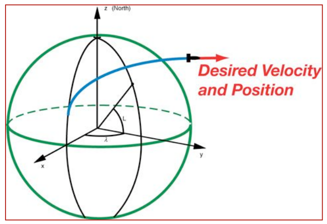
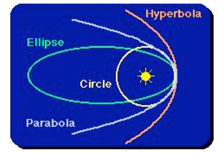
Launch Vehicle Design: Trajectories and Aerodynamics

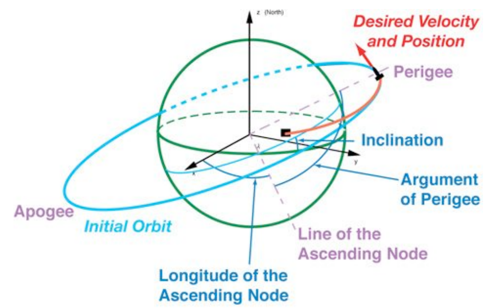
Space System Design, MAE 342, Princeton University
Robert Stengel

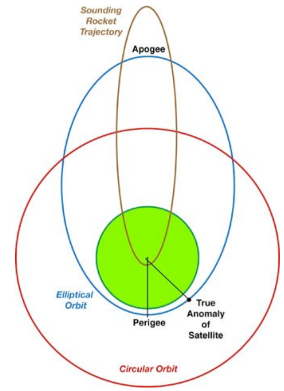
- Launch trajectories and effects
- Forces, moments, and stability
- Point mass, rigid body, and body bending





Final Position and Velocity Determine Orbital Elements





Identical major axes = Identical orbital periods

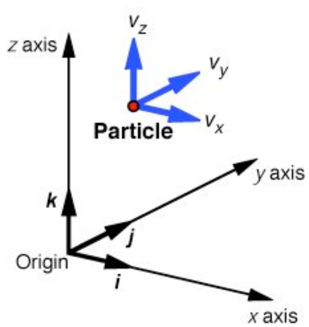
Position and Velocity

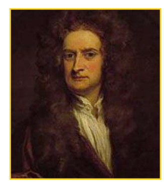
- **Position of a particle**

$$\mathbf{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

- **Velocity of a particle**

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$





Newton's Laws of Motion: Dynamics of a Particle

- **First Law**
  - If no force acts on a particle, it remains at rest or continues to move in a straight line at constant velocity, as observed in an inertial reference frame -- Momentum is conserved

$$\frac{d}{dt}(m\mathbf{v}) = 0 \quad ; \quad m\mathbf{v}\big|_{t_1} = m\mathbf{v}\big|_{t_2}$$

- **Second Law**
  - A particle of fixed mass acted upon by a force changes velocity with an acceleration proportional to and in the direction of the force, as observed in an inertial reference frame; the ratio of force to acceleration is the mass of the particle: F = *m* a

$$\frac{d}{dt}(m\mathbf{v}) = m\frac{d\mathbf{v}}{dt} = \mathbf{F} \quad ; \quad \mathbf{F} = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}$$

$$\therefore \quad \frac{d\mathbf{v}}{dt} = \frac{1}{m}\mathbf{F} = \begin{bmatrix} 1/m & 0 & 0 \\ 0 & 1/m & 0 \\ 0 & 0 & 1/m \end{bmatrix} \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}$$

- **Third Law**
  - For every action, there is an equal and opposite reaction

Equations of Motion for a Point Mass

$$\frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}} = \mathbf{v} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$

$$\frac{d\mathbf{v}}{dt} = \dot{\mathbf{v}} = \frac{1}{m}\mathbf{F} = \begin{bmatrix} 1/m & 0 & 0 \\ 0 & 1/m & 0 \\ 0 & 0 & 1/m \end{bmatrix} \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}$$

Equations of Motion for a Point Mass

- Written as a single equation

$$\dot{\mathbf{x}}(t) = \frac{d\mathbf{x}(t)}{dt} = \mathbf{f}[\mathbf{x}(t), \mathbf{F}]$$

- With

$$\mathbf{x} \equiv \begin{bmatrix} \mathbf{r} \\ \mathbf{v} \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ v_x \\ v_y \\ v_z \end{bmatrix}$$

Combined Equations of Motion for a Point Mass

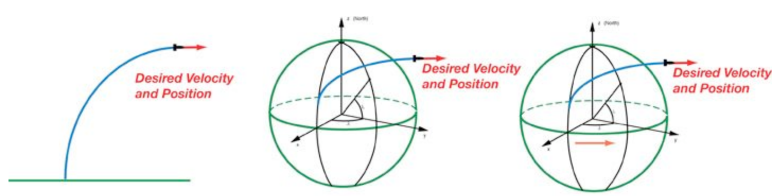
$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \end{bmatrix}_I = \begin{bmatrix} v_x \\ v_y \\ v_z \\ f_x/m \\ f_y/m \\ f_z/m \end{bmatrix}_I = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ v_x \\ v_y \\ v_z \end{bmatrix}_I + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1/m & 0 & 0 \\ 0 & 1/m & 0 \\ 0 & 0 & 1/m \end{bmatrix} \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}_I$$

- With

$$\mathbf{F}_I = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}_I = \left[\mathbf{F}_{gravity} + \mathbf{F}_{aerodynamics} + \mathbf{F}_{thrust}\right]_I$$

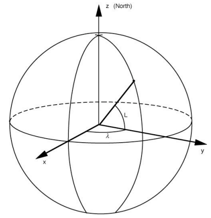
Newtonian Frame of Reference

- Newtonian (Inertial) Frame of Reference
  - Unaccelerated Cartesian frame whose origin is referenced to an inertial (non-moving) frame
  - Origin can translate at constant linear velocity
  - Frame cannot be rotating with respect to inertial origin
- ... but the Earth is rotating
  - Different approximations to "inertial" suit different problems



Flat Earth — Round Earth — Round, Rotating Earth

Force due to Gravity



- **Flat-earth approximation**
  - **g** is gravitational acceleration
  - *m***g** is gravitational force
  - Independent of position

$$m\mathbf{g}_f = m\begin{bmatrix}0\\0\\g_o\end{bmatrix} \quad ; \quad g_o = 9.807\ m/s^2$$

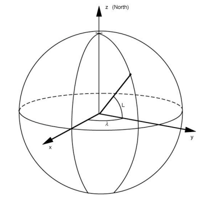
- **Round, rotating earth**
  - Inverse-square gravitation
  - "Centrifugal acceleration"
  - Non-linear function of position
  - μ = 3.986 x 10^{14} m/s^2
  - Ω = 7.29 x 10^{-5} rad/s

$$\mathbf{g}_r = \begin{bmatrix}g_x\\g_y\\g_z\end{bmatrix} = \mathbf{g}_{gravity} \quad [\text{non-rotating frame}]$$

$$\mathbf{g}_r = \mathbf{g}_{gravity} + \mathbf{g}_{rotation} \quad [\text{rotating frame}]$$

$$= \frac{-\mu}{r^3}\begin{bmatrix}x\\y\\z\end{bmatrix} - \Omega^2\begin{bmatrix}x\\y\\0\end{bmatrix} \quad ; \quad r = \left[x^2+y^2+z^2\right]^{1/2}$$

Equations of Motion with Round-Earth Gravity Model (Inertial, Non-Rotating Frame)

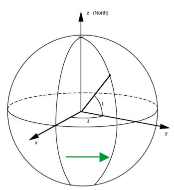


$$\begin{bmatrix}\dot{x}\\\dot{y}\\\dot{z}\\\dot{v}_x\\\dot{v}_y\\\dot{v}_z\end{bmatrix}_E = \begin{bmatrix}0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\-\mu/r^3&0&0&0&0&0\\0&-\mu/r^3&0&0&0&0\\0&0&-\mu/r^3&0&0&0\end{bmatrix}\begin{bmatrix}x\\y\\z\\v_x\\v_y\\v_z\end{bmatrix}_I + \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\\1/m&0&0\\0&1/m&0\\0&0&1/m\end{bmatrix}\left[\begin{pmatrix}f_x\\f_y\\f_z\end{pmatrix}_{aero} + \begin{pmatrix}f_x\\f_y\\f_z\end{pmatrix}_{thrust}\right]_I$$

- **Position of the vehicle (in spherical coordinates)**

$$\mathbf{r} = \begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}\cos L\cos\lambda\\\cos L\sin\lambda\\\sin L\end{bmatrix}(R+h)$$

R : Earth's radius
h : Altitude (height)
L : Latitude
λ : Longitude

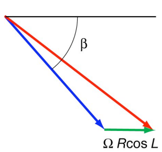


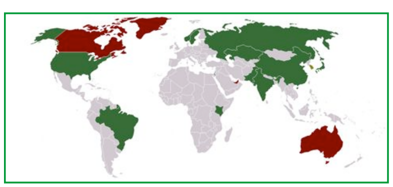
Effect of Launch Site on Launch Velocity

- **Launch site and azimuth**
  - **Earth's rotation adds up to 465 m/s to final inertial velocity**
  - **Function of launch latitude and azimuth angles**

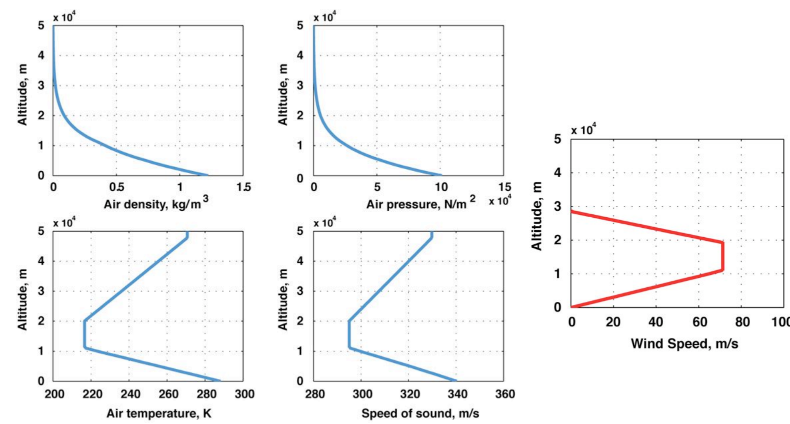
$$\Delta V_{launch} \approx \Omega R\cos L\cos\beta$$

β : Launch azimuth angle (from East)





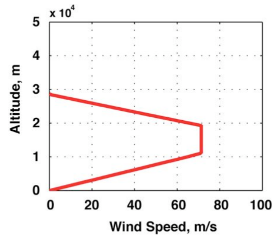
Properties of the Lower Atmosphere



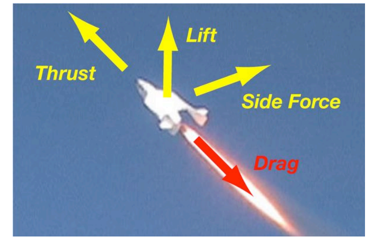
- **Air density and pressure decay exponentially with altitude**
- **Air temperature and speed of sound are linear functions of altitude**
- **Jet stream magnitude typically peaks at 10-15-km altitude**

Lower Atmosphere Rotates With The Earth

- Zero wind at Earth's surface = Inertially rotating air mass
- Wind measured with respect to Earth's rotating surface



Aerodynamic Forces



$$\begin{bmatrix} Drag \\ Side\ Force \\ Lift \end{bmatrix} = \begin{bmatrix} C_D \\ C_Y \\ C_L \end{bmatrix} \frac{1}{2}\rho V^2 S \approx \begin{bmatrix} Axial\ Force \\ Side\ Force \\ Normal\ Force \end{bmatrix} = \begin{bmatrix} C_A \\ C_Y \\ C_N \end{bmatrix} S\bar{q}$$

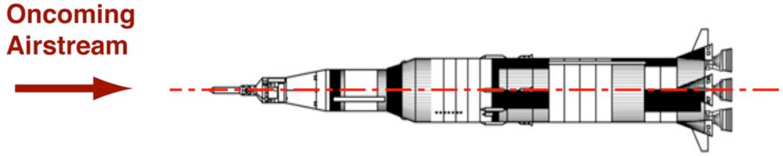
- *V* = air-relative velocity = velocity w.r.t. air mass
- Drag measured opposite to the air-relative velocity vector
- Lift and side force are perpendicular to the velocity vector

Aerodynamic Forces

$$\rho = air\ density, function\ of\ height$$
$$= \rho_{sea\ level} e^{-\beta z}$$
$$\rho_{sea\ level} = 1.225\ kg/m^3; \quad \beta = 1/9{,}042\ m$$
$$V = \left[v_x^{\ 2} + v_y^{\ 2} + v_z^{\ 2} \right]^{1/2} = \left[\mathbf{v}^T \mathbf{v} \right]^{1/2}$$
$$Dynamic\ \ pressure = \bar{q} = \frac{1}{2}\rho V^2$$
$$S = reference\ area,\ m^2$$
$$\begin{bmatrix} C_D \\ C_Y \\ C_L \end{bmatrix} = non-dimensional\ aerodynamic\ coefficients$$

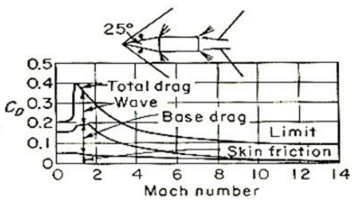
Aerodynamic Drag

$$Drag = C_D \frac{1}{2}\rho V^2 S$$

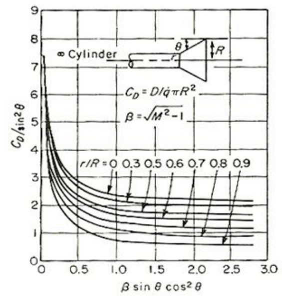


- Drag components sum to produce total drag
  - Skin friction
  - Base pressure differential
  - Forebody pressure differential (M > 1)

Drag Coefficients of Cones and Cone Frustums

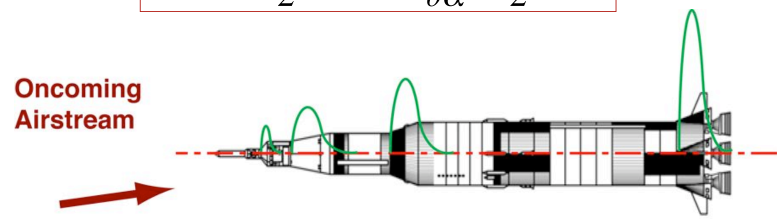


$$M = \frac{Air - Relative\,Velocity}{Speed\ of\ Sound}$$



Aerodynamic Lift Force

$$Lift = C_L \frac{1}{2}\rho V^2 S \approx \frac{\partial C_L}{\partial \alpha}\alpha \frac{1}{2}\rho V^2 S$$



- Angle between x axis and airstream = angle of attack, α
- Lift components integrate over length to produce net lift
  - Increase in cross-sectional area
  - Tail fins
- For symmetric vehicle, lift = 0 if $\alpha = 0$

2-D Equations of Motion for a Point Mass

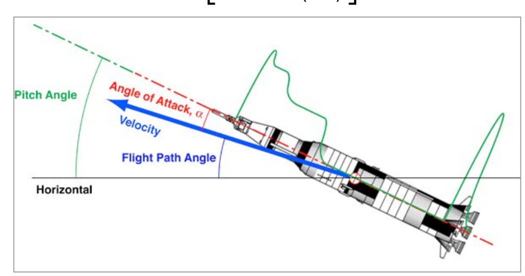
- Restrict motions to a vertical plane (i.e., motions in *y* direction = 0)

$$\begin{bmatrix} \dot{x} \\ \dot{z} \\ \dot{v}_x \\ \dot{v}_z \end{bmatrix} = \begin{bmatrix} v_x \\ v_z \\ f_x/m \\ f_z/m \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ z \\ v_x \\ v_z \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1/m & 0 \\ 0 & 1/m \end{bmatrix} \begin{bmatrix} f_x \\ f_z \end{bmatrix}$$

2-D Equations of Motion for a Point Mass

- Transform velocity from Cartesian to polar coordinates

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} V\cos\gamma \\ -V\sin\gamma \end{bmatrix}; \quad \begin{bmatrix} V \\ \gamma \end{bmatrix} = \begin{bmatrix} \sqrt{\dot{x}^2 + \dot{z}^2} \\ -\sin^{-1}\left(\frac{\dot{z}}{V}\right) \end{bmatrix} = \begin{bmatrix} Velocity \\ Flight\ path\ angle \end{bmatrix}$$



Flat-Earth Model

- Ignore round, rotating Earth effects (!)
- i.e., assume that flat-Earth-relative frame is inertial

$$\begin{bmatrix}\dot{x}\\ \dot{z}\\ \dot{v}_x\\ \dot{v}_z\end{bmatrix}=\begin{bmatrix}v_x\\ v_z\\ f_x/m\\ f_z/m\end{bmatrix}=\begin{bmatrix}0&0&1&0\\0&0&0&1\\0&0&0&0\\0&0&0&0\end{bmatrix}\begin{bmatrix}x\\ z\\ v_x\\ v_z\end{bmatrix}+\begin{bmatrix}0&0\\0&0\\1/m&0\\0&1/m\end{bmatrix}\begin{bmatrix}f_x\\ f_z\end{bmatrix}$$

$$\begin{bmatrix}\dot{x}\\ \dot{z}\end{bmatrix}=\begin{bmatrix}V\cos\gamma\\ -V\sin\gamma\end{bmatrix};\quad \begin{bmatrix}V\\ \gamma\end{bmatrix}=\begin{bmatrix}\sqrt{\dot{x}^2+\dot{z}^2}\\ -\sin^{-1}\left(\frac{\dot{z}}{V}\right)\end{bmatrix}=\begin{bmatrix}Velocity\\ Flight\ path\ angle\end{bmatrix}$$

- Adequate model for investigating early phase of launch

Simplified Launch Trajectory Equations of Motion

- Gravity-turn, flat earth, vertical plane
  - Thrust aligned with velocity vector ($\alpha = 0$)
  - Lift = 0
  - Round, rotating earth effects neglected

$$\dot{V}(t)=\frac{Thrust-\left[Drag+m(t)g\sin\gamma(t)\right]}{m(t)}=\left[\left(Thrust-C_D\frac{1}{2}\rho(h)V^2(t)\right)\Big/m(t)-g\sin\gamma(t)\right]$$

$$\dot{\gamma}(t)=-g\cos\gamma(t)/V(t)$$

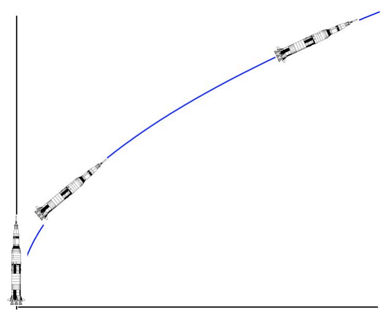
$$\dot{h}(t)=-\dot{z}(t)=V(t)\sin\gamma(t)$$

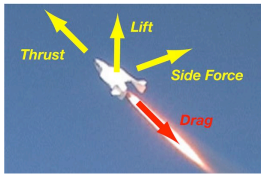
$$\dot{r}(t)=\dot{x}(t)=V(t)\cos\gamma(t)$$

V = velocity
γ = flight path angle
h = height (altitude)
r = range

Gravity-Turn Flight Path

- For vertical launch,
  - trajectory is vertical unless
  - vehicle is pitched over via thrust-vector control
- Following pitch-over,
  - if thrust is aligned with the velocity vector,
  - the result is called a gravity turn
- Gravity-turn flight path is a function of 3 variables
  - Initial pitch-over angle (from vertical launch)
  - Velocity at pitch-over
  - Acceleration profile, $T(t)/m(t)$

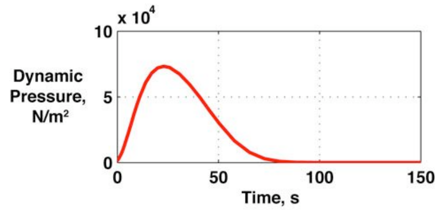




Typical Velocity Loss due to Drag During Launch

- Aerodynamic effects on launch vehicle are most important below ~50-km altitude
- Maintain angle of attack and sideslip angle near zero to minimize side force and lift
- Typical velocity loss due to drag for vertical launch
  - Constant thrust-to-weight ratio
  - $C_D S/m$ = 0.0002 m^2/kg
  - Final altitude above 80 km

Thrust-to-Weight Ratio	Velocity Loss, m/s
2	336
3	474
4	581

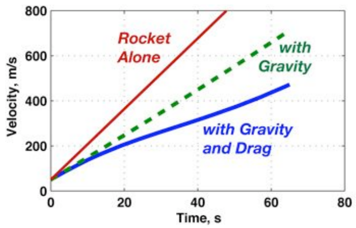


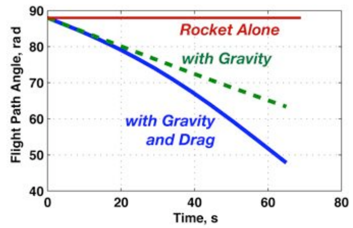
Effects of Gravity and Drag on the Velocity Vector

Thrust/Weight = T/W = 2
Thrust = 1960
C_D = 0.2
S = 0.1
Mass = 100

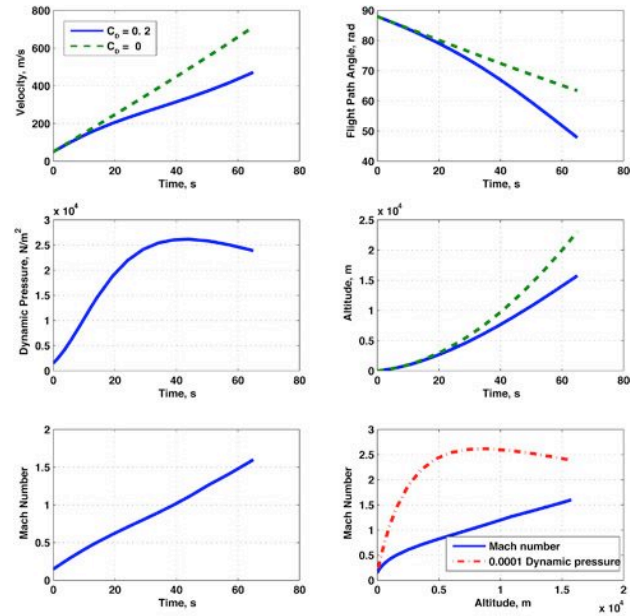
$$\dot{V}(t) = \frac{Thrust - \left[C_D S \frac{1}{2}\rho(h)V^2(t) + mg\sin\gamma(t)\right]}{m}$$

$$\dot{\gamma}(t) = -g\cos\gamma(t)/V(t)$$





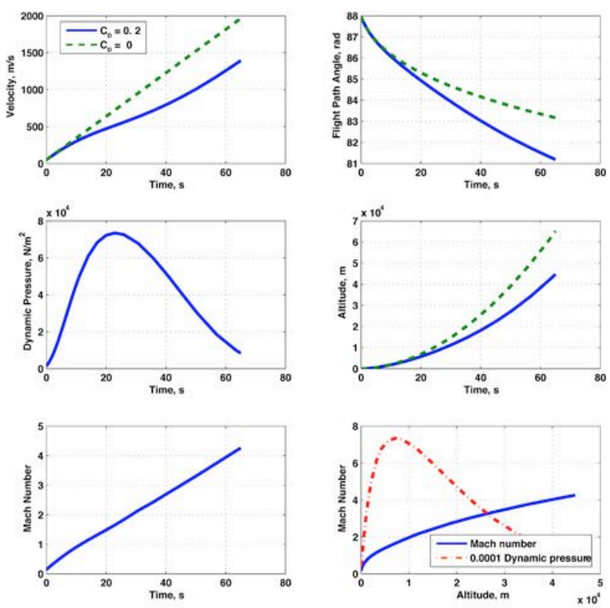
- **Significant reduction in velocity magnitude**
- **Strong curvature of the flight path**



Typical Properties of Launch Trajectories

Thrust/Weight = 2

- **Maximum dynamic pressure, Mach = 1, maximum drag, and maximum jet stream magnitude tend to occur at similar altitudes**
- **Aerodynamic effects on launch vehicle become negligible above ~50-km altitude**

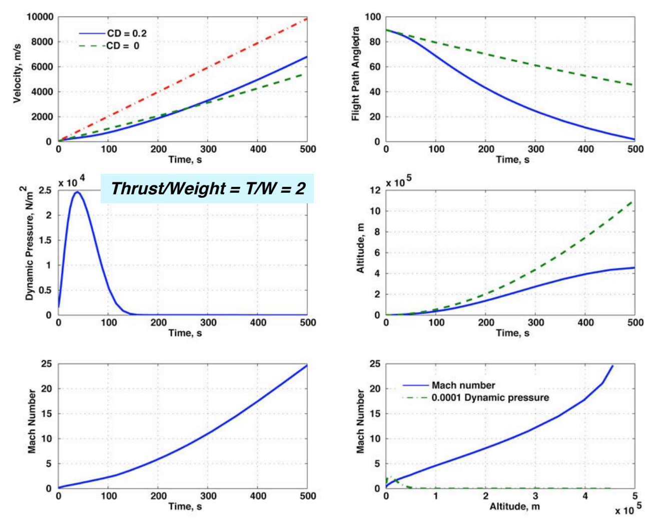


Typical Properties of Launch Trajectories

Thrust/Weight = 4

- **Maximum dynamic pressure, Mach = 1, maximum drag, and maximum jet stream magnitude tend to occur at similar altitudes**
- **Aerodynamic effects on launch vehicle become negligible above ~50-km altitude**

Gravity and Drag Effects during Single-Stage Orbital Launch

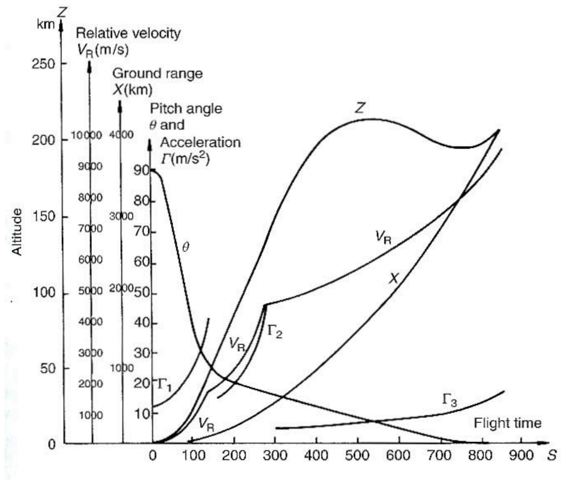


- **Pseudo-orbital launch trajectory (using flat-earth model)**
- **Red line signifies velocity due to rocket alone**
- **Several km/s lost to gravity and drag**

- **With higher T/W**
  - **Shorter time to orbit**
  - **Increased loss due to drag**
  - **Decreased loss due to gravity**

Typical Ariane 4 Launch Profile

(*Spacecraft Systems Engineering*, 2003)



Mass-Ratio Effect on Final Load Factor

- **Thrust-to-weight ratio = load factor**

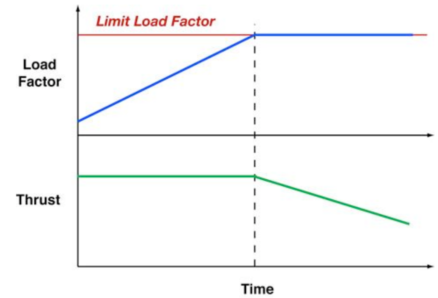
$$\frac{Thrust}{Weight} = n\,(load\ factor) = \frac{Thrust}{mg_o}$$

$$n_{initial} = \frac{Thrust}{m_{initial}g_o}; \quad n_{final} = \frac{Thrust}{m_{final}g_o}$$

- **If thrust is constant**

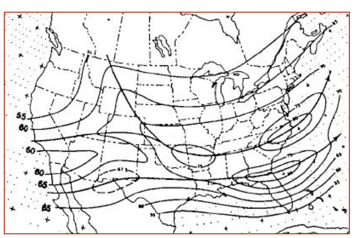
$$\frac{n_{final}}{n_{inital}} = \frac{m_{initial}}{m_{final}} = \mu$$

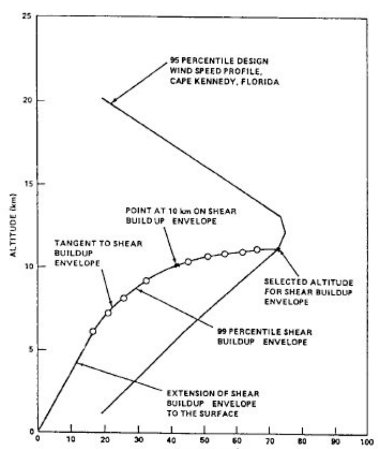
Initial Load Factor	Final Load Factor: Mass Ratio = 2	Mass Ratio = 5
1.3	2.6	6.5
2	4	10
3	6	15



Jet Stream Profiles

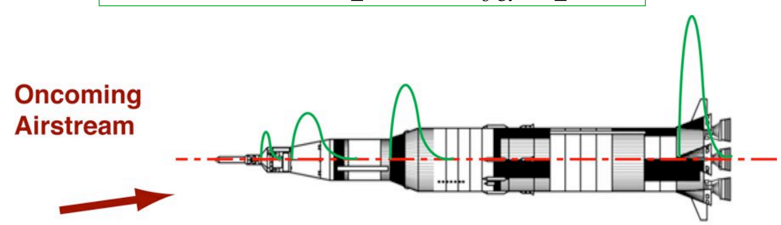
- **Launch vehicle must able to fly through strong wind profiles**
- **Design profiles assume 95th-99th-percentile worst winds and wind shear**





Aerodynamic Normal Force

$$Normal\ Force = C_N \frac{1}{2}\rho V^2 S \approx \frac{\partial C_N}{\partial \alpha}\alpha \frac{1}{2}\rho V^2 S$$

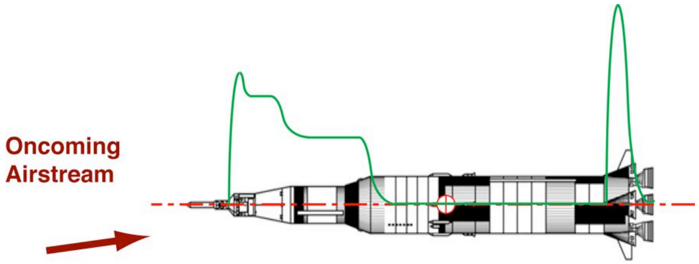


- **For small angle of attack, normal force is approximately the same as lift**

Aerodynamic Pitching Moment

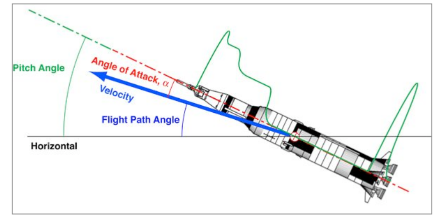
$$Pitching\ Moment = C_m \frac{1}{2}\rho V^2 Sr \approx \frac{\partial C_m}{\partial \alpha}\alpha \frac{1}{2}\rho V^2 Sr$$

$$r = Reference\ Length$$



- Pitching moment components integrate over length to produce net pitching moment
  - Increase in cross-sectional area
  - Tail fins
- ... plus pitching moment due to thrust vectoring for control

Angular Attitude Perturbations



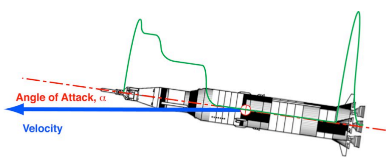
- Pitch-angle perturbation, $\Delta\theta$, is about the same as angle-of-attack perturbation, $\Delta\alpha$

$$\Delta\ddot{\theta} \approx \Delta\ddot{\alpha} = \frac{Net\ Pitching\ Moment}{Pitching\ Moment\ of\ Inertia}$$

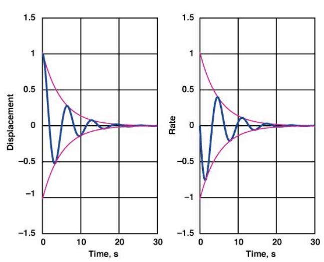
- Then

$$\Delta\ddot{\alpha} = \frac{M_{y_{aero}} + M_{y_{thrust}}}{I_{yy}} \equiv \frac{M_{y_{net}}}{I_{yy}} \approx \frac{1}{I_{yy}}\left[\frac{\partial M_{y_{net}}}{\partial \dot{\alpha}}\Delta\dot{\alpha} + \frac{\partial M_{y_{net}}}{\partial \alpha}\Delta\alpha\right]$$

Attitude Stability



$$\Delta\ddot{\alpha} = \frac{M_{y_{aero}} + M_{y_{thrust}}}{I_{yy}} \equiv \frac{M_{y_{net}}}{I_{yy}} \approx \frac{1}{I_{yy}}\left[\frac{\partial M_{y_{net}}}{\partial \dot{\alpha}}\Delta\dot{\alpha} + \frac{\partial M_{y_{net}}}{\partial \alpha}\Delta\alpha\right]$$



- Attitude perturbations are stable if

$$\frac{\partial M_{y_{net}}}{\partial \dot{\alpha}} < 0, \quad \frac{\partial M_{y_{net}}}{\partial \alpha} < 0$$

- Oscillatory divergence if

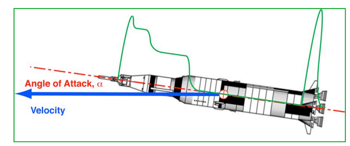
$$\frac{\partial M_{y_{net}}}{\partial \dot{\alpha}} > 0 \quad Dynamic\ Instability$$

- Non-oscillatory divergence if

$$\frac{\partial M_{y_{net}}}{\partial \alpha} > 0 \quad Static\ Instability$$

Thrust-vector feedback control normally required to provide static and dynamic stability

Typical Thrust-Vector Angle Requirements



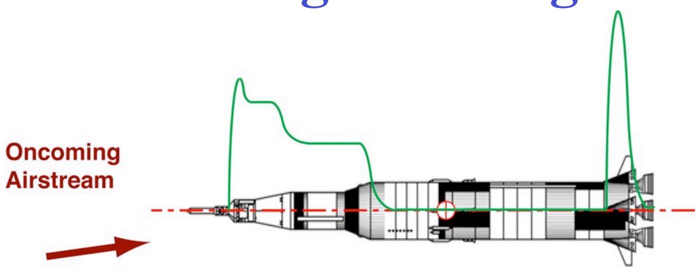
- Example: Concept study for solid-fueled Saturn-class vehicles (NASA TN D-4662, 1968)

TABLE II. - THRUST-VECTOR DEFLECTION ANGLE REQUIREMENTS

Parameter	Variation	Apollo	Voyager	SSO PM
		Deflection angle, deg		
1. Steady stage winds	99 percent	1.35	2.30	1.17
2. Wind gusts	3σ	.15	.26	.13
3. Thrust misalinement	3σ	.25	.25	.25
4. Thrust and weights	3σ	.15	.15	.15
5. Pitch program	maximum	.50	.50	.50
[a]Total		1.68	2.69	1.49

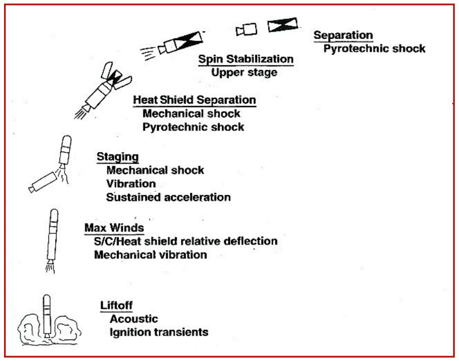
[a]Total consists of item 1 plus root sum square of items 2, 3, and 4.

Pitching Moment Distribution Causes Large Bending Effects



- Aerodynamic and thrust-vectoring effects bend the vehicle
- More on this in a later lecture

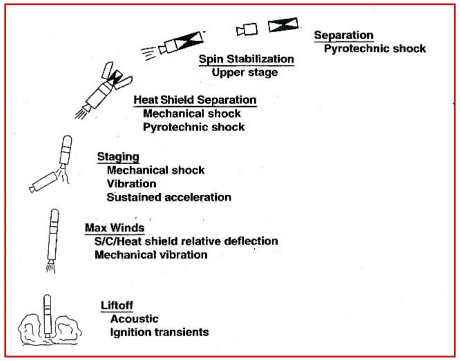
Launch Phases and Loading Issues-1



- Liftoff
  - Reverberation from the ground
  - Random vibrations
  - Thrust transients
- Winds and Transonic Aerodynamics
  - High-altitude jet stream
  - Buffeting
- Staging
  - High sustained acceleration
  - Thrust transients

Launch Phases and Loading Issues-2

- Heat shield separation
  - Mechanical and pyrotechnic transients
- Spin stabilization
  - Tangential and centripetal acceleration
  - Steady-state rotation
- Separation
  - Pyrotechnic transients



Next Time: Launch Vehicle Design: Configurations and Structures