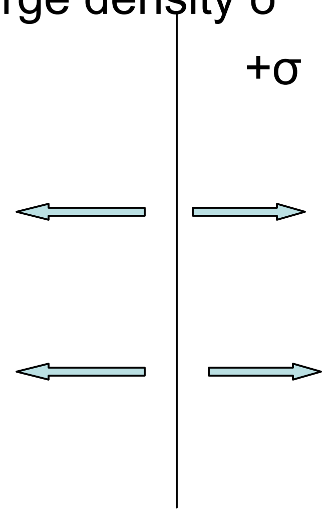
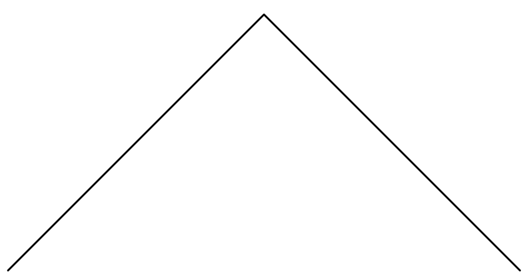
Boundary conditions example

Infinite plane with charge density σ





Is the E field continuous across the plane ?

No !

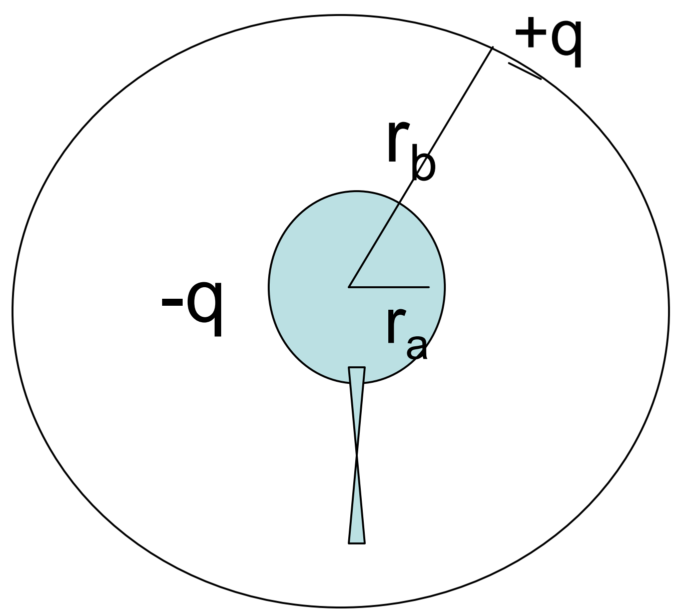
Is the electric potential V continuous across the plane ?

Yes ! (energy is not created or destroyed)

Conclusion is general

Hints for 23.49

Assume the potential at infinity is zero



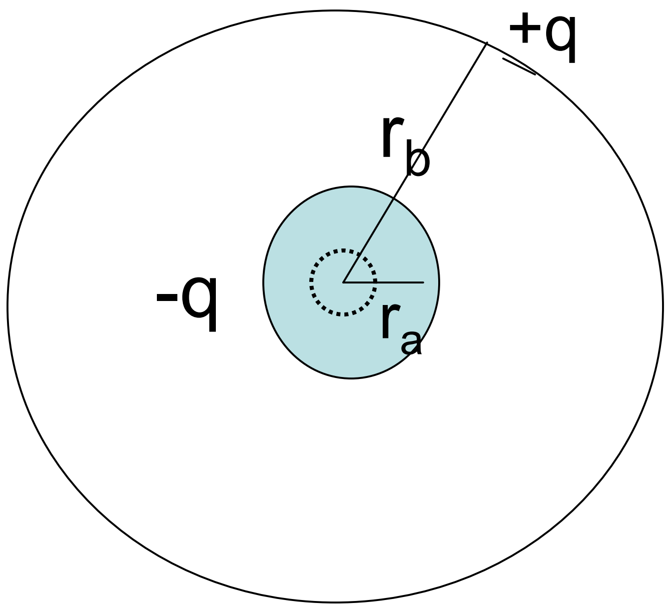
A metal sphere with radius r is supported on an insulating stand in the center of a hollow metal spherical shell of radius b.

There is a charge –q on the inner sphere and a charge +q on the outer shell.

<u>One approach: find the E fields and then the potentials. Match the boundary conditions</u>

Hints for 23.49 cont’d

Make a Gaussian surface inside the sphere. What is E inside ?

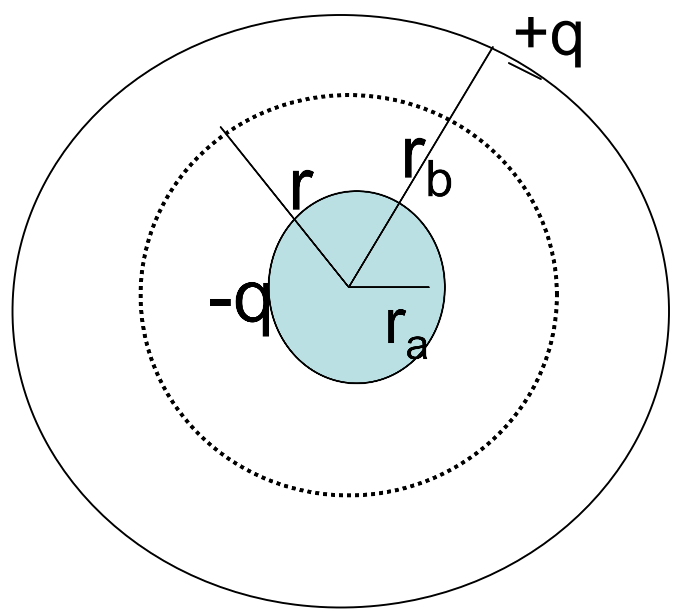


What is the electric potential for $r<r_a$?

$V=V_0$ we will determine this constant later.

Hints for 23.49 (cont'd)

Now make a spherical Gaussian surface at radius r



What is the charge enclosed ?

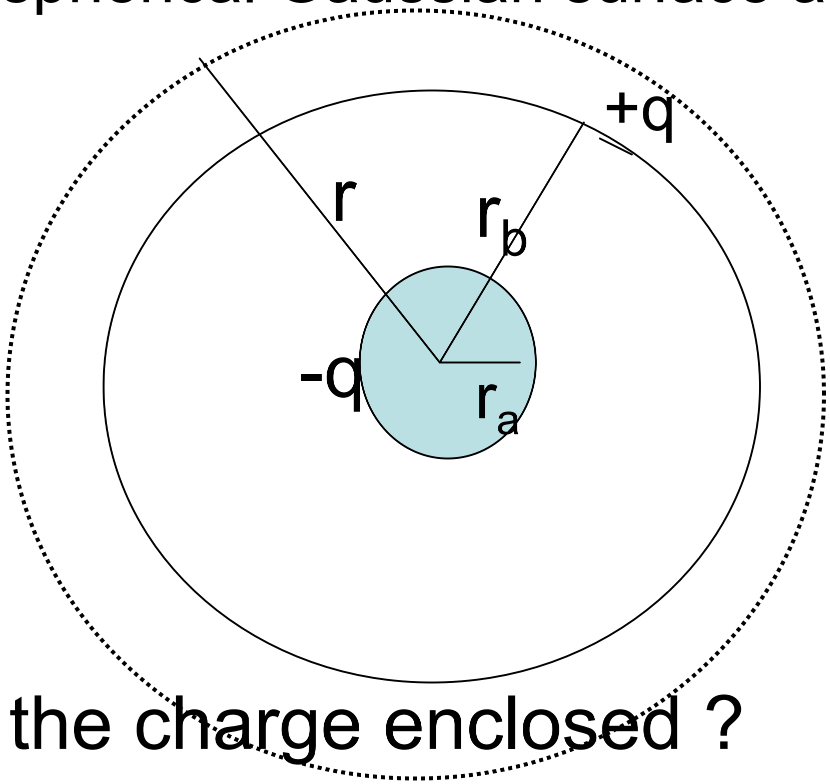
What is the E field ?

$$E = \frac{-q}{4\pi\varepsilon_0 r^2}$$

$$V(r) = \frac{-q}{4\pi\varepsilon_0 r} + V_1$$

Hints for 23.49 (cont'd)

Now make a spherical Gaussian surface at radius r outside.



What is the charge enclosed ?

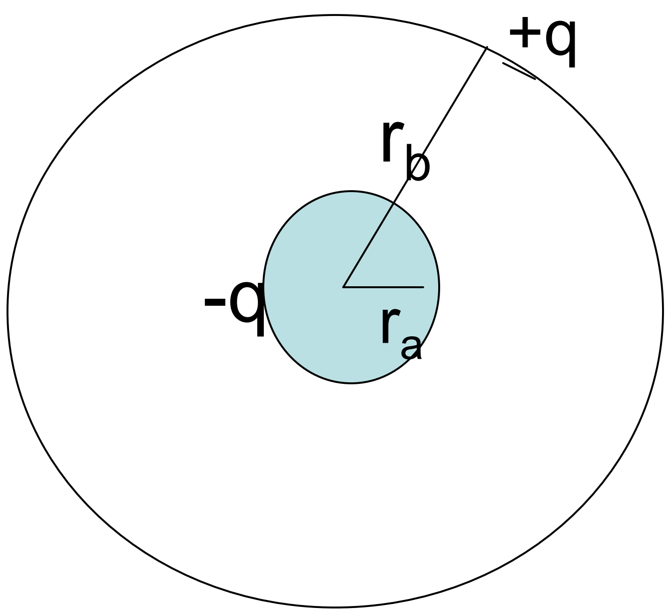
What is the E field ?

$$V(r) = V_2 = 0$$

What is the potential ?

Hints for 23.49 (cont'd)

Now put all the electric potentials together.



What happens at the boundaries ?

r<r a $$V(r) = V_0$$

r_a <r<r_b $$V(r) = \frac{-q}{4\pi\varepsilon_0 r} + V_1$$

r>r_b $$V(r) = V_2 = 0$$

$$r = r_a; V = V_0 = \frac{-q}{4\pi\varepsilon_0 r_a} + V_1$$

$$r = r_b; V = 0 = \frac{-q}{4\pi\varepsilon_0 r_b} + V_1$$

Hints for 23.49 (cont'd)

$$r = r_b; V = 0 = \frac{-q}{4\pi\varepsilon_0 r_b} + V_1 \qquad V_1 = \frac{q}{4\pi\varepsilon_0 r_b}$$

$$r = r_a; V = V_0 = \frac{-q}{4\pi\varepsilon_0 r_a} + V_1 \qquad V_0 = \frac{-q}{4\pi\varepsilon_0 r_a} + \frac{q}{4\pi\varepsilon_0 r_b}$$

r<a $V(r) = V_0$

a<r<b $V(r) = \frac{-q}{4\pi\varepsilon_0 r} + V_1$

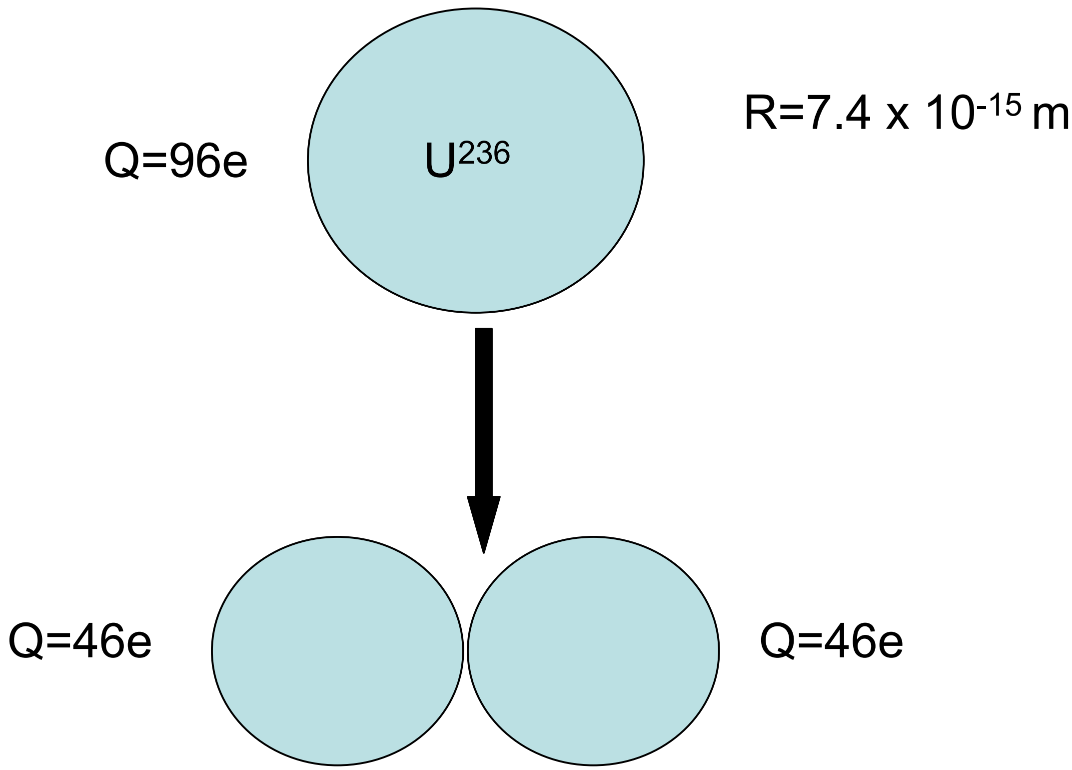
r>b $V(r) = V_2 = 0$

Calculate $V_{ab}=V_b-V_a$

$$V_{ab} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = V_0$$

Hints for 23.87

Nuclear Fission

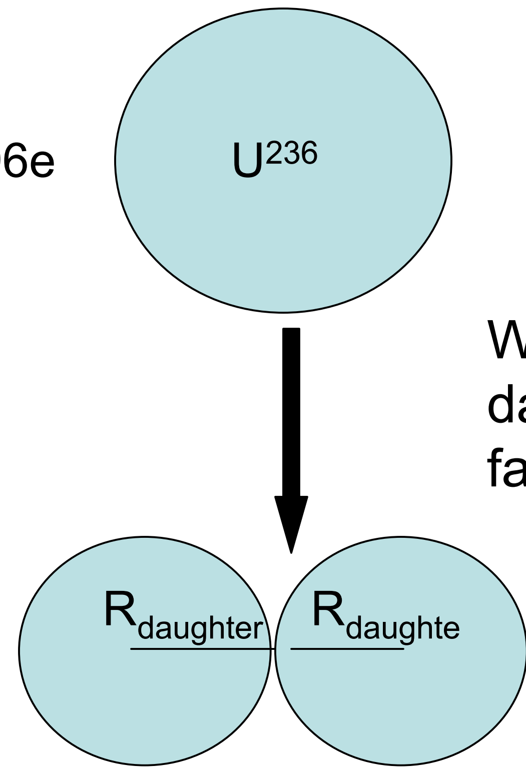


Half the volume and half the charge of the original nucleus

$$V/2 = \frac{4\pi}{3} R_{daughter}^3$$

What is the kinetic energy of each daughter nucleus when they are far apart ?

Hints for 23.87



What is the kinetic energy of each daughter nucleus when they are far apart ?

$$U = \frac{k(46e)^2}{2R_{daughter}} \quad \Rightarrow \quad K = \frac{U}{2}$$

Numerical final result: 253 kilotons of TNT from 10 kg of uranium !!

This is "electric potential energy"