Reminders:

1) All assignments must already be completed

2) Review Practice Final today

3) Will post grades for final HWs, including online, clicker scores, etc by end of week
Final exam information:

1) 1 page of notes

2) 8 problems, 2 hours (~15 minutes/problem)

3) Roughly equal balance of material throughout the course

4) Don't miss/arrive late

5) Suggest writing key equation when starting each problem/problem sub-part

6) Units can be your best friend
Problem 1: 25 points

A very long conducting tube (hollow cylinder) has inner radius $a$ and outer radius $b$. It carries charge per unit length $+\alpha$, where $\alpha$ is a positive constant with units of C/m. A line of charge lies along the axis of the tube. The line of charge has charge per unit length $+\alpha$.

(a) Calculate the electric field in terms of $\alpha$ and distance $r$ from the axis of the tube for

1. $r < a$
2. $a < r < b$
3. $r > b$

(b) Show your results in graph of $E$ as a function of $r$, making sure to indicate the relevant radii.

(b) What is the charge per unit length on

1. the inner surface of the tube
2. the outer surface of the tube
Gauss' Law

\[ \oint E \cdot d\mathbf{A} = \Phi_E = \frac{q_{\text{enclosed}}}{\varepsilon_0} \]
Infinite Line of Charge

- Symmetry \( \Rightarrow E \)-field must be \( \perp \) to line and can only depend on distance from line
- Therefore, CHOOSE Gaussian surface to be a cylinder of radius \( r \) and length \( h \) aligned with the \( x \)-axis.

Apply Gauss’ Law:
- On the ends, \( \vec{E} \cdot d\vec{A} = 0 \)
- On the barrel, \( \int \vec{E} \cdot d\vec{A} = 2\pi rhE \) AND \( q = \lambda h \) \( \Rightarrow \)

\[
E = \frac{\lambda}{2\pi \varepsilon_0 r}
\]
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(b) What is the charge per unit length on

1. the inner surface of the tube
2. the outer surface of the tube

\[ \int \vec{E} \cdot d\vec{A} = 2\pi rhE \]

\[ q = \alpha h \]

\[ E = \frac{\alpha}{2\pi\varepsilon_0 r} \]
Problem 2: 25 points

An electron (charge \( -e = 1.6 \times 10^{-19} \text{C} \)) moves in a straight line from point \( a \) to point \( b \) inside an old Cathode Ray Tube television set, a total distance of \( d = 0.5 \text{m} \). The electric field is uniform along this line with magnitude \( E = 1.7 \times 10^4 \text{N/C} \) in the direction from \( a \) to \( b \). Determine

(a) the force on the electron?

(b) the work done on it by the field?

(c) the potential difference \( V_a - V_b \)?
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(a) the force on the electron?

(b) the work done on it by the field?

\[ W_{a \to b} = \int_a^b \vec{F} \cdot d\vec{l} = \int_a^b F \cdot dl \cos \theta \]

(c) the potential difference \(V_b - V_a\)?

\[ \Delta V = V_b - V_a = \frac{\Delta U}{q} = \frac{U_b - U_a}{q} = -\frac{W_{a \to b}}{q} = -\int_a^b \vec{E} \cdot d\vec{l} \]
Problem 3: 25 points

A single charged particle of charge $q$ is moving in the $x$ direction at time $t = 0$ with instantaneous velocity $\vec{\sigma} = vi$ in the presence of a magnetic field $\vec{B} = Bj$.

(a) Sketch the motion of the charged particle.

(b) If the magnetic field strength is increased, what will happen to the motion of the charged particle?

(c) Will the charged particle speed up or slow down?

(d) When the magnetic field is stable, how much work is being done on the charged particle?
Charged particle motion in a constant $B$ field - velocity in plane $\perp$ to $B$.

Suppose we have a magnetic field given by $\vec{B} = -B_0 \hat{z}$ and a particle starts out at the origin moving in the $+x$ direction. The particle will move in a circle with the radius $R$ and angular velocity $\omega$:

$$\vec{F} = m\vec{a} = qvB = m\frac{v^2}{R}$$

magnetic force provides centripedal force

$$R = \frac{mv}{qB}$$

independent of $r$ and $v$. 

$$\omega = \frac{v}{R} = \frac{qB}{m}$$
Problem 3: 25 points

A single charged particle of charge \( q \) is moving in the \( x \) direction at time \( t = 0 \) with instantaneous velocity \( \vec{v} = v\hat{j} \) in the presence of a magnetic field \( \vec{B} = B\hat{j} \).

(a) Sketch the motion of the charged particle.

\[
\vec{F} = q\vec{v} \times \vec{B}
\]

(b) If the magnetic field strength is increased, what will happen to the motion of the charged particle?

\[
R = \frac{mv}{qB}
\]

(c) Will the charged particle speed up or slow down?

\[
\omega = \frac{v}{R} = \frac{qB}{m} \quad \text{independent of } r \text{ and } v.
\]

(d) When the magnetic field is stable, how much work is being done on the charged particle?

\[
W_{a \rightarrow b} = \int_{a}^{b} \vec{F} \cdot d\vec{l} = 0
\]
Problem 4: 25 points

A conducting bar with mass $m$ and length $L$ slides over horizontal rails that are connected to a voltage source. The voltage source maintains a constant current $I$ in the rails and bar, and a constant, uniform vertical magnetic field $\vec{B}$ fills the region between the rails.

(a) Find the magnitude and direction of the net force on the conducting bar. (ignore friction, air resistance, and electrical resistance)

(b) If the bar has mass $m$, find the distance that the bar must move to attain speed $v$

(c) It has been suggested that rail guns based on this principle could be used to accelerate payloads into earth orbit or beyond. Find the distance the bar must travel along the rails if it is to reach the escape speed for earth (11.2 km/s). Let $B = 0.5$ T, $I = 2.0 \times 10^3$ A, $m=25$ kg, and $L = 50$ cm.
The area is $LS(t)$ and $dS/dt$ equals velocity of slider

\[ \mathcal{E} = -\frac{d}{dt} \Phi_B = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A} = -\frac{d}{dt} \int B \, dA \]

\[ = -\frac{d}{dt} BA = -\frac{d}{dt} BLS = -BL \frac{d}{dt} S = -BLv \]
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A conducting bar with mass \( m \) and length \( L \) slides over horizontal rails that are connected to a voltage source. The voltage source maintains a constant current \( I \) in the rails and bar, and a constant, uniform vertical magnetic field \( \vec{B} \) fills the region between the rails.

(a) Find the magnitude and direction of the net force on the conducting bar. (ignore friction, air resistance, and electrical resistance)

\[
\vec{F} = L\vec{I} \times \vec{B}
\]

(b) If the bar has mass \( m \), find the distance that the bar must move to attain speed \( v \)

\[
F = ma \Rightarrow a = \frac{F}{m} = \frac{ILB}{m}
\]

\[
v^2 = v_0^2 + 2a(z - z_0)
\]

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Problem 5: 25 points

You are using a loop antenna with area 0.1 $m^2$ to detect EM waves for which $B_{rms} = 10^{-9}$ T.

\[ B_{max} = \sqrt{2} \cdot B_{rms} \]

(a) If the wave frequency is 1 MHz, what is the maximum value of the emf induced in the antenna?

\[ \varepsilon = \frac{1}{2} \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A} = -\frac{d\Phi_B}{dt} \]

(b) What is the $rms$ E field of these waves?

\[ E_{max} = \sqrt{2} \cdot E_{rms} \]
\[ E_{max} = c \cdot B_{max} \]

(c) What is the velocity of these EM waves?

\[ c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \]

(d) What is the intensity of these EM waves?

\[ I = \frac{1}{2} \varepsilon_0 c E_{max}^2 \]
Problem 6: 25 points

Evil Mr. Laserhands has tracked him down and found Mister Bond hiding in an aquarium filled with water (n=1.33). The window glass (n=1.5) is 3 cm thick and our secret agent is 5m below and 10m away from the window.

(a) Sketch the scene. Our villain uses a laser in the place of his right hand to finish off his victims. At what angle should he fire to hit 007?

(b) How does this differ from the direction Mr. Bond appears to be?

(c) How deep would our hero have to dive (staying same distance from window), to avoid getting blasted?
Snell’s Law (law of refraction)

\[ n_a \sin \theta_a = n_b \sin \theta_b \]

Medium a, \( v_a = \frac{c}{n_a} \)

Medium b, \( v_b = \frac{c}{n_b} \)

Angles are usually given relative to normal to plane.
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(a) Sketch the scene. Our villain uses a laser in the place of his right hand to finish off his victims. At what angle should he fire to hit 007?

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]
\[ n_2 \sin \theta_2 = n_3 \sin \theta_3 \]
\[ n_1 \sin \theta_1 = n_3 \sin \theta_3 \]

(b) How does this differ from the direction Mr. Bond appears to be?

(c) How deep would our hero have to dive (staying same distance from window), to avoid getting blasted?
Problem 7: 25 points

(a) Copper sphere A has radius 5 cm, while copper sphere B has a radius of 10 cm. The two spheres are connected by a conducting wire. Is the magnitude of the electric potential of sphere A (larger than, smaller than, or the same as) that of sphere B? (explain).

Conductor = equipotential

(b) Consider a series R-L-C \((R = 1 \, \Omega, \, L = 1 \, \text{mH}, \, \text{and} \, C = 1 \, \text{pF})\) circuit with an ac generator that runs at 10 kHz. Which is larger, the resistance, capacitive reactance, inductive reactance (pick one)?

\[ R = 1\Omega \quad X_C = \frac{1}{\omega C} \Omega \quad X_L = \omega L \Omega \]

(c) Far away from a dipole, electric field falls off like \(1/r^2, \, 1/r, \, 1/r^3\) (pick one)

\[ E_y(x,0) = -2 \frac{1}{4\pi \varepsilon_0} \frac{q}{(x^2 + a^2)^{3/2}} \]

(d) According to Ampere’s Law, the magnetic field of a long straight current-carrying wire falls off like \(1/r^2, \, 1/r, \, 1/r^3\) (pick one) and is in the radial, azimuthal or along the wire (pick one) direction.

\[ |\vec{B}| = \frac{\mu_0 I}{2\pi r} \hat{r} \]
TIR will occur for $\theta >$ critical angle. Snell’s law says $\sin \theta_c = n_2/n_1$.

Next slide
Why is the sky **blue**?

• Light from Sun scatters off of air particles—“Rayleigh scattering”
  – Rayleigh scattering is wavelength-dependent.
  – Shorter wavelengths (blue end of the visible spectrum) scatter more.

![Rayleigh scattering diagram](image)

• This is also why sunsets are **red**!
  – At sunset, the light has to travel through more of the atmosphere.
  – If longer wavelengths (red and orange) scatter less…
  – The more air sunlight travels through, the redder it will appear!
  – This effect is more pronounced if there are more particles in the atmosphere (e.g., sulfur aerosols from industrial pollution).
Problem 8: 25 points

(a) Will total internal reflection occur for light going from air towards water? How about from water towards air? Explain.

(b) Why is the sky red/orange at sunset? (give a short explanation).

(c) Given supplies/equipment you can find at a hardware store, explain how you might make a magnet to generate a 1 T magnetic field.

**Next slide**

(d) Arrange the following types of electromagnetic radiation in order of increasing energy: infrared light, ultraviolet light, microwaves, x-rays, FM radio.

(e) Explain how a mass spectrometer is used to separate isotopes of different materials.
B Field of an ∞ Solenoid

• To calculate the B field of the solenoid using Ampere's Law, we need to justify the claim that the B field is nearly 0 outside the solenoid (for an ∞ solenoid the B field is exactly 0 outside).

• To do this, view the ∞ solenoid from the side as 2 ∞ current sheets.

• The fields are in the same direction in the region between the sheets (inside the solenoid) and cancel outside the sheets (outside the solenoid).

• B is uniform inside solenoid and zero outside.

• Draw square path of side w:

\[ \oint \vec{B} \cdot d\vec{l} = Bw \]

\[ I = nwi \quad \Rightarrow \quad B = \mu_0ni \]

Note: \( B \propto \frac{\text{Amp}}{\text{Length}} \)
Problem 8: 25 points

(a) Will total internal reflection occur for light going from air towards water? How about from water towards air? Explain.

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The ElectroMagnetic Spectrum

PROPERTIES of ELECTROMAGNETIC WAVES

Increasing energy

Electromagnetic Spectrum

- Radio
- Microwaves
- Infrared
- Ultraviolet
- X-ray
- Gamma

λ (meters)

f (Hz)

10^3 1 10^{-3} 10^{-6} 10^{-9} 10^{-12}

longer shorter

10^6 10^9 10^{12} 10^{15} 10^{18} 10^{21}

lower higher
Problem 8: 25 points

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Next slides
A charged particle enters a region with perpendicular electric and magnetic fields. The electric and magnetic forces will cancel if the velocity is just right. Particles with this velocity will go through undeflected. Others will be deflected.

\[ \vec{F}_E = q\vec{E} = -\vec{F}_m = -q\vec{v} \times \vec{B} \]

\[ q|\vec{E}| = q|\vec{v}|\vec{B}|1 \]

\[ |\vec{v}| = \frac{|\vec{E}|}{|\vec{B}|} \]

Choosing a particular E/B ratio will select the desired velocity.
Mass Spectrometer

Measure m to identify substances:
1.) Ionize atoms by hitting them with accelerated electrons.
2.) Accelerate ions through known potential V. q=e if singly ionized.
   \[ U=qV \]
3.) Velocity select.
4.) Pass ions through known B field and measure R.

\[ R = \frac{mv}{qB} \]
\[ v = \frac{qB}{R}; \quad v^2 = \frac{q^2B^2}{R^2} \]
\[ \frac{1}{2}mv^2 = qV \]
\[ m = \frac{qB^2R^2}{2V} \]
Again, suggestions:

1) Review all equations/material (just because I didn’t cover it today doesn’t mean it won’t appear)

2) 8 problems, 2 hours (~15 minutes/problem)

3) Suggest writing key equation when starting each problem/problem sub-part

4) Units can be your best friend
Onward to Final

• Good luck! (study hard and you won’t need it)

• Suggest review 1 chapter/day

• Review problems, quizzes, examples (all lectures posted)