

Electric Potential Energy

Start “new” topic...

- Conservative Forces and Energy Conservation
 - Total energy is constant and is sum of kinetic and potential
- Electric Potential Energy

Conservation of Energy of a particle from Phys 170

- **Kinetic Energy (K)** $K = \frac{1}{2}mv^2$
 - non-relativistic
- **Potential Energy (U)** $U(x, y, z)$
 - determined by force law
- **for Conservative Forces: K+U is constant**
 - total energy is always constant
- **examples of conservative forces**
 - gravity; gravitational potential energy
 - springs; coiled spring energy (Hooke's Law): $U(x) = \frac{1}{2}kx^2$
 - electric; electric potential energy (today!)
- **examples of non-conservative forces (heat)**
 - friction
 - viscous damping (terminal velocity)

Example: Gravity at surface of earth.

Gravitational force conservative.

$$F = mg = \text{const.}$$

$U = mgy$ gravitational potential energy

$$U + K = \text{constant}$$

drop ball from a:

U large; K small

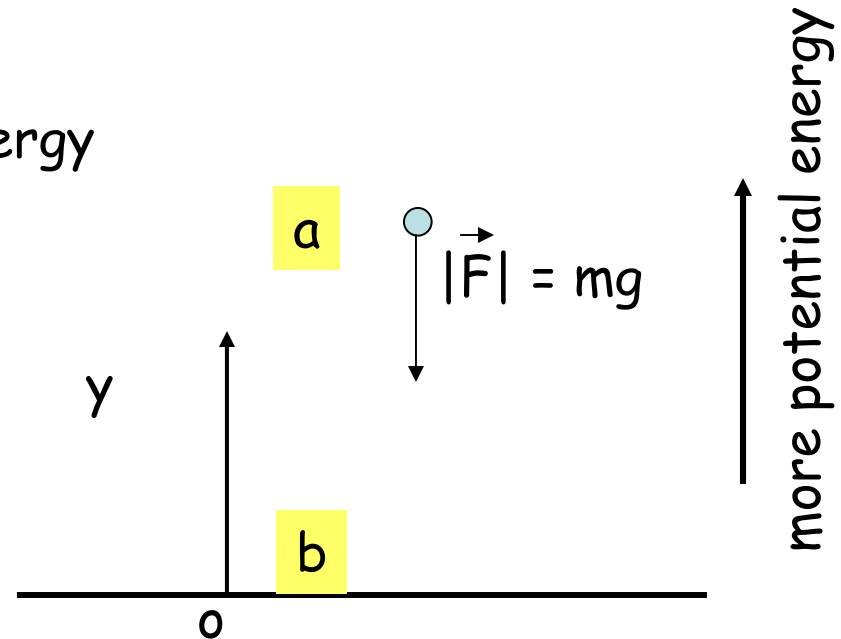
at b: K large; U small

everywhere:

$$U + K = \text{const.}$$

$$U_a + K_a = U_b + K_b$$

Energy conserved.



Can solve this problem two ways:

- using forces and Newton's Laws
- using C. of E.

Example: Charge q in uniform E field.

Electrical force also conservative.

For uniform electric field:

$$F = qE = \text{const.}$$

$$U = qEy \quad \text{electrical potential energy}$$

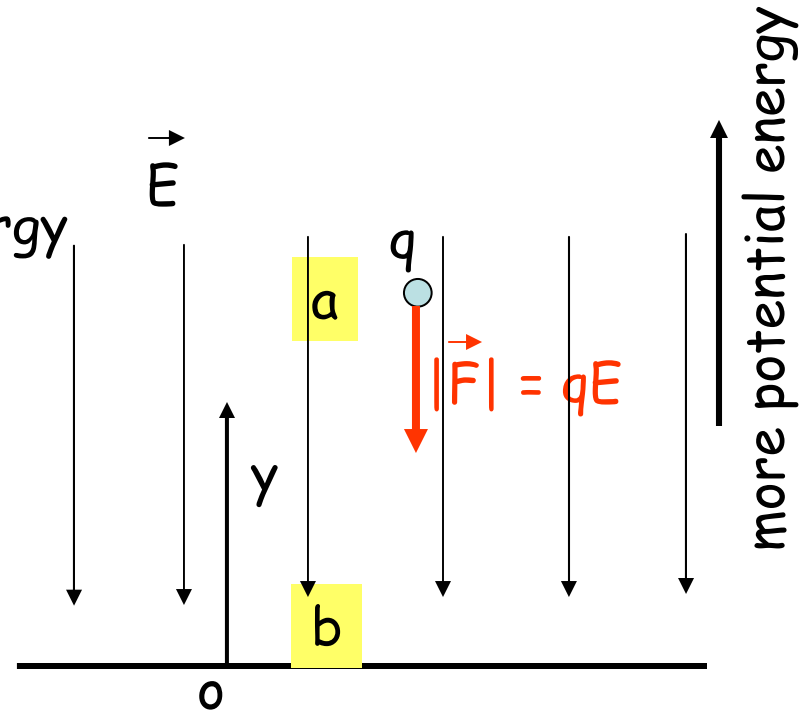
release charge from rest at a:

E and F downward

as it accelerates down,

K increases; U decreases.

$$U + K = \text{constant}$$



Uniform gravity	Uniform E field
$U = mgy$	$U = qEy$

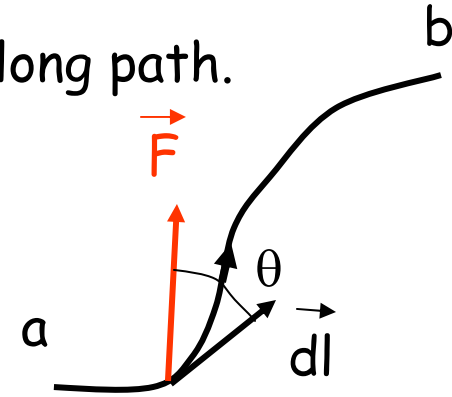
Two problems very similar.

Work and potential energy.

$W_{a \rightarrow b}$ = work done by force in going from a to b along path.

$$W_{a \rightarrow b} = \int_a^b \vec{F} \cdot d\vec{l} = \int_a^b F dl \cos \theta$$

$$\Delta U = (U_b - U_a) = - W_{a \rightarrow b} = + W_{b \rightarrow a}$$



For our example:

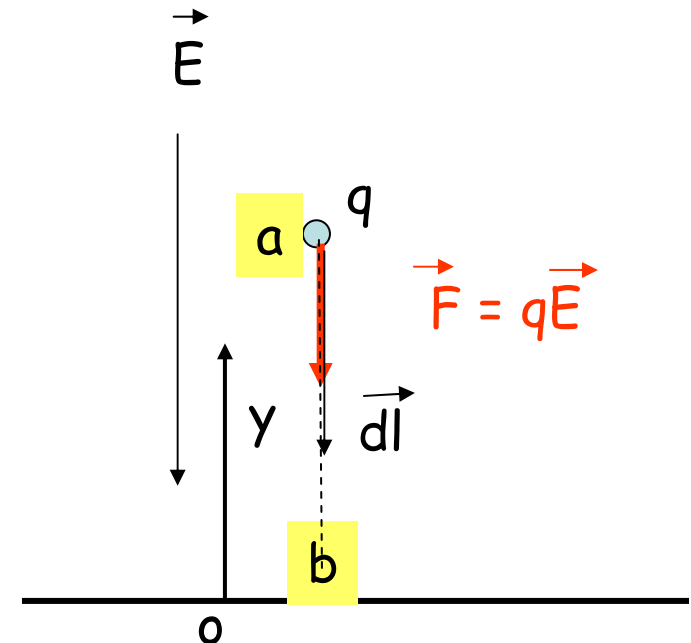
$$W_{a \rightarrow b} = \int_a^b \vec{F} \cdot d\vec{l} = q \int_a^b \vec{E} \cdot d\vec{l} = q \int_a^b E dl \cos 0^\circ$$

$$= q \int_a^b E dl = qE \int_a^b dl = qE(y_a - y_b)$$

$$U_a - U_b = + W_{a \rightarrow b} = qE(y_a - y_b)$$

let $y_b = 0$ and choose $U = 0$ at $y = 0$

$$U = qEy$$



Electric potential energy

Imagine two positive charges, one with charge q , the other with charge q_o :

q

q_o

Initially the charges are very far apart, so we say that the initial potential energy U_i is zero

(we are free to *define* the energy zero *somewhere*)

Consider q fixed and q_o moves radially from r_a to r_b

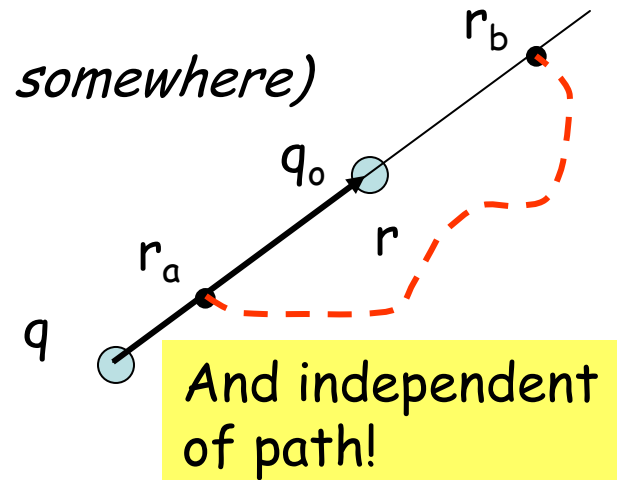
$$\begin{aligned} W_{a \rightarrow b} &= \int_a^b \vec{F} \cdot d\vec{l} = q_o \int_{r_a}^{r_b} \vec{E}_r \cdot d\vec{r} = q_o \int_{r_a}^{r_b} E_r dr \cos 0^\circ \\ &= q_o \int_{r_a}^{r_b} \frac{1}{4\pi\epsilon_o} \frac{q}{r^2} dr = \frac{qq_o}{4\pi\epsilon_o} \left(\frac{1}{r_a} - \frac{1}{r_b} \right) = U_a - U_b \end{aligned}$$

Can identify:

$$U = \frac{qq_o}{4\pi\epsilon_o} \frac{1}{r}$$

Like:

$$U = -\frac{Gmm_o}{r}$$



Electric potential energy

What if q is positive and q_o is negative?

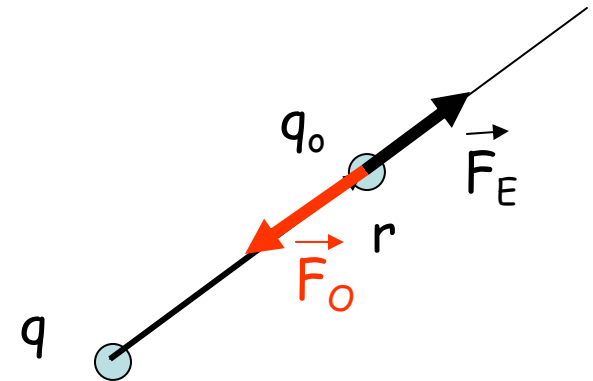
$$U = \frac{qq_o}{4\pi\epsilon_o} \frac{1}{r}$$

Particles tend to move to smaller U .

- like charges repel.
- unlike charges attract.

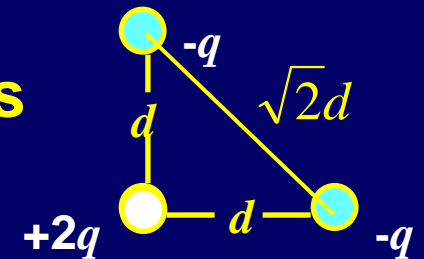
Can also think of the U as the work done by an outside agent to assemble the charge distribution starting with the particles at infinity.

$$\begin{aligned}\vec{F}_O &= -\vec{F}_E \\ W_O &= -W_E = U\end{aligned}$$



Electric Potential Energy

- Example: What is the potential energy of this collection of charges?



Step 1: Bring in $+2q$ from infinity. This costs nothing.

Step 2: Bring in one $-q$ charge. The force is attractive! The work required is negative:

$$U = \frac{(2q)(-q)}{4\pi\epsilon_0 d}$$

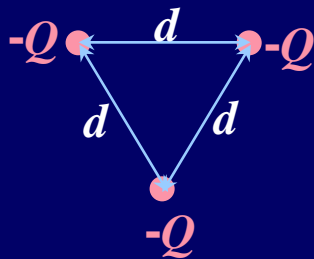
Step 3: Bring in 2nd $-q$ charge. It is attracted to the $+2q$, but repelled from the other $-q$ charge. The total work (all 3 charges) is

$$U = \left\{ \frac{(2q)(-q)}{4\pi\epsilon_0 d} + \frac{(2q)(-q)}{4\pi\epsilon_0 d} + \frac{(-q)(-q)}{4\pi\epsilon_0 \sqrt{2}d} \right\} = -\frac{q^2}{4\pi\epsilon_0 d} \left\{ 4 - \frac{1}{\sqrt{2}} \right\}$$

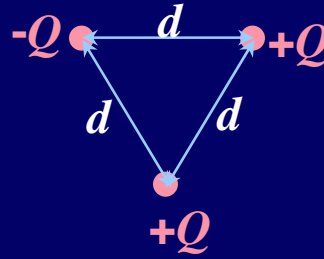
A *negative* amount of work was required to bring these charges from infinity to where they are now (*i.e.*, the attractive forces between the charges are larger than the repulsive ones).

Example 3

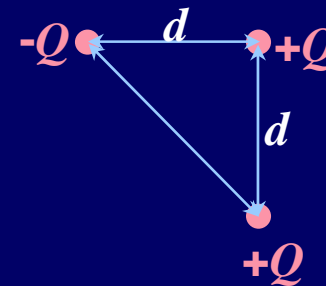
- Consider the 3 collections of point charges shown below.
 - Which collection has the smallest potential energy?



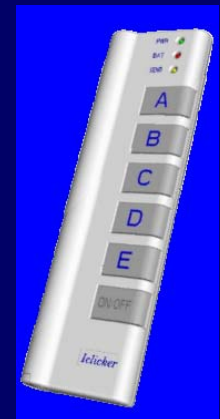
(a)



(b)

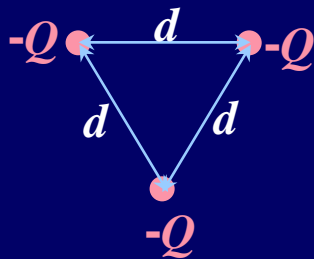


(c)

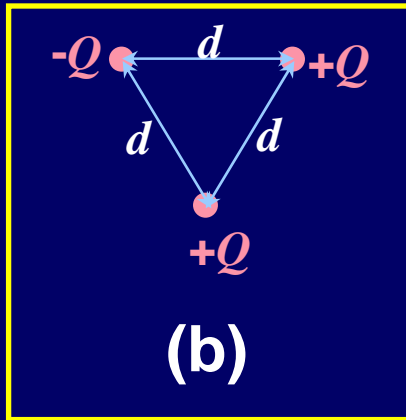


Example 3

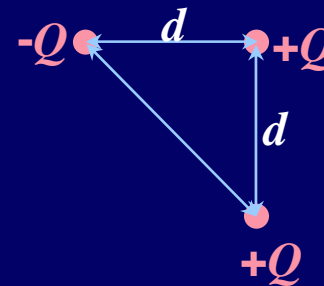
- Consider the 3 collections of point charges shown below.
 - Which collection has the least potential energy?



(a)



(b)



(c)

- We have to do positive work to assemble the charges in (a) since they all have the same charge and will naturally repel each other. In (b) and (c), it's not clear whether we have to do positive or negative work since there are 2 attractive pairs and one repulsive pair.

$$(a) \quad U = +3 \frac{1}{4\pi\epsilon_0} \frac{Q^2}{d}$$

$$(b) \quad U = - \frac{1}{4\pi\epsilon_0} \frac{Q^2}{d}$$

$$(c) \quad U = - \frac{1}{4\pi\epsilon_0} \frac{Q^2}{\sqrt{2}d}$$



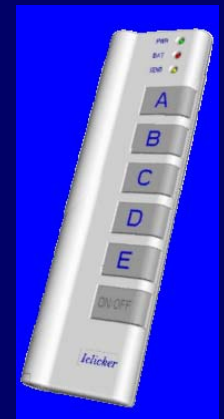
Example 4:

Two charges which are equal in magnitude, but opposite in sign are placed at equal distances from point A.



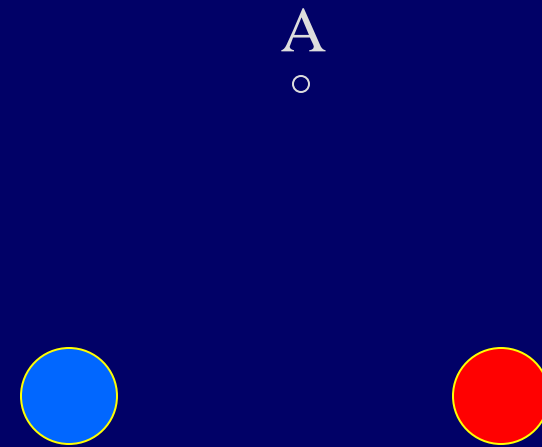
If a third charge is added to the system and placed at point A, how does the electric potential energy of the charge pair change ?

- a) increases
- b) decreases
- c) doesn't change



Example 4:

Two charges which are equal in magnitude, but opposite in sign are placed at equal distances from point A.



If a third charge is added to the system and placed at point A, how does the electric potential energy of the charge collection change ?

a) increases

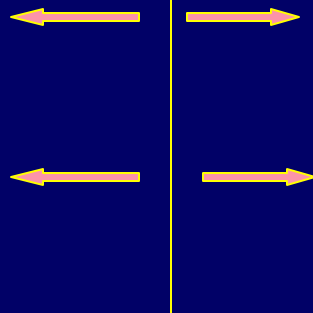
b) decreases

c) doesn't change

Boundary conditions example

Infinite plane with
charge density σ

$+\sigma$

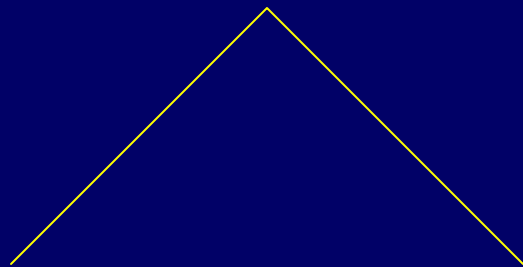


Is the E field continuous
across the plane ?

No !

Is the electric potential V
continuous across the plane ?

Yes ! (energy is not
created or destroyed)



Conclusion is general

For next time

- HW #3 → start on (Hints posted)
- Office Hours immediately after this class (9:30 – 10:00) in WAT214 (1-1:30 WF)
- Don't fall behind – next 2nd Quiz coming

