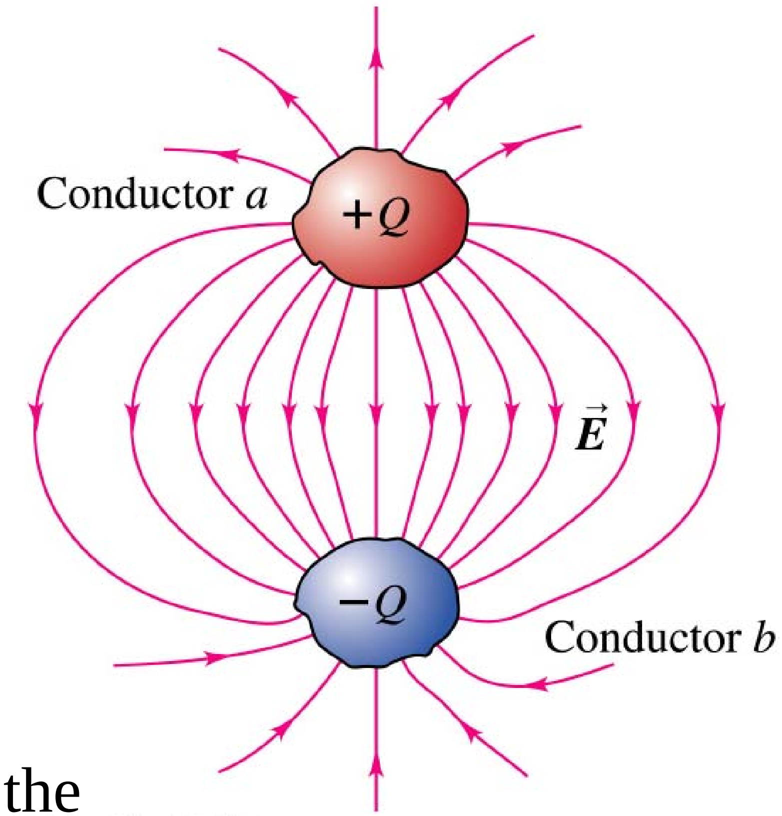
CAPACITORS

• C - Capacitance
  $V = V_a - V_b$ - voltage difference

$$\boxed{C = \frac{Q}{V}}$$



• Capacitance, depends on the geometry of the two conductors (size, shape, separation) and capacitance is always a positive quantity by its definition (voltage difference and charge of + conductor)

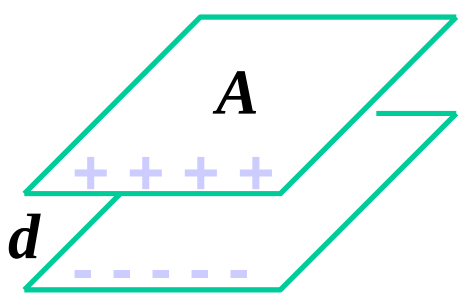
• UNITs: Coulomb/Volts or Farads, after Michael Faraday

Capacitor Summary

- A Capacitor is an object with two spatially separated conducting surfaces.
- The definition of the capacitance of such an object is:

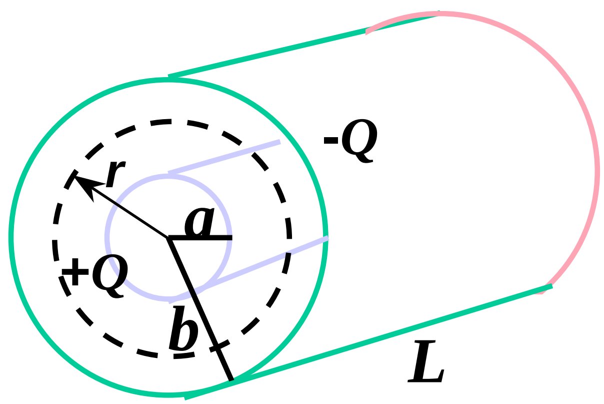
$$\boxed{C \equiv \frac{Q}{V}}$$

- The capacitance depends on the geometry :



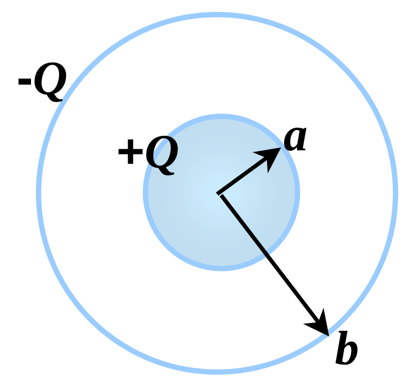
Parallel Plates

$$C = \frac{A\varepsilon_0}{d}$$



Cylindrical

$$C = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{b}{a}\right)}$$



Spherical

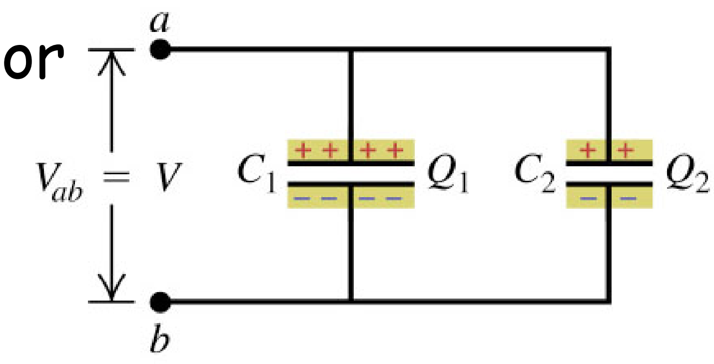
$$C = \frac{4\pi\varepsilon_0 ab}{b-a}$$

CAPACITORs in parallel

Want to find the equivalent capacitance C_{eq}:

- Voltage is same across each capacitor

$$V_{ab} = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}; \quad C_1 = \frac{Q_1}{V_{ab}}; \quad C_2 = \frac{Q_2}{V_{ab}}$$



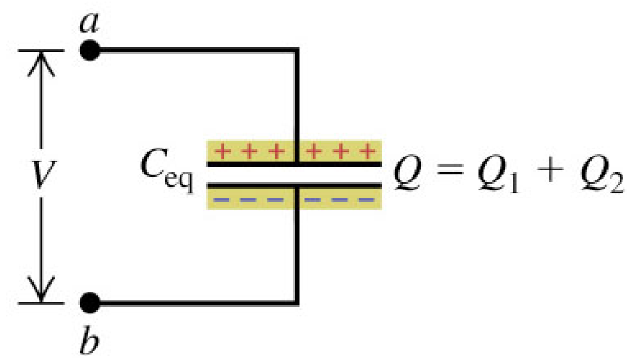
(a)

- Total charge and voltage ratios for parallel capacitor,

$$\frac{Q_1 + Q_2}{V_{ab}} = \boxed{C_{parallel} = C_1 + C_2}$$

For more parallel capacitors:

$$\boxed{C_{parallel} = \sum_i C_i}$$

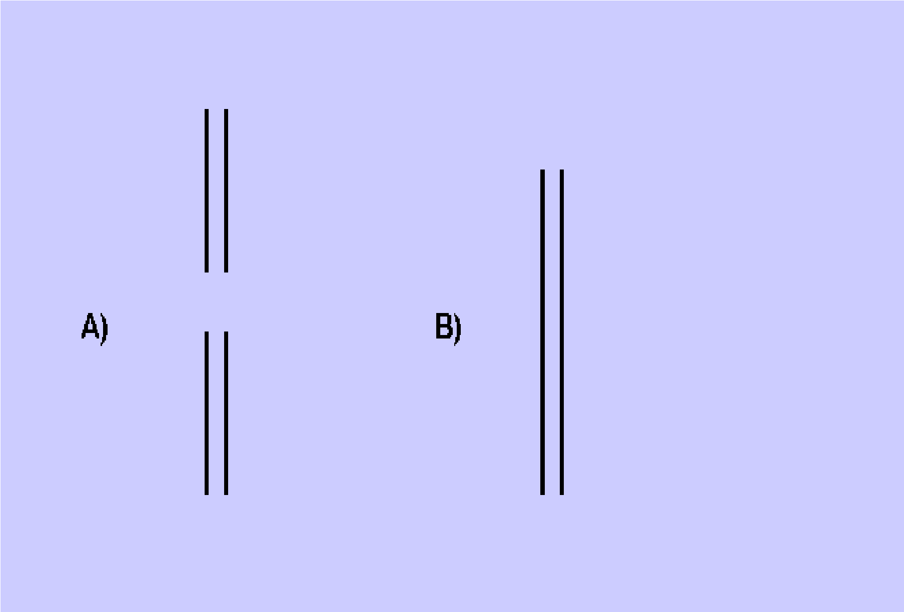


(b)

Note; 2 parallel capacitors C, doubles capacitance.

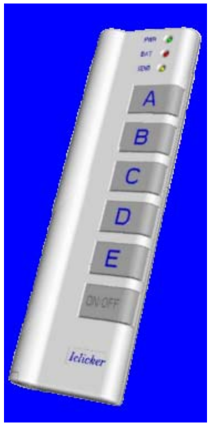
Example 1:

Two identical parallel plate capacitors are shown in an end-view in A) of the figure. Each has a capacitance of C.



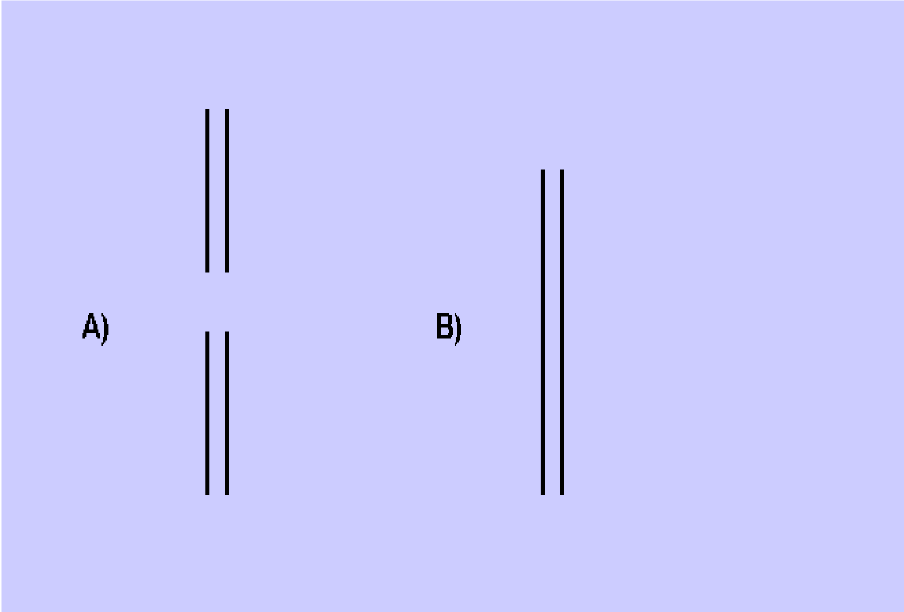
4) If the two are joined together as shown in B), forming a single capacitor, what will be the final capacitance?

a) $C/2$ b) C c) $2C$



Example 1:

Two identical parallel plate capacitors are shown in an end-view in A) of the figure. Each has a capacitance of *C*.



4) If the two are joined together as shown in B), forming a single capacitor, what will be the final capacitance?

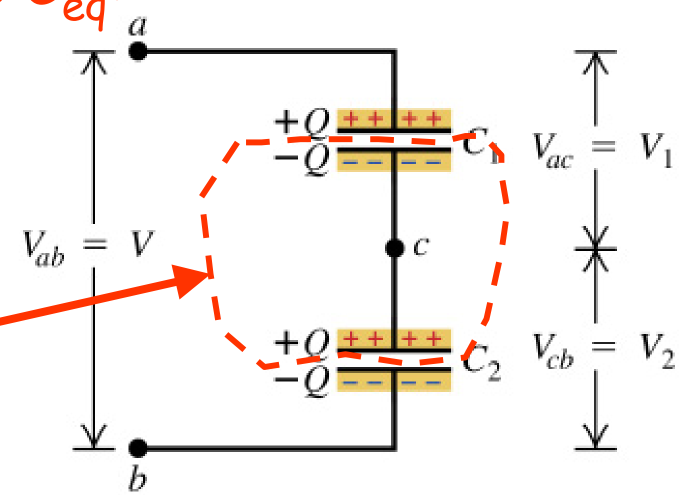
a) $C/2$ b) C c) $2C$

CAPACITORs in series

Want to find the equivalent capacitance C_{eq}:

• If a voltage is applied across a and b, then a +Q appears on upper plate and -Q on lower plate.

• A -Q charge is induced on lower plate of C_1 and a +Q charge is induced on upper plate of C_2. The total charge in circuit c is neutral.

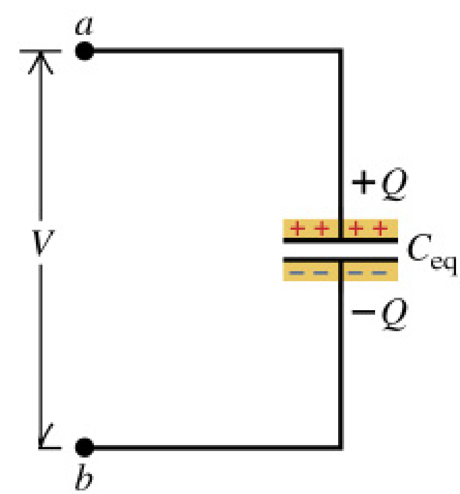


(a)

$$V_1 = \frac{Q}{C_1} \quad ; \quad V_2 = \frac{Q}{C_2}$$

$$V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2}$$

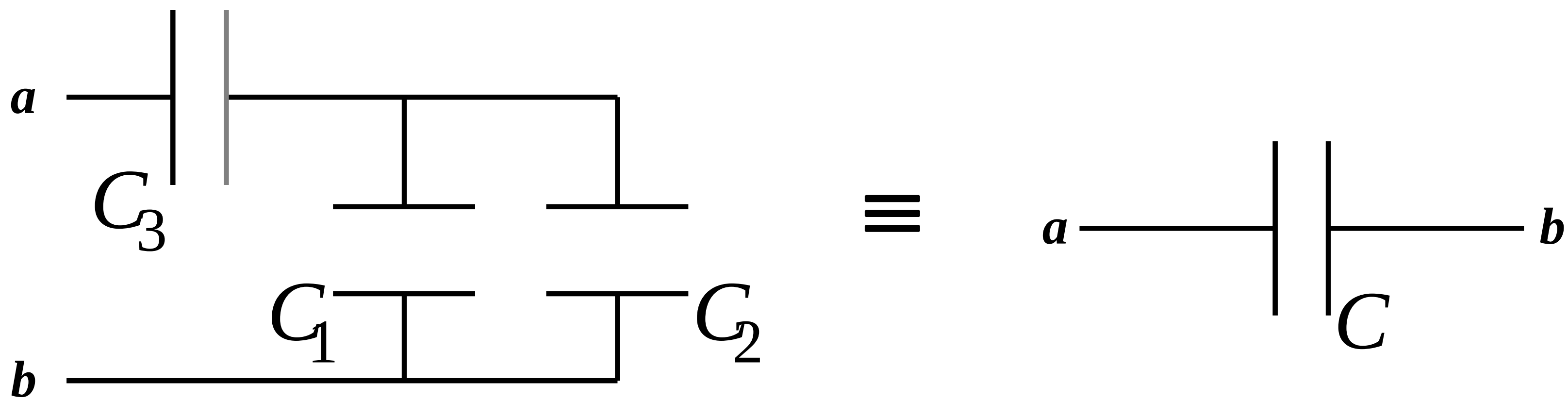
$$\frac{V}{Q} = \boxed{\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2}}$$



(b)

Note; 2 series capacitors C, halves capacitance.

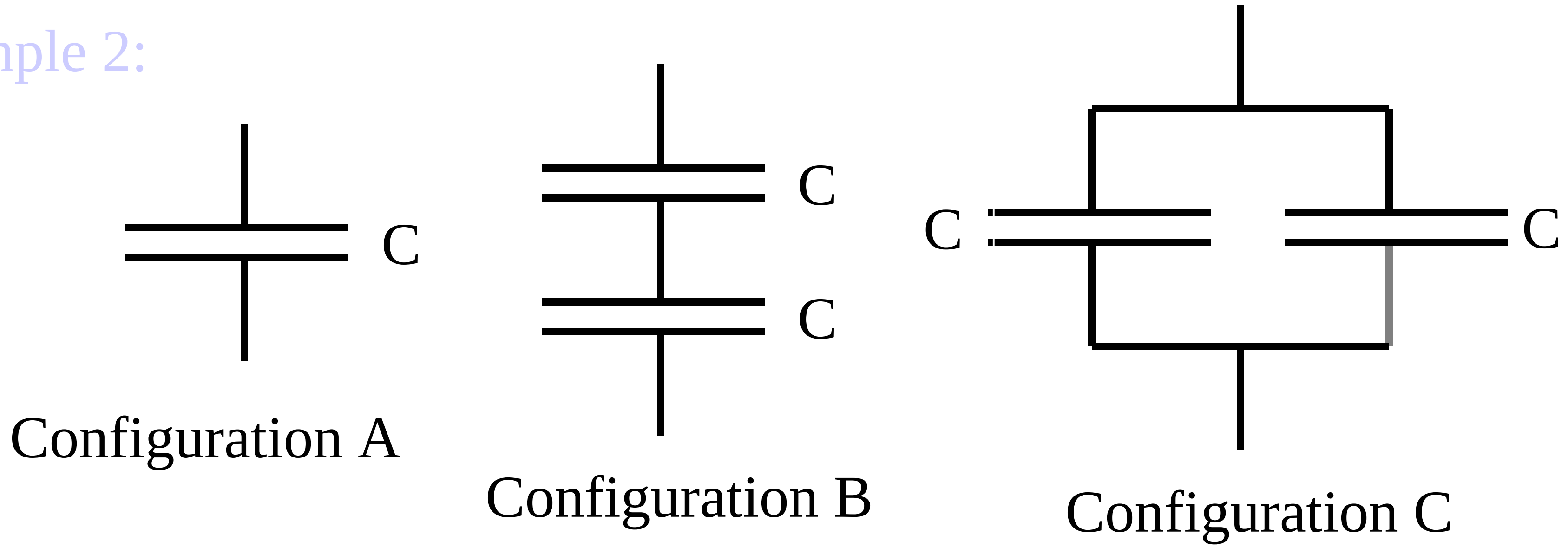
Examples:
Combinations of Capacitors



- **How do we start??**
- **Recognize C_3 is in series with the parallel combination on C_1 and C_2. i.e.,**

$$\frac{1}{C} = \frac{1}{C_3} + \frac{1}{C_1 + C_2} \quad \Rightarrow \quad \boxed{C = \frac{C_3(C_1 + C_2)}{C_1 + C_2 + C_3}}$$

Example 2:



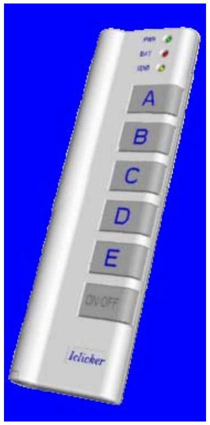
Three configurations are constructed using identical capacitors

➔ Which of these configurations has the lowest overall capacitance?

a) Configuration A

b) Configuration B

c) Configuration C



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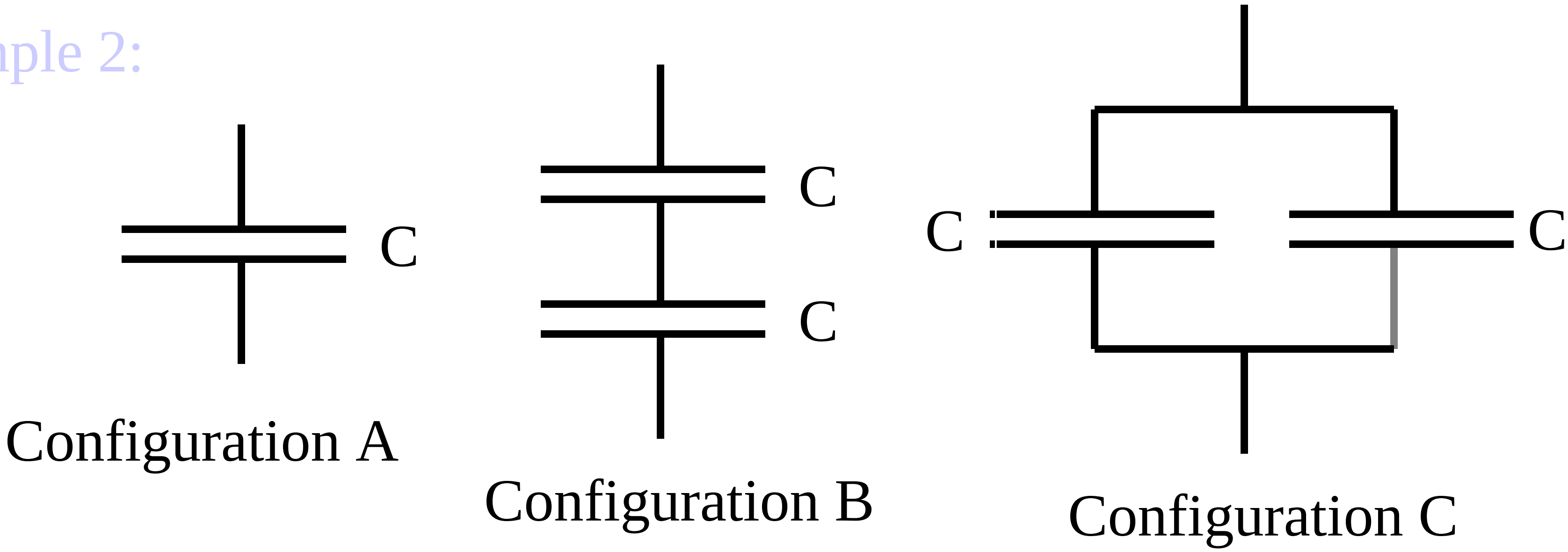

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➔ Which of these configurations has the lowest overall capacitance?

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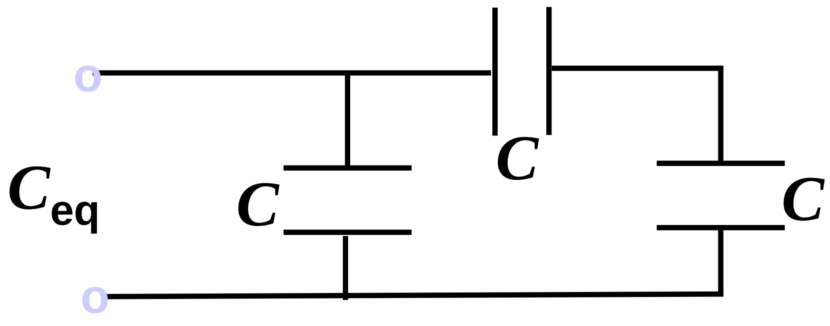
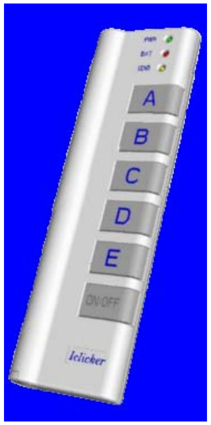
Example 4

- **What is the equivalent capacitance, C_{eq}, of the combination shown?**



(a) $C_{eq} = (3/2)C$ **(b)** $C_{eq} = (2/3)C$ **(c)** $C_{eq} = 3C$



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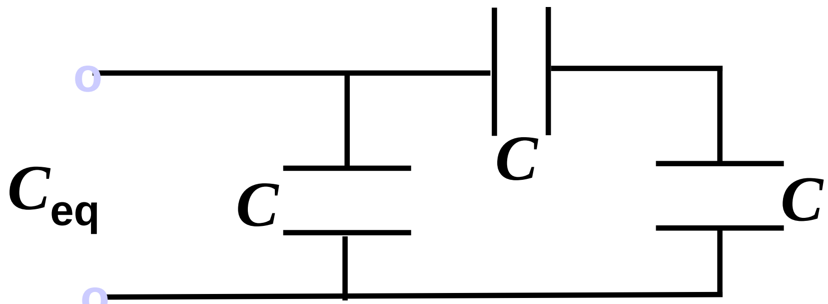
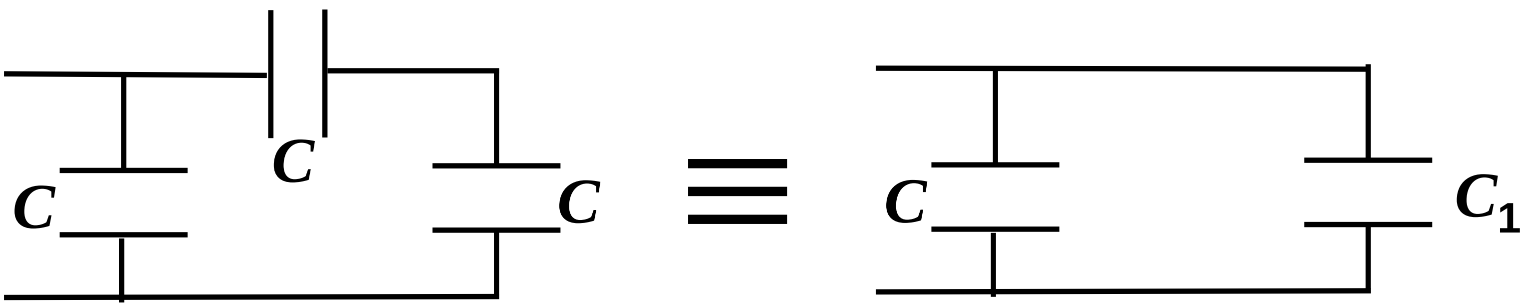


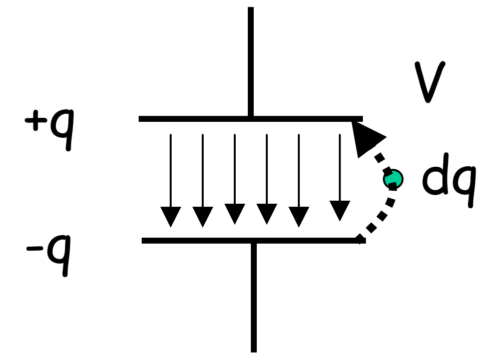
$\boxed{\textbf{(a) } C_{eq} = (3/2)C}$ **(b)** $C_{eq} = (2/3)C$ **(c)** $C_{eq} = 3C$



$$\frac{1}{C_1} = \frac{1}{C} + \frac{1}{C} \Rightarrow C_1 = \frac{C}{2} \Rightarrow C_{eq} = C + \frac{C}{2} = \frac{3}{2}C$$

Energy storage in CAPACITORs

Charge capacitor by transferring bits of charge dq at a time from bottom to top plate. Can use a battery to do this. Battery does work which increase potential energy of capacitor.



q is magnitude of charge on plates

V= q/C V across plates

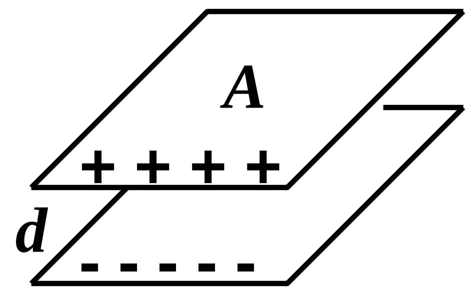
dU = V dq increase in potential energy

$$U = \int_0^U dU = \int_0^Q \frac{q}{C}\,dq = \frac{Q^2}{2C} = \frac{(CV)^2}{2C} = \frac{1}{2}CV^2$$

Y&F, eqn. 24.9

two ways to write

Question!



- **Suppose the capacitor shown here is charged to Q and then the battery is *disconnected*.**
- **Now suppose I pull the plates further apart so that the final separation is d_1.**
- **How do the quantities Q, C, E, V, U change?**

- Q: **remains the same.. no way for charge to leave.**
- C: **decreases.. since capacitance depends on geometry**
- E: **remains the same... depends only on charge density**
- V: **increases.. since $C\downarrow$, but Q remains same (or $d\uparrow$ but E the same)**
- U: **increases.. add energy to system by separating**

- **How much do these quantities change?.. exercise for student!!**

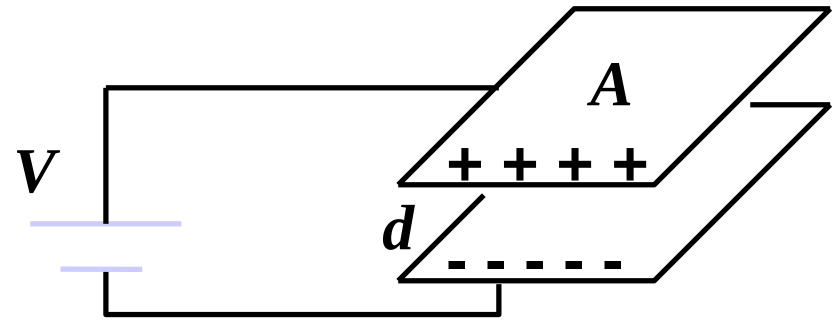
Answers:

$$\boxed{C_1 = \frac{d}{d_1} C}$$

$$\boxed{V_1 = \frac{d_1}{d} V}$$

$$\boxed{U_1 = \frac{d_1}{d} U}$$

Related Question

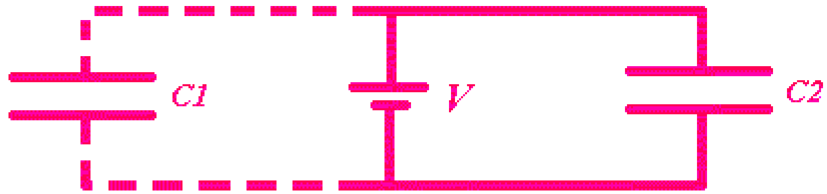


- **Suppose the battery (*V*) is kept attached to the capacitor.**
- **Again pull the plates apart from *d* to d_1.**
- **Now what changes?**
- C**:** **decreases (capacitance depends <u>only</u> on geometry)**
- V**:** **must stay the same - the battery forces it to be** V
- Q**:** **must decrease,** $Q=CV$ **charge flows off the plate**
- E**:** **must decrease (**$E=\frac{V}{D}$ **,** $E=\frac{\sigma}{E_0}$**)**
- U**:** **must decrease (**$U=\frac{1}{2}CV^2$**)**
- **How much do these quantities change?**

Answers:

$$\boxed{C_1=\frac{d}{d_1}C}\qquad\boxed{E_1=\frac{d}{d_1}E}\qquad\boxed{U_1=\frac{d}{d_1}U}$$

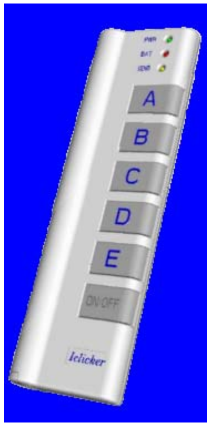
Example 5:



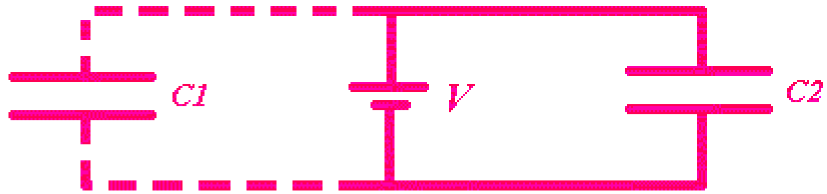
Two identical parallel plate capacitors are connected to a battery, as shown in the figure. C_1 is then disconnected from the battery, and the separation between the plates of both capacitors is doubled.

➔ What is the relation between the charges on the two capacitors ?

a) $Q_1 > Q_2$ b) $Q_1 = Q_2$ c) $Q_1 < Q_2$



Example 5:



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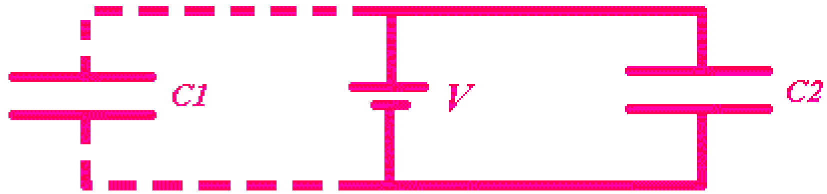
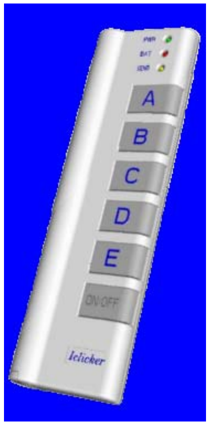


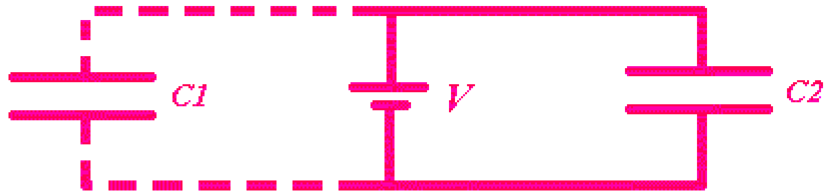
Two identical parallel plate capacitors are connected to a battery, as shown in the figure. C_1 is then disconnected from the battery, and the separation between the plates of both capacitors is doubled.

➔ How does the electric field between the plates of C_2 change as separation between the plates is increased ? The electric field:

a) increases b) decreases c) doesn't change



Example 6:



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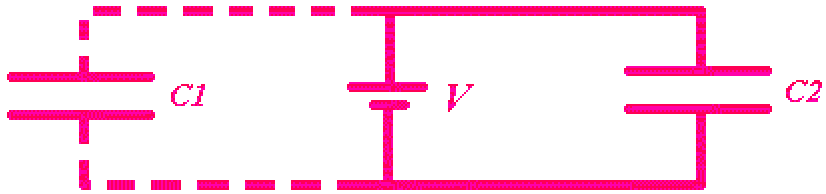
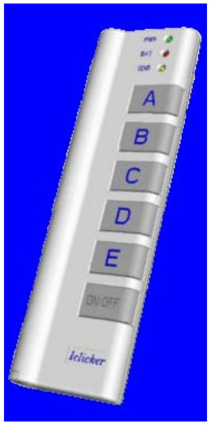


Two identical parallel plate capacitors are connected to a battery, as shown in the figure. C_1 is then disconnected from the battery, and the separation between the plates of both caps is doubled.

➔ What is the relation between the voltages on the two capacitors?

a) $V_1 > V_2$ b) $V_1 = V_2$ c) $V_1 < V_2$



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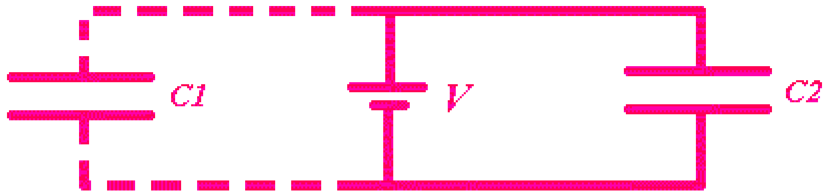

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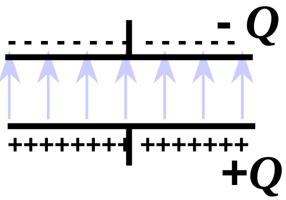
Where is the Energy Stored?

- **Claim: energy is stored in the electric field itself. Think of the energy needed to charge the capacitor as being the energy needed to create the field.**

- **To calculate the energy density in the field, first consider the constant field generated by a parallel plate capacitor, where**



$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q^2}{(A\varepsilon_0/d)}$$

- **The electric field is given by:**

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \quad \Rightarrow \quad U = \frac{1}{2}\varepsilon_0 E^2 A d$$

This is the energy density, *u*, of the electric field….

- **The energy density *u* in the field is given by:**

$$\boxed{u = \frac{U}{volume} = \frac{U}{Ad} = \frac{1}{2}\varepsilon_0 E^2}$$

Units: $\frac{\mathrm{J}}{\mathrm{m}^3}$

Energy Density

Claim: the expression for the energy density of the electrostatic field

$$\boxed{u = \frac{1}{2}\varepsilon_0 E^2}$$

is general and is not restricted to the special case of the constant field in a parallel plate capacitor.

- **Example (and another exercise for the student!)**
  - Consider $\boldsymbol{E}$- field between surfaces of cylindrical capacitor:
  - Calculate the energy in the field of the capacitor by integrating the above energy density over the volume of the space between cylinders.

$$U = \frac{1}{2}\varepsilon_0 \int E^2 dV = \frac{1}{2}\varepsilon_0 \int\int E^2 \pi r\, dr\, dl = etc.$$

  - Compare this value with what you expect from the general expression:

$$\boxed{W = \frac{1}{2}CV^2}$$

For next time

- HW #3 → turn in if haven't
- HW #4 → available
- Office Hours immediately after this class (9:30 – 10:00) in WAT214 [M 1:30-2; WF 1-1:30]
- Don't fall behind – 2nd Quiz Friday